I have designed these Rates thus: The Columns of 3, 4 or 5 *per Cent.* for Incomes of Lands, *&c.* that are certain; That Table at 6 *per Cent.* for Houses new or in good Repair, Brick; That of 8 *per Cent.* for the Purchase, *&c.* of Houses that are pretty old; That of 10 *per Cent.* for very old Houses, or not in very good Repair. And they may be used as every one's Discretion directs, according to the foregoing Rules. And as to this Table of Lives, if the Years Purchase be multiply'd by the Annuity, the Product shews the Value of such Estate for such Life. Examples follow.

The Value of a Life at the several Ages thereof.

Age.	*Years Purchase.*	*Age.*	*Years Purchase.*	*Age.*	*Years Purchase.*
1	10.28	25	12.27	50	9.21
5	13.4	30	11.72	55	8.51
10	13.44	35	11.12	60	7.60
15	13.33	40	10.57	65	6.54
20	12.78	45	9.91	70	5.32

The Use of these Tables of Compound Interest.

Prop. 1.] To find the Amount of any Sum of Money for any Number of Years. This is done by the first Column towards the left hand; so the Amount of *l.*50 in 50 Years Compound Interest allow'd at 5 *per Cent.* is *l.* 573 : 7 : 5 : found by multiplying the Tabular Number against 50 Years by *l.*50 thus.

	11.4674091
	50
l.	573.3704550

Prop. 2.] An Office is worth *l.* 500: What may be paid in present Money to enter upon it 15 Years hence? This is done by the 2d Column at 5 *per Cent.* by multiplying the present Worth of *l.* 1 due 15 Years hence by 500, as in the Margin, where the Answer is *l.* 240 : 10 : 2.

	.4810171
	500
l.	240.5085500

Prop. 3.] An Annuity of 70 *l. per ann.* is forborn 33 Years, what is the Improvement, Compound Interest being

being allow'd the Owner at 5 *per Cent*? By multiplying the Amount of *l.* 1 *per ann.* in 33 Years by the Annuity *l.* 70, you have the Answer *l.* 5604 : 9 : 4 $\frac{1}{2}$.

	Years.	
l. 1 *per ann.*	33 =	80.0638328
		70
		l. 5604.4682960

Prop. 4.] There are 13 Years to come in the Lease of an House that is pretty old, but in good Repair, and is *l.* 50 *per ann.* clear; what is a farther Lease for 31 Years worth in present Money, at the Rate of 8 *per Cent*?

I add the 31 Years Reversion to the 13 in *Esse* makes 44.

The present Worth of *l.* 1 due 44 Years hence, is 12.0770735; from which deduct the present Worth for 13 Years, and the Rest is multiply'd by 50 for Answer: *All by the Table of* 8 *per Cent.*

	Years.	
l. 1 *per ann.*	44 =	*l.* 12.0770735
l. 1	13 =	*l.* 7.9037759
	Rests =	*l.* 4.1732976
		50
	Answ.	*l.* 208.66488

And if you multiply any of the Numbers in the 4th Columns (according as the Estate is in Land or Houses, as aforesaid) by the Annuity you would know the present Worth of, the Product shews the present Value of such Annuity for any Years required under 62.

To find the Present Worth of Estates for Lives.

Prop. 5.] To find the Value of an Estate of *l.* 35 *per ann.* in Land for a Life of 45 Years. By the little Table above, as computed by the Learned Dr. *Haley*, that Life is Years Purchase —— 9.91
Which being multiply'd by the Annuity 35, the 35
Product is *l.* 346 : 17 : — = the Answer. ——
Prod. *l.* 346.85

Prop. 6.] It many times happens, that a Life and so many Years certain to come, are proposed in the same Question; therefore it becomes necessary, as in Computations for two Lives also, *to reduce the Years Purchase of any Life into Years certain to come, or Years in a Lease*: which is done thus.

I find a Life of 50 worth 9.21 Years Purchase. Now if you turn to any of the four Columns in the Tables above, which shew the present Worth of Annuitys according to the several Rates, you will find at 5 *per Cent.* the next to the said 9.21 is 9.393573, against which in the Column under Years is 13 Years to come. At 6 *per Cent.* the next to 9.21 is 9.294984, against which under Years are 14 Years. At 8 *per Cent.* are 9.1216381, against which under Years are

are 17 Years Leaſe. And at 10 *per Cent.* ſtands 9.2372232, (as next to the ſaid 9 21) and againſt that are 27 Years Leaſe.

Example. What is the Value of an Houſe of *l.* 96 *per ann.* clear Rent for 23 Years after the Death of a Perſon 60 Years old?

The Age being found = 12 Years to come; therefore by the Table of 8 *per Cent.* Column 4. the Anſwer, as under *Prop.* 4. is found to be as *per* Marg. *l.*395.37504 = *l.* 395 : 7 : 6.

	Years.	*Years Purch.*	*Col.* 4. 8 *per C.*	*Years to come.*
	60 =	7.60 ——	or 7.536078	or = 12

The preſ. Worth of l.1 *per ann.* —

	Years.	*l.*
for	35 =	11.654568
for	12 =	7.536078
of *l.*1 *Reverſion* =		4.118490
		96
Anſw. =		395.37504

Prop. 7.] *In Computations of the Value of Eſtates for two Lives*; as ſuppoſe 50 and 55. Firſt, find by the little Table of the Value of Lives the Years Purchaſe that the elder Life is worth; then find in the fourth Column at 5 *per Cent.* (ſuppoſing the Purchaſe to be a clear certain Income) the 8.51 found in the ſaid little Table, or the next to it, which is = 8.3064142; right againſt which, in the Column of Years, is 11 Years: to which add the Difference in the Ages (5) and the Sum is 16 Years; againſt which, in the 4th Column from the Years, is 10.8377695, the Value of *l.* 1 *per ann.* for the ſaid two Lives: which multiply'd by the Annuity, gives the Anſwer or Value thereof for the ſaid two Lives, near enough the Truth. Farther Directions might be given on this head; but I have not room to enlarge on things ſo uncertain, and which at beſt depend on many caſual Circumſtances.

A TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe of their Lands, to make up ſuch Leaſe* 21 *Years.*

Annual Rent of Lands.	1 *Year lapſed.*				2 *Years lapſ.*				3 *Years lapſ.*				4 *Years lapſ.*				5 *Years lapſ.*				6 *Years lapſ.*			
	Fines Payable.				*Fines Payable.*				*Fines Payable.*				*Fines Payable.*				*Fines Payable.*				*Fines Payable.*			
l.	*l.*	*s.*	*d.*	*q*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q*	*l.*	*s.*	*d.*	*q.*
1		2	0	0		4	3	0		6	9	0		9	6	2		12	7	3		16	1	2
2		4	0	1		8	6	0		13	6	0		19	1	0	1	5	3	3	1	12	3	0
3		6	0	1		12	9	0	1	0	3	1	1	8	7	2	1	17	11	2	2	8	4	2
4		8	0	2		17	0	0	1	7	0	1	1	18	2	0	2	10	7	2	3	4	6	0
5		10	0	2	1	1	3	1	1	13	9	1	2	7	8	3	3	3	3	1	4	0	7	2
6		12	0	3	1	5	6	1	2	0	6	1	2	17	3	1	3	15	11	1	4	16	9	1
7		14	1	0	1	9	9	1	2	7	3	1	3	6	9	3	4	8	7	0	5	12	10	3
8		16	1	0	1	14	0	1	2	14	0	1	3	16	4	1	5	1	3	0	6	9	0	1
9		18	1	1	1	18	3	1	3	0	9	2	4	5	10	3	5	13	10	3	7	5	1	3
10	1	0	1	1	2	2	6	1	3	7	6	2	4	15	1	1	6	6	6	3	8	1	3	1
20	2	0	2	2	4	5	0	3	6	15	1	0	9	10	10	2	12	13	1	2	16	2	6	3
30	3	0	3	3	6	7	7	0	10	2	7	2	14	6	4	0	18	19	8	1	24	3	10	0
40	4	0	5	0	8	10	1	1	13	10	2	0	19	1	9	1	25	6	3	0	32	5	1	1
50	5	0	6	1	10	12	7	3	16	17	8	2	23	17	2	2	31	12	9	3	40	6	4	3
60	6	0	7	2	12	15	2	0	20	5	3	0	28	12	7	3	37	19	4	2	48	7	8	0
70	7	0	8	3	14	17	8	1	23	12	9	1	33	8	1	1	44	5	11	1	56	8	11	1
80	8	0	10	0	17	0	2	3	27	0	3	3	38	3	6	2	50	12	6	0	64	10	2	3
90	9	0	11	1	19	2	9	0	30	7	10	1	42	18	11	3	56	19	0	3	72	11	6	0
100	10	1	0	2	21	5	3	2	33	15	4	3	47	14	5	1	63	5	7	2	80	12	9	1
200	20	2	0	1	42	10	7	0	67	10	9	3	95	8	10	1	126	11	3	0	161	5	7	0
300	30	3	1	2	63	15	10	2	101	6	2	2	143	3	3	1	189	16	10	2	241	18	4	1
400	40	4	2	0	85	1	1	2	135	1	7	2	190	17	8	2	253	2	6	0	322	11	1	2
500	50	5	2	2	106	6	5	0	168	17	0	0	238	12	1	1	316	8	1	1	403	3	11	0

A TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe*, &c.

Annual Rent of Lands.	7 *Years lapſ.* *Fines Payable.*	8 *Years lapſed.* *Fines Payable.*	9 *Years lapſed.* *Fines. Payable.*	10 *Years lapſ.* *Fines Payable.*	11 *Years lapſ.* *Fines Payable.*
l.	*l. s. d. q.*	*l. s. d. q.*	*l. s. d. q.*	*l. s. d. q.*	*l. s. d. q.*
1	1	1 4 4 0	1 9 1 3	1 14 6 1	2 0 6 0
2	2	2 8 7 3	2 18 3 1	3 9 1 0	4 1 0 1
3	3	3 12 11 2	4 7 5 0	5 3 6 2	6 1 6 2
4	4	4 17 3 2	5 16 6 2	6 18 0 3	8 2 0 2
5	5	6 1 7 1	7 5 8 1	8 12 6 3	10 2 6 3
6	6	7 5 11 0	8 14 10 0	10 7 1 0	12 3 0 3
7	7	8 10 3 0	10 3 11 2	12 1 7 1	14 3 7 0
8	8	9 14 7 0	11 13 1 1	13 16 1 2	16 4 1 1
9	9	10 18 10 2	13 2 2 3	15 10 7 2	18 4 7 1
10	10	12 3 2 2	14 11 4 2	17 5 1 3	20 5 1 2
20	20	24 6 5 0	29 2 9 0	34 10 3 2	40 10 2 3
30	30	36 9 7 1	43 14 1 2	51 15 5 1	60 15 4 1
40	40	48 12 9 3	58 5 6 0	69 0 7 0	81 0 5 3
50	50	60 16 0 0	72 16 10 2	86 5 8 3	101 5 7 0
60	60	72 19 2 2	87 8 3 0	103 10 10 2	121 10 8 2
70	70	85 2 5 0	101 19 7 2	120 16 0 1	141 15 10 0
80	80	97 5 7 1	116 11 0 0	138 1 2 0	162 0 11 1
90	90	109 8 9 3	131 2 4 3	155 6 3 3	182 6 0 3
100	100	121 12 0 0	145 13 9 0	172 11 5 2	202 11 2 1
200	200	243 4 0 0	291 7 6 2	345 2 11 1	405 2 4 2
300	300	364 16 0 0	437 1 3 2	517 14 4 3	607 13 6 3
400	400	486 8 0 0	582 15 0 2	690 5 10 1	810 4 9 0
500	500	608 0 0 0	728 8 10 0	862 17 3 3	1012 15 11 1

A TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe, &c.*

Annual Rent of Lands.	12 Years lapſed Fines Payable.				13 Years lapſed. Fines Payable.				14 Years lapſed. Fines. Payable.				15 Years lapſed. Fines Payable.				16 Years lapſed. Fines Payable.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	2	7	2	2	2	14	8	0	3	3	0	0	3	12	3	2	4	2	7	3
2	4	14	4	3	5	9	4	0	6	5	11	3	7	4	6	3	8	5	3	2
3	7	1	7	1	8	4	0	0	9	8	11	3	10	16	10	1	12	7	11	1
4	9	8	9	3	10	18	8	0	12	11	11	3	14	9	1	3	16	10	7	1
5	11	16	0	0	13	13	4	0	15	14	11	2	18	1	5	0	20	13	3	0
6	14	3	2	2	16	8	0	0	18	17	11	2	21	13	8	2	24	15	10	3
7	16	10	5	0	19	2	8	0	22	0	11	2	25	5	11	3	28	18	6	2
8	18	17	7	1	21	17	4	0	25	3	11	1	28	18	0	3	33	1	2	1
9	21	4	9	3	24	12	0	0	28	6	11	1	32	10	4	1	37	3	10	0
10	23	12	0	1	27	6	8	0	31	9	11	1	36	2	10	0	41	6	5	3
20	47	4	0	2	54	13	4	0	62	19	10	1	72	5	8	0	82	12	11	3
30	70	16	0	3	82	0	0	0	94	9	9	2	108	8	6	1	123	19	5	3
40	94	8	1	0	109	6	8	0	125	19	8	3	144	11	4	1	165	5	11	2
50	118	0	1	1	136	13	4	0	157	9	8	0	180	14	2	1	206	12	5	2
60	141	12	1	2	164	0	0	0	188	19	7	1	216	17	0	1	247	18	11	1
70	165	4	1	3	191	6	8	0	220	9	6	2	252	19	10	1	289	5	5	1
80	188	16	2	0	218	13	4	0	251	19	5	3	289	0	8	2	330	11	11	0
90	212	8	2	1	246	0	0	0	283	9	4	3	325	3	6	2	371	18	5	0
100	236	0	2	3	273	6	8	0	314	19	4	1	361	8	4	2	413	4	10	3
200	472	0	5	2	546	13	3	0	629	18	8	1	722	16	9	1	826	9	10	0
300	708	0	8	1	819	19	10	2	944	18	0	1	1084	5	1	3	1239	14	8	2
400	944	0	11	0	1093	6	6	0	1259	17	4	2	1445	13	6	2	1652	19	7	1
500	1180	1	1	1	1366	13	1	1	1574	16	8	2	1807	1	11	0	2066	4	6	1

A TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe*, &c.

Annual Rent of Lands.	17 *Years lapſed.*	18 *Years lapſed.*	19 *Years lapſed.*	20 *Years lapſed.*	21 *Years Leaſe.*
	Fines Payable.	*Fines Payable.*	*Fines Payable.*	*Fines Payab'e.*	*Fines Payable.*
l.	*l. s. d. q*	*l. s. d. q.*	*l. s. d. q.*	*l. s. d. q.*	*l. s. d. q.*
1	4 14 2 2	5 7 1 2	6 1 6 1	6 17 6 0	7 15 6 0
2	9 8 5 0	10 14 2 3	12 3 0 1	13 14 11 3	15 11 0 0
3	14 2 7 3	16 1 4 1	18 4 6 2	20 12 5 3	23 6 6 0
4	18 16 10 1	21 8 5 2	24 6 0 3	27 9 11 3	31 2 0 0
5	23 11 0 3	26 15 7 0	30 7 6 3	34 7 5 3	38 17 6 0
6	28 5 3 2	32 2 8 2	36 9 1 0	41 4 11 2	46 13 0 0
7	32 19 6 0	37 9 9 3	42 10 7 1	48 2 5 2	54 8 6 0
8	37 13 8 2	42 16 11 1	48 12 1 1	54 19 11 2	62 4 0 0
9	42 7 11 0	48 4 0 2	54 13 7 2	61 17 5 2	69 19 6 0
10	47 2 1 3	53 11 2 0	60 15 1 3	68 14 11 1	77 15 0 0
20	94 4 3 1	107 2 4 0	121 10 3 1	137 9 10 3	155 10 0 1
30	141 6 5 0	160 13 6 0	182 5 5 0	206 4 10 0	233 5 0 2
40	188 8 6 3	214 4 8 1	243 0 6 3	274 19 9 2	311 0 0 3
50	235 10 8 1	267 15 10 1	303 15 8 1	343 14 8 3	388 15 0 3
60	282 12 10 0	321 7 0 1	364 10 10 0	412 9 8 0	466 10 1 0
70	329 14 11 2	374 18 2 1	425 5 11 2	481 4 7 2	544 5 1 1
80	376 17 1 1	428 9 4 1	486 1 1 1	549 19 6 3	622 0 1 1
90	423 19 3 0	482 0 6 1	546 16 3 0	618 14 6 1	699 15 1 2
100	471 1 4 2	535 11 8 2	607 11 4 2	687 9 5 2	777 10 1 3
200	942 2 9 1	1071 3 5 0	1215 2 9 1	1374 18 11 1	1555 0 3 2
300	1413 4 1 3	1606 15 1 1	1822 14 1 3	2062 8 4 3	2332 10 5 1
400	1884 5 6 2	2142 6 10 0	2430 5 6 2	2749 17 10 1	3110 0 7 0
500	2355 6 11 0	2677 18 6 1	3037 16 11 0	3437 7 3 3	3887 10 8 3

The Use of this Table.

Example 1.] What Fine is to be paid, to make up a Lease 21 Years, when 9 Years of 21 are lapsed (or let slip) supposing the Rent *l.*147 *per annum*?

	per ann.	*Fine.*
In the Table under 9 Years lapsed against	*l.* 100 is	*l.*145 : 13 : 9
against	40 ...	58 : 5 : 6
	7 ...	10 : 3 : 11½
Sums, *Rent*	147 *Fine*	214 : 3 : 2½

There are two Books that afford us a Table of this kind: The first was written by the Reverend *John Newton* D. D. one of the King's Chaplains, *Anno* 1668. which contains only Fines, or Value of 1 *l. per ann.* Rent, all in Decimals. The other is said to be approved of by a very great Author, (who perhaps never saw it) which is very tedious in its Use; as I shall shew by the Example above, which is performed by my Table: To find the Fine payable to make up 9 Years lapsed 21 Years; Rent 147 *l. per ann.* his Answer is 1 : 1 : 2 : 5 Purchase, taken out of one Table; which he values by another thus:

	Year.	*Qr.*	*Mon.*	*Tenths.*
per ann.				
l. 147 for 1 Year =	*l.* 147 :	— :	—	
100 for 1 Quarter =	*l.* 25 :	— :	—	
2 Months =	16 :	13 :	4	
5 Tenths of a Mon.	4 :	3 :	4	
l. 40 for 1 Quarter	10 :	— :	—	
2 Months	6 :	13 :	4	
5 Tenths of a Month	1 :	13 :	4	
7 for 1 Quarter	1 :	15 :	—	
2 Months	1 :	3 :	4	
5 Tenths of a Month	— :	5 :	10	

Sums.

l. per ann.	*Fine.*
147.	*l.* 214 : 7 : 6

But 'tis plain the Use of my Table (as above) is much more brief, and will be allow'd more accurate.

Example 2.] To make up a Lease of 14 Years any Numbers of Years less than 21: As suppose I would add 5 Years to 14 in a Lease of *l.* 40 *per ann.* take 5 from 7 there rests 2; then take the Fine payable for 2 Years lapsed = *l.* 8 : 10 : 1 : 1 from the Annual Rent, (here 40) and the Remainer is = *l.* 31 : 9 : 10 : 3 = the Sum to be paid to add 5 Years.

Example 3.] To find the Value of a Church or College full Leaſe of Lands of 21 Years: as ſuppoſe *l.* 80 *per ann.* The laſt Column in the Table againſt *l.* 80 gives you *l.* 622 : — : 1 : 1, the Anſwer.

I need not tell the Reader, That in caſe of Improvement by the Seller, he may advance the Rent to the Buyers in proportion.

Sect. IX. *To Extract the Square and Cube Roots of Decimals and Mixt Numbers.*

1ſt, For the Square Root. What is the Square Root of .000976541?

To what is ſaid under the Extraction of the Square Root for Intire Numbers, it is only neceſſary to add for Decimals, that you begin to point over Seconds place of the Decimal given, and ſo proceed over every other towards the right hand: and if the Decimal Places given are not an even Number, you muſt make them ſo, by adding a Cypher towards the right hand.

. . . . .
.0009765410 (.03124 = Root.
. . . .
6) 76
62) 1554
624) 31010
6034 reſts.

2. That for every Decimal Place you would have in the Root, you muſt (by adding Cyphers towards the right hand, if need be) make twice ſo many Decimal Places in that given to have its Root extracted.

There are ſeveral Ways of Extracting the Roots, but I chuſe that natural one, according to the Algebraic Canon, as in whole Numbers foregoing.

2dly, For the Cube Root.

Example 1.] *Of a Mixt Number.* What is the Cube Root of 32.934168093464? Anſwer 3.2054.

In theſe Caſes you muſt point over the Thirds place of the Decimal, *&c.* over every Third toward the right hand adding Cyphers, if the Point fall not over the place next the right hand.

Example 2.] What is the Cube Root of 91? See the Work.

In Examples of this kind, where the whole Number is not a compleat Cube, you muſt add ſo many times 3 Cyphers towards the right hand, as you intend to have Decimal Places in the Root: as

as here are 6 Cyphers for the 2 Decimal Places in the Root. And if you desire more, it is but putting 3 Cyphers towards the right hand of the Remainer 481151, and proceed to make that Root as exact as you please.

3. If you have the Square or Cube Roots of a Vulgar Fraction to Extract, you may reduce it to a Decimal, and then proceed as in the Example above.

4. And if the Decimals have Cyphers before next the Point, keep to the Rule under the last Example of the Cube Root above for the Points over: as .014672, point thus .014672; .0014672, point thus, .0014672000; and .00014672 point thus, .00014672000, *&c.* and extract as in Whole Numbers: For in this Case of the Cube of Decimals, their Places must always be, or be made, 3, 6, or 9, *&c* by putting Cyphers toward the right hand of the Decimal whose Root is required.

Having already shewn how to extract the Square, Cube, *&c.* Roots of Intire Numbers, and Fractions Vulgar and Decimal; I shall in the Use of Logarithms give a much more easy and short Way of Extraction, and also shew the Use of Logarithms in resolving Questions of Interest: on both which accounts, Logarithms (among many other of their Uses) are extremely to be valued.

```
                .   .   .
91.000000 (4.49
-----------
27000
-----
 12
 48
-----
 492
-----
  64
 192
 192
-----
21184
-------
5816000
-------
  132
 5808
-----
 58212
-----
  729
 10692
52272
-----
5334849
-------
481151 rests.
```

Here ends Decimal Arithmetic.

CHAP.

CHAP. IV.

Duodecimal Arithmetic.

I. *Umeration.*] I have ſhew'd under *Duodecimals*, in the Definition of Numbers, *Chap.* 1. what theſe Fractions are; as in *Foot.* *I* *II* *III* *IIII*
1 : 1 : 1 : 1 : 1. This is read, 1 Foot, 1 Prime (or 12th of a Foot) 1 Second (or 12th of a Prime) 1 Third (or 12th of a Second) 1 Fourth (or 12th of a Third) *&c.* which is ſufficient to know how to read or write Duodecimals. I ſhall proceed to

II. *Addition.*] This is no more of conſequence to what is above, but to divide the Sum of each Series by 12, putting down the Remainer, and carrying the Quotes to the next. But that next the left hand is done as all other things that have but one Denomination. The Example makes it clear.

Feet. *I* *II* *III* *IIII* *IIIII*
321 : 11 : 10 : 9 : 8 : 7
210 : 10 : 9 : 8 : 7 : 6
109 : 9 : 8 : 7 : 6 : 5
98 : 8 : 7 : 6 : 5 : 4
87 : 7 : 6 : 5 : 4 : 3
76 : 6 : 5 : 4 : 3 : 2
65 : 5 : 4 : 3 : 2 : 1
Sum 971 : 0 : 4 : 9 : 1 : 4

III. *Subſtraction of Duodecimals.*] A Joiner having lined ſeveral Rooms with Cedar, as above; finds the Deductions for the Apertures, as Chimneys, Windows, Beauvets, Arches, Viſtoes, *&c.* to be 89 : 9 : 8 : 7 : 6 : 5

How many Feet muſt he be paid for?

Subſtracting the leſſer from the greater, adding 12 where the upper is too little, or 10 in that to the left, the Remainder is the Anſwer 881 : 2 : 8 : 1 : 6 : 11

IV. *Multiplication.*] This is the principal Rule to be inſiſted on in this kind of Arithmetic, as being much the oftneſt in uſe, and conſequently the moſt to be obſerved. *Ex-*

Example 1.] Multiply 9 *F.* 10′ by 8 : 8′ Feet. See the Work in the Margin: Where I say, 8 times 10 Primes is 80 or 6′ to carry, and 8″ to put down; then 8 times 9 is 72, and 6 is 78 Primes, which divided mentally by 12 is 6 *F.* 6′ which I put down as you see. Then I multiply 9 : 10′ by the 8 *F*, saying 8 times 10 is 80 Primes, which is 8′ and 6 Foot to carry; then 8 times 9 is 72, and 6 carried is 78: the Sum of which 2 Lines or Products is 85 : 2′ : 8″ = the Answer.

Foot.	*I*	
9	10	
8	8	
6 :	6 :	8
78 :	8	
	I	*II*
Foot 85 :	2 :	8 = Answer.

By which Example you may perceive the Necessity of having in your mind the Product of any 2 Numbers under 13, or not exceeding 12, by 12, as in the Multiplication-Table.

Example 2.] Multiply 16 : 10 : 6 by 9 : 3. See the Operation. Here 'tis plain I multiply, 1st, the Seconds, then the Primes, and then the Integers by the 3, carrying the Quote of each Product divided by 12 to that towards the left hand; placing the Remainer in each Division under the Degrees of the Factors, as the Example indicateth.

	F.	*I*	*II*
	16 :	10 :	6
	9 :	3 :	—
	4 :	2 :	7 : 6
	151 :	10 :	6
Prod.	156 :	1 :	1 : 6

Examp. 3.] Multiply 372 : 11′ : 4″ by 25 : 6′ : 3″. I have inserted the whole Work. As,

1st, I multiply 4″ by 3 produceth 12, so I carry 1; 11′ by 3 is 33, and 1 is 34, I put down 10, and carry 2; then I multiply 372 by the 3, adding the 2 makes 1118″, all which I put down as you see. Then I multiply 372 : 11 : 4 by 6′, and it produceth 2237′ : 8″ : 0‴, as you see.

		I	*II*	*III*	*IIII*
	372	11	4		
	25	6	3		
1st *Product*	....	....	1118	10	
2d *Product*		2237	8		
3d *Product*		283	4		
Sum		2520	1130	110	
Quote	.....	94	12		
Sum	.....	2614			
Quote	217	1 10			
Prod. 372 *by* 25 =	9300				
Add.					
Prod. or Answ. =	9517	10	2	10	

And this I take to be the most easy and natural Way of Working, when the Integers are large Numbers.

3*dly*, I multiply 372 : 11 : 4 by 25 : as 4 times 25 is 100, or $8^{l} : 4^{ll}$; put 4 down, and carry 8; 11 times 5 is 55, and 8 is 63^{l}, put 3 down, and carry 6; 11 times 2 is 22^{l}, and 6 is 28, or $283^{l} : 4^{ll}$.

4*thly*, I sum up those 3 Lines makes $2520^{l} : 1130^{ll} : 10^{lll}$.

5*thly*, I divide the 1130 Seconds mentally by 12, produceth 94^{l}, which makes $2614^{l} : 2^{ll}$.

6*thly*, I divide, as before, 2614^{l} by 12, produceth 217 Integers, and 10^{l} remains.

7*thly*, I multiply the 372 by the 25 at once, as taught in Decimals, and the Product is 9300: So the Sum or Answer is 95 17 *Integ.* $10^{l} : 2^{ll} : 10^{lll} : 0^{llll}$.

Notes to be observed as appear by the above Examples.

If you multiply 2ds by 2ds, the Product is 4ths; Primes by 2ds place gives 3ds, Primes by Primes gives Seconds, and Integers by Primes gives Primes, and by 2ds gives 2ds, *&c.* And accordingly it is proper to place them Degrees towards the right hand, that so the Degrees of each part of the Operation may fall under the like in the given Duodecimals.

So also 2 Places given in order, as Integers and Primes, to be multiplied by 2 Places or Denominations, produce 3 Places; 3 Places by 2 produce 4 Places; and 3 Places by 3 produceth 5 Denominations, or Degrees in the Product; *i. e.* 1 less than the Sum of the Exponents or Places, or Names in both the Factors given.

V. *Division of Duodecimals.*] Divide 156 *F.* $1^{l} : 1^{ll} : 6^{lll} : 9^{llll}$ by 9. See the Operation: Where what rests is always reduced into the next Denomination or Degree, and the respective Figures of each Degree added.

F.									
						l	*ll*	*lll*	*llll*
9	156	1	1	6	9	17	4	1	6	1 = Quote.
	66								
	37								
	13								
	54								
	9								

There is rarely occasion ('tis to be supposed) to divide by a Number of more than 1 Degree or Name: yet in case there should,

ſhould, it may be done as in the Margin, where 85 : 2′ : 8″ is divided by 8 : 8′. Where I ſay, the Eights in 85 are 9 times, which put in the Quote, and ſay 9 times 8′ is 72, (which is 6 to carry, and put down 0) 9 times 8 Integers is 72, and 6 is 78, which put down, and deducted, the Remainer is 7 : 2′ or 86′; to which bring down the 8″, and ſay 8 in 86 is 10 times; 10 times 8′ is 80, or 6′ : 8, put the 8 down, and carry the 6, ſaying 10 times 8 is 80, and 6 is 86, which deducted, 0 remains: which proves the Truth of the firſt Example in Multiplication.

F. ′ ′ ″ ′

8 : 8) 85 : 2 : 8 (9 : 10 = the Quote.
78 : 0

7 : 2
86 : 8
86 : 8

0 reſts.

And by the ſame Rules the Example in the Margin, or any other, is performed. But it may be obſerved, that in this Diviſion more than one Place of Figures may, and often muſt be put in the Quote at once.

′ ′ ″ ‴ ′ ″

9 : 3) 156 : 1 : 1 : 6 (16 : 10 : 6 = Quote.
148 : 0

8 : 1
97 : 1
92 : 6

4 : 7
55 : 6
55 : 6

0 reſts.

CHAP. V.

Sexageſimal Arithmetic.

THESE Fractions, or 60th Parts, are by ſome called Aſtronomicals, becauſe uſed in the Menſuration of Time and Motion in Aſtronomy: as $1^h : 1^I : 1^{II} : 1^{III} : 1^{IIII}$, that is, 1 Hour, 1 Minute (or 60th part of an Hour) 1 Second (or 60th part of a Minute) 1 Third (or 60th part of a Second) 1 Fourth (or 60th of a Third) *&c.* of Time. And in Motion or Meaſure in the Heavens, Earth, or Sea, $1^o : 1^I : 1^{II}$ is 1 Degree (or 60, but according to ſome 'tis *69 Engliſh* Miles upon the Terraqueous Globe) 1^I is 1 Minute (or 60th part of a Degree) 1^{II} is 1 Second (or 60th of a Minute) 1^{III} is the 60th of a Second, *&c.* See *Sexageſimal Numbers*, at the beginning of *Chap.* 1.

Addition is only to ſum up each Column, and divide them by 60, carrying the Quotes, *&c.* And *Subſtraction* is ſo eaſy, that neither of them need an Example. But

In Multiplication

I think the beſt way is to work as *per* Margin; as being a methodical and eaſy Way, which does not charge the Memory: and I am ſure 'tis more brief than any Way I have ſeen. The Rule is plain; for having multiplied the Denominations one in another, and placed the whole Products as you ſee, I add them; and find the Sum as under the ſecond black Line.

$h.$	I	II	
57 :	29 :	52	} Multiply.
6 :	12 :	10	
		570 :	290 : 520
	684 :	348 :	624
342 :	174 :	312	
342 :	858 :	1230 :	914 : 520
	14 :	20 : 15 :	8
Anſwer = 356 :	38 :	45 :	22 : 40

Then I begin at the right hand to divide by 60, and find 520^{IIII} to be $8^{III} : 40^{IIII}$, placed as you ſee; and ſo I proceed with the reſt, not making a Digit more than what is down. *Ex-*

Example 2.] $75^{h} : 33^{I} : 52^{II} : 27^{III}$ by $32 : 35^{I} : 24^{II} : 42^{III}$. See the Operation just in the same Method with the last Example above.

	h.	*I*	*II*	*III*	*IIII*	*IIIII*	*IIIIII*
Multiply }	75	33	52	27			
	32	35	24	42			
Multiplication of Sexagesimals.				3150	1386	2184	1134
			1800	792	1248	648	
		2625	1155	1820	945		
	2400	1056	1664	864			
Sums Totals	2400	3681	4619	6626	3579	2832	1134
Quotes	= 62	78	111	60	47	18	
Product =	2462	39	50	26	26	30	54 *Answ.*

Here it may be noted, that I multiply 27^{III}, 52^{II}, &c. by 42^{III}, without making 2 Lines, (as is taught in Multiplication of Decimals.)

2*dly*, I begin at 1134^{IIIIII} to divide the Line of Totals by 60, (or all except Units place by 6) so the Quote is 18, and Remainer 54; then I divide $28\frac{2}{3}$ by 60, and 47 is the Quotient, and 30 the Remainer, &c.

I shall give the Reader the Proof of this by Division, for a Conclusion; *viz.*

Division of Sexagesimals.

I II III — *I II III IIII IIIII IIIIII* — *I II III*

$32 : 35 : 24 : 42$) $2462 : 39 : 50 : 26 : 26 : 30 : 54$ ($75 : 33 : 52 : 27$ = Quote.

$2444 : 15 : 52 : 30$ = the Divisor multiplied in the 75 in the (Quote, and divided by 60.

$18 : 23 : 57 : 56$ the Difference berween the 4 first places in (the two last Lines.

$1103 : 57 : 56 : 26$ ditto with the Integ. reduc'd to Primes, &c.

$1075 : 28 : 35 : 6$ the Divisor multiplied in 33^{I}.

$28 : 29 : 21 : 20$ the Difference between the 2 last.

$1709 : 21 : 20 : 30$ ditto with the 28^{I} reduced to ^{II}, &c.

$1694 : 41 : 24 : 24$ the Divisor multip. in 52^{II} in the Quote.

$14 : 39 : 56 : 6$ the Difference between the 2 last.

$879 : 56 : 6 : 54$ ditto with the 14^{II} reduced to ^{III}.

$879 : 56 : 6 : 54$ the Divisor mult. in 27 in Quote.

0 Remains.

CHAP. VI.

Political Arithmetic.

THIS Specie of Arithmetic has nothing new in it, as to the Nature of the Numbers themſelves, nor as to the Manner of Operation; but only in the Application or Subject about which the Numbers are employ'd, which is Political (or relating to Polity or Government:) As to diſcover the Riches and Strength of a Nation, by eſtimating its Income from Land, Trade, Commerce, Art and Labour, Exports, Imports, its Number of People, Males, Females, Fighting Men, *&c.*

And there have been ſeveral Pieces written on this Subject, as Mr. *Grant*'s Obſervations on the Bills of Mortality of *London* in 1676, done at the requeſt of the Royal Society; Sir *William Petty*'s Political Arithmetic in 1687, and his Book of the Uſe of Duplicate Proportion; the Learned Dr *Haley*'s Obſervations on the Bills of Mortality of *Breſlaw* in 1692, whereby he computes the Value of Annuitys for Lives at different Ages thereof, *&c.* as mentioned a little farther; and Dr. *Davenant* and *Gregory King* Eſq; on Revenues and Trade, *&c.* I ſhall give ſome Examples of this Way of Computing: As,

Prop. 1. To find the Number of People in *England*, and how many are Males, Females, and Fighting Men.

Firſt, I find the Number of Houſes within the Weekly Bills of *London*, by conſidering the Number of thoſe Families who pay to the Poor in each Pariſh, and of the Poor themſelves; or by the Poll-Books, King's-Tax-Books, *&c.* Which ſuppoſe I find 108000 (as they are thereabout) theſe at 5 in each Family, one with another, makes 540000 People in the ſaid Compaſs. Now admit that by a Poll-Tax, *&c.* it has been found that the Heads in *London* are to thoſe in all the reſt of *England* as 1 to 9; there muſt at this rate be 4860000 Souls in *England*. And it having been found by many

Years

Years Obſervation upon the Weekly Bills, that there are 14 Males to 13 Females; ſo that of the ſaid 4860000 Perſons, there muſt be 2520000 Males, and 2340000 Females: for

27. 14 :: 4860000. 2520000. Or 27. 13 :: 4860000. 2340000

And hence to find the Fighting Men, it has been computed, That 37.4 of 100 Souls are above 16, and under 60; therefore

100. 37.4 :: 2520000. 942480 = the Fighting Men.

Prop. 2. To find the Number of People by the Coals imported: I find, ſuppoſe, by the Coal-Meters Books 404200 Chaldron imported in one Year, *com. Ann.* It may eaſily, by conſidering the Medium of the ſeveral Claſſes of Houſekeepers, be found that each Houſe, one with onother, burneth about $3\frac{3}{4}$ Chaldron *per ann.* So that by Diviſion I find 107786 Houſes, which is but 214 ſhort of the Number of Houſes in the Weekly Bills, found as in the laſt Propoſition.

Prop. 3. To find the Number of Houſes within the Bills, by the Yards Square of the Ground. I find (ſuppoſe, as *per* Mr. *Grant*) 54 Houſes in every hundred Yards Square, and 2000 ſuch Squares built upon within the Bills of Mortality; which multiplied together gives 108000 Houſes, as *per* the firſt Method.

From theſe and ſuch-like Reaſonings by Political Arithmetic, the Vitality *per Cent.* of ſeveral Ages of Perſons is found.

As that 64 of 100 born are alive at 6 Years old.
40 of 100 —— at 16
25 of 100 —— at 26
16 of 100 —— at 36
10 of 100 —— at 46
6 of 100 —— at 56
3 of 100 —— at 60
1 of 100 —— at 76

And that the Income of *England per ann.* is, from Trade *l.* 9000000
Land —— 14000000
Art and Labour 20000000
So that the whole Annual Income is 43000000 *l. Sterl. per ann.*
The Income of *France l.* 81000000
Of *Holland l.* 18000000

Alſo that the Exports of *England* in *Engliſh* Commodities are about *l.* 5000000 *per ann.*

The Current Coin of *England* is about *l.*7000000.

The

The Expence for the aboveſaid Number of People at 8 *l. per* Head *per ann.* is *l.* 38,880000 *per ann.* So that *England* increaſeth in Riches at that rate, *l.* 4120000 *per ann.*

The Number of Acres of the Land of *England* has been by the very Judicious eſteemed (of the ſeveral ſorts of Land) as follows: *viz.*

	N° of Acres.
Of Arable Land	9,000000
Paſture and Meadow	12,000000
Wood and Coppice	3,000000
Foreſts, Parks, and Commons	3,000000
Houſes, Orchards, and Church-yards	1,000000

So that by the above ſaid Eſtimate there is 8 $\frac{1}{2}\frac{2}{8}$ Acres for each Head in *England*, and about *l.* 1 : 8 : 10 each in Caſh.

	N° of Acres.
In the laſt Column	28,000000
Heaths and Barren Lands	10,000000
Rivers, Lakes and Ponds	500000
Roads and Waſte-Land	500000
Sum = the Acres in *England*	39,000000
In *Scotland*	24,000000
Ireland	29,000000

	N° of Acres.
There is computed to be in *France*	117,000000
The 17 Provinces	13,000000
Spain and *Portugal*	78,000000
Italy, *Venice*, and Iſles thereabout	66,000000
Germany	120,000000
Sweden, *Norway*, and *Denmark*	156,000000
Poland about	116,000000
Muſcovy	468,000000
Turkey in *Europe*	192,000000
Turkey in *Aſia*	234,000000
Arabia	585,000000
Perſia	507,000000

	N° of Acres.
Arces in *Great Britain* and *Ireland*	92,000000
In laſt Column	2652,000000
Mogul's Empire	741,000000
China and *Tonquin*	702,000000
Tartary	2730,000000
Georgia	117,000000
Madagaſcar	78,000000
Sumatra, and Iſles thereabout	80,000000
Japan	117,000000
All *Africa*	4680,000000
America	3510,000000
Iſles *Wight*, *Man*, *Guernſey*, *Jerſey*; & in the *Weſt-Indies*	50,000000

So that the Acres on the whole Earth are computed	15549,000000
Area of the Earth and Sea, as the Superficies of a Sphere in Acres	127675,955000

So

So that the Area of the Sea in Acres is —— 112126,000000
And the People of the whole World are computed at —— 300,000000

But the People in the World, in proportion as the Acres of *England* is to the People of *England*, would be 1937,600000. For

39,000000. 4,860000 : : 15549,000000. 1937,600000

This Difference above 300,000000 (which is about $\frac{5}{6}$ of the whole 1937, *&c.*) is caused by the vast quantitys of Barren Ground, as $\frac{9}{10}$ of *Africa*, $\frac{2}{3}$ *America*, $\frac{5}{7}$ *Tartary*, $\frac{2}{3}$ *Russia*, $\frac{7}{8}$ *Arabia*, &c. which are probably desert and unpeopled.

The very Ingenious and Accurate Dr. *Haley*, in his said judicious Remarks upon the *Breslaw* Bills of Mortality, (wherein both the Ages and Sexes of all that died were monthly delivered and compared with the Number of Births, for the Years 1687, 1688, 1689, 1690, and 1691) hath calculated a Table, (published in the *Philosophical Transactions* in 1692, and in *Miscellanea Curiosa*, Vol. 1.) and hath shew'd these Uses thereof; as, 1*st*, In finding the Proportion of Men able to bear Arms in any Multitude, from 18 to 56 Years of Age. 2*dly*, The different Degrees of Vitality in all Ages; as at what Number of Years it is an even Lay, that a Person of any Age shall die: for instance, that a Man of 30 Years of Age liveth between 27 and 28 Years; That 'tis 80 to 1, that a Person of 25 Years does not die in a Year; That 'tis $5\frac{1}{2}$ to 1, that one of 40 lives 7 Years. 3*dly*, He computes the Value of Annuitys for Lives, (as in the curious little Table after the Tables of Compound Interest foregoing) and the Price of Insurances; and that one half of those who are born, do not live above 17 Years, *&c.*

Mr. *Grant* (besides what he computeth, as above, of the Vitality *per Cent.*) saith, That the People of *London* are about one 14th of the People of *England*. And

Sir *William Petty* saith, That there are more People living between the Ages of 16 and 26 than any other Age; and thence infers, That the Square Root of every Person's Age under 16 (whose Square Root is 4) sheweth the Proportion of Probability of such Person's living to 70 Years: *i. e.* It is 4 times more likely that one of 16 Years of Age lives to be 70, than a Child of 1 Year; 'Tis thrice as probable that one of 9 Years lives to 70, than that a new-born Child does; That the odds is 5 to 4, that one of 25 dieth before 1 of 16 Years old; That 'tis 6 to 5, that one of 36 dieth before 1 of 25 Years: These Proportions being the Roots of 25,

16,

16, and 36 = the Ages, as is above ſaid. He alſo ſays, That the Shipping of *Europe* is about 2000000 Tons, of which *England* hath 500000, the *Dutch* 900000, the *French* 100000, *Spain*, *Portugal*, and *Italy* 250000, *&c.*

And beſides what is ſaid above of the Number of Acres of each kind of Land in *England*, Meſſieurs *King* and *Davenant* ſay farther, That the Increaſe of the People of *England* is 9000 *per annum*, Allowances being made for War, Plague, Shipping, and the Plantations. They reckon the Souls of *London* 530000, in the Citys and Market-Towns in *England* 870000, the Villages and Hamlets 410000. The Annual Produce by Cattel in Butter, Cheeſe, and Milk, about *l.* 2500000; the Value of the Wool yearly ſhorn, about *l.* 2000000: of Horſes bred yearly *l.* 250000; Value of Fleſh yearly ſpent as Food *l.* 3350000; of the Tallow and Hides about *l.* 600000; Hay yearly conſumed by Horſes *l.* 1300000; by other Cattel 1000000, *&c.*

CHAP. VII.

Logarithmical Arithmetic.

IN this kind of Arithmetic the Work of Multiplication, Diviſion, and Extraction of Roots, are performed not by the Numbers themſelves, but by Artificial Numbers adapted to thoſe given; ſo that their Addition performs Multiplication; Subſtraction does the buſineſs of Diviſion; and Dividing by 2, 3, 4, *&c.* of theſe Artificial Numbers, gives the Square, Cube, Biquadrate, *&c.* Roots of the reſpective Natural Numbers.

2. There are to every Natural or Common Number, an Artificial one proportioned; which being formed into a Table, where each Natural Number has its own Artificial ſtanding right againſt it, 'tis called a Table of Artificial Numbers or Logarithms.

3. Of theſe Tables ſome have the Logarithms of all Natural Numbers from 1 to 1000, ſome to 10000, and ſome to 100000. And

And again, theſe Logarithms conſiſt of Places from 5 to 14; but one and the ſame Table has always but one Number of Places: and the more extenſive any Table is, the more it is univerſally uſeful; and therefore I ſhall by and by give Rules for enlarging a Table.

4. There may be ſeveral kinds of Logarithms contrived; for any Series of Numbers in Arithmetical Progreſſion are the Logarithms of thoſe right againſt them in Geometrical Proportion: As

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024	Natural Numbers in Geomet. Progr.
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.	Their Logarithms in Arith. Progreſ.

And the aforeſaid Uſes of Logarithms will appear even in this ſmall Example of a Table: As,

1. *In Multiplication.* As 32 by 16. Againſt theſe in the upper Series, you have 5 and 4 in the lower (or that of Logarithms) therefore add 5 to 4 maketh 9, which ſee in the Logarithms, and over it is 512 = the Product of 32 by 16, done by only adding their reſpective Logarithms 5 and 4. So 32 by 32 is 1024; 8 by 16 = 128, *&c.*

2. *In Diviſion.* To divide 1024 by 64: Under 1024 ſtands the Logarithm 10; and under 64 ſtands the Log. = 6, therefore take 6 from 10, and the Remainer is the Log. = 4, over which ſtands the Quotient 16. And ſo 512 divided by 16, quotes 32, and proves the firſt Example in Multiplication, *&c.*

3. *In the Extraction of the Square Root.* Admit you would have the Square Root of 1024: Under that you have the Logar. 10. the half of which is 5, over which Log. is 32 = the Square Root of 1024, by only taking half its Logarithm; ſo the Root of 256 is 16, *&c.*

4. *In Extracting the Cube Root:* If you would have the Cube Root of 512, under that is the Log. 9. a 3d of which is 3; over which Log. ſtands the Anſwer 8 = the Cube-Root ſought, *&c.*

5. By this very ſmall Specimen it may be obſerved how neceſſary (becauſe exceeding uſeful) a Table of Artificial Numbers muſt needs be, when they are made ſuch a univerſal Series, as to be fitted for all Numbers from 1 to 100000; and this was the happy Contrivance or Thought of the Famous Lord *Neper*, a *Scotiſh* Baron, *Anno* 1614. whoſe Memory will always be precious, eſpecially to all Trigonometrical Calculators. But the firſt Table, as aboveſaid, was made

by that Learned Mathematician of our own Country, Mr. *Henry Briggs*, ſometime *Savilian* Profeſſor of Geometry in the Univerſity of *Oxford*, for Natural Numbers to 100000, and the Log. to 14 places; which new Invention was extremely approved of and promoted by *Kepler* and other Foreigners.

6. Theſe Logarithms being a Series of Numbers in Arithmetical Proportion to anſwer 1, 2, 3, 4, *&c.* to 10 or 100 Thouſand, muſt needs be Decimals, except the Index or firſt Figure towards the left hand, ſeparated from the Logarithm by a Point: for theſe ſhew how many Places the Natural Number conſiſts of, if it be not a Decimal; but if a Decimal, the Index is marked with a Note of Negation, as —, put commonly under the Index or Characteriſtic: which Index, in caſe of Logarithms of Decimals, does ſhew how many Places from the Point the firſt ſignificant Figure ſtands.

The Index of the Log. of all Intire Numbers.	*In Decimals the Index of*
from	
1, to 10 excl. is 0	.1, .2, .3, *&c.* is $\underline{1}$
10, to 100 .... is 1	.01, .02, .03, *&c.* $\underline{2}$
100, to 1000 ... is 2	.001, .002, *&c.* $\underline{3}$
1000, to 10000 is .. 3	.0001, .0002, *&c.* $\underline{4}$
10000, to 100000 = 4 *&c.*	.00001, *&c.* *&c.* $\underline{5}$

And 'tis plain, that the Indexes of Logarithms of intire Numbers are always 1 leſs in Value, than the Number of Places in thoſe of whole Numbers.

Note, That for taking the Logarithm out of my Table, *&c.* you may ſee *Sect.* 2, & 4. of this Chap.

7. Indefatigable muſt the Trouble and Pains be ſuppoſed, which Mr. *Briggs* had in his firſt Table; as muſt be granted, when the Method is underſtood of making but one commonly uſed, conſiſting of half the Number of Places. Which is this: They firſt found ſo many Mean Proportionals between 10 and 1, by the continual adding Cyphers, about 28, and half ſo many afterward towards the right hand of the 10, and then extracting the Roots, till at laſt the Root was ſo ſmall, as to have as many Cyphers next the Point in the Decimal, as they intended the Log. ſhould conſiſt of Places; which could not be done under 27 very long Extractions: theſe ſeveral

veral Means they put down in order, from the extreme 10 downward.

2*dly*, They found Logarithms to theſe Extreme and Means, by putting down againſt the Extreme 10.000, *&c.* the Log. 1.000, *&c.* (as is beforeſaid) and proceeding to take half that Log. continually, half of that half, *&c.* and placing them orderly againſt their reſpective Mean Proportionals found as before.

3*dly*, Then when they had a mind to make the Log. of any Number, they by extracting the Square Root, found ſo many continual Means between the Number given and 1, that the laſt Mean was the leaſt Root that had ſo many Cyphers next the Point, as the Logarithm was to have Places.

4*thly*, Then to find Logarithms to theſe Means; the Proportion is

As the ſignificant Figures of the Mean Proportional found between 10 and 1, (of the like place with the laſt continual Mean in theſe you are upon)
Is to its reſpective Logarithm (omitting the Cyphers)
So is the laſt Mean in the Series you are now a working,
To its Logarithm.

5*thly*, This Logarithm of the laſt continual Mean between the Number you are inveſtigating the Log. of, and 1, being found, is to be doubled, and that, *&c.* Products doubled continually; putting the ſeveral Products againſt the reſpective Means laſt found, till at laſt the Product falling againſt the Number whoſe Log. is ſought, gives you that Logarithm, by which ſeveral others are eaſily found: for the Log. of 4 multiplied by 2, gives the Log. of 16; $\frac{1}{2}$ Log. of 4 is that of 2: ſo have you the Logarithms of all the Powers of 2; and the Log. of 10 (or 1.0) leſs the Log. of 2. leaveth the Log. of 5. So you have the Log. of all the Powers of 5, and Product by 2 and 5.

I did intend to have inſerted here the Whole of finding the Log. of any Number between 1 and 10; but there being lately ſo many briefer Ways of finding the Logarithms, publiſhed by Mr. *Mercator*, Mr. *Gregory*, Mr. *Speidal*, and eſpecially by Dr. *Haley*, in *Philoſophical Tranſactions*, N°216. that I thought it not worth while to give an Example of the old tedious Way above-mentioned.

It could hardly be expected, that in one Treatiſe handling ſo many different Parts of Arithmetic, I ſhould inſert a Table of Logarithms; which alone, with its Uſe, makes a conſiderable Volume: And eſpecially conſidering there are ſo many Tables already publiſhed; for beſides Mr. *Briggs*, Original, there are Tranſcripts by

Norwood, Gunter, Wingate, Moor, Newhouſe, Leyburn, Sturmy, Taylor, Harris, Ward, &c. *Engliſhmen*; beſides thoſe of *Kepler, Ulacq, Urſin,* &c. Foreigners. But I have however (the better to make this Book anſwer to the Title) inſerted a Table to 10000, and Rules to make it to 100000000.

SECT. II. *To Add, Subſtract, Multiply, and Divide Logarithms; or to perform the Work of Multiplication, Diviſion, to Square and Cube, and Extraction of Roots of Intire Numbers by the Logarithms.*

BUT it muſt firſt be obſerved, That

I. *To take any Logarithm in my Table belonging to any Natural Number:*

If the Natural Number conſiſt of three Places only, you have them in the Column under *Natural Numbers*; and in the next Column toward the right hand are their reſpective Logarithms under (o).

But if the Number given conſiſt of four places, look for all but Units place under *Natural Numbers* as before, right againſt which, and under the Digit in Units place, is your Logarithm required. And in like manner for any Logarithm under 100, ſee for that in Tens place under *Natural Numbers*; and for that in Units at the head of the Column, and in the common Angle of meeting ſtands the Log. ſought.

II. *To find the Natural Number to a Logarithm.*

This you will eaſily do by the help of your Index, and eſpecially by conſidering the Nature of the Numbers which increaſe from the left hand towards the right; as the Digits at the head of the Columns, and thoſe in the left hand Columns reſpectively increaſe where four Places is in the Natural Number. But downwards the Logarithms are to ſuch Natural Numbers as increaſe by Ten: Thus the Logarithm 8123785 I find under 2, and againſt 649; ſo that it is the Logarithm of 6492: but if it were the Logarithm 8122447, I find that under (o) and againſt 649; ſo that 6490 is the Natural Number ſought. And the ſame Logarithm is alſo for 649: As all are Logarithms for the Natural Number expreſſed in the Table, or for that Number multiplied by 1 or more Tens; but you are to be guided by your Index what places your Number is.

So

So much for the Logarithm of Integers: And for thoſe of Fractions and Mixt Numbers, ſee *Sect.* 3. following, and more of the above, *Sect.* 4.

I. *Multiplication of Intire Numbers by the Logarithms.*

Example 1.] Multiply 91 by 93.

Rule. To the Logarithm of 91, which is = 1.9590414
Add the Logarithm of 93 ..... = 1.9684829

Product = 8463; the Nat. Numb. to the Log. 3.9275243 Sum.

Example 2.] Multiply 4 by 177.

Rule. To the Log. of 177, which is = 2.2479733 } Add.
Add the Log. of 4, which is = 0.6020600 }

Product 708; its Logarithm = 2.8500333 Sum.

Theſe two Examples are ſufficient to ſhew how to multiply any Intire Number by another; by the Logarithms, where I need not give many words to ſhew that againſt the Natural Numbers given ſtand their Logarithms; and againſt the Sum of thoſe Logarithms ſtand the reſpective Natural Numbers 8463 and 708: *i. e.* In my Table you have the Log. 9275243 under 3, and againſt 846 (the Units place, where there are 4) being always over the Logarithm. See *Sect.* 4. and the Rules above for finding the Logarithms, and their Natural Numbers.

II. *Diviſion by Logarithms.*

Example 1.] Divide 8463 by 93.

Rule. From the Logarithm of 8463, which is = 3.9275243
Deduct the Log. of 93, which is = 1.9684829

Reſts the Log. of the Quote 91, which is = 1.9590414

Example 2.] Divide 708 by 177.

From the Logarithm of 708, which is = 2.8500333
Deduct the Log. of 177, which is = 2.2479733

Reſts the Log. of the Quote 4; which is = 0.6020600

III. *To Square any Intire Number, and Extract the Square Root by the Logarithms.*

1. To ſquare (or involve in itſelf) any whole Number.

Exam-

Example.] What is the Square of 99?
Rule. Multiply the Logarithm of 99, which is = 1.9956352
by 2

9801, the Square ſought; being the Natural Number to the Product ——— 3.9912704

2. To evolve, or extract the Square Root of any *Intire Number.*
Example.] What is the Square Root of 9801?
Rule. Take half of the Logarithm of 9801. Log is 3.9912704
Half that Log. of 9801 is = 1.9956352
Which half is the Log. of 99 = the Root ſought. And this proves the laſt Queſtion.

IV. *To Cube, or Extract the Cube Root of any Intire Number, by the Logarithms.*

1. To Cube any *Whole Number.*
Example.] What is the Cube of 21?
Rule. Multiply the Log. of 21, which is ..... = 1.3222193
by 3

And the Cube required is 9261 = the Natural Number of the Log. ——— 3.9666579

2. To extract the Cube Root of any *Intire Number.*
Example.] What is the Root of 9261?
The Log. of 9261 is as before = 3.9666579
$\frac{1}{3}$ of that Logarithm is = 1.3222193
Whoſe Natural Number is 21 = the Cube Root required.

3. If you deſire the Biquadrate of any Number, multiply its Log. by 4, gives the Log. of the Anſwer: And if you would have the Biquadrate or Surſolid Roots, *&c.* of a Number, divide its Logarithm by 4 or 5, *&c.* Examples are needleſs; but note, that if the Index is leſs than the Diviſor, take ſo often as the Diviſor can be had in 2 places.

SECT. III. *To perform the Work of Multiplication, Diviſion, Square or Cube, &c. or Extract the Roots of Fractions and Mixt Numbers, by the Logarithms.*

BEFORE I give Examples of Logarithms, it will be neceſſary to give a Rule for *Adding or Subſtracting Negative Indices or Characteriſtics.*

Note,

Note, That when the Index is mark'd under thus (—) it is ſo much leſs than nothing as thoſe beforegoing, or 2, 3, 1, *&c.*

1. The Rules for adding and ſubſtracting the Indices of Logarithms are numerous, and not eaſy to retain in mind: I have therefore ſuppoſed (which taking the Reaſon of the thing along with it, eaſily ſolves all, without charging the Memory) when the Index is negative, that it is a Perſon owing ſo many *l.*100 more than he is worth, as the Index-Digit expreſſeth.

Addition of Indices.	*Subſtraction.*
Thus, 1. Add the Index 3	From — 2
to — 1	Take — 1
Sum = 2	Reſts 3
Exam. 2. Add — 3	2*dly*, From — 2
to — 1	Take — 1
Sum = 2	Reſts 3
Exam. 3. Add — 2	3*dly*, From — 5
to — 3	Take — 2
Sum = 5	Reſts 3
	Theſe prove the Truth of thoſe in Addition.

This being done, the adding to or taking from the upper Index according to the Nature of each, makes the Sum or Remainer plainly appear.

1. So in the firſt Example of Addition above, I conſider that if I add *l.*100 to a Perſon who is 3 worſe than nothing, the Sum muſt be 2; that is, I then make him worth but 2 leſs than nothing.

2. In the ſecond Example, if I add 1 leſs than (o) to; that is, if I take 1 from a Perſon who hath 3, the Sum is but 2; for that is all that is left. And if I add 3 leſs than nothing to him who before was 2 worſe than (o) I leave him 5 leſs than (o), as in Example 3.

So in Subſtraction of Indices:

3. If (as in the firſt Example) I take 1 from him who already is 2 worſe, or leſs than (o), I leave him 3 leſs than nothing; and ſo 3 is the Remainer.

4. If

4. If (as in the 2d Example) I take $\underline{1}$ (or 1 leſs than nothing) from him that hath 2 in poſſeſſion already, I then leave him worth 3, becauſe I take 1 leſs than nothing from him: therefore I add 1 to him, that is, I make 2 into 3, which is therefore the Remainer. (See the words *Negative Arithmetic* in Algebra.) And all this conſider'd right, is but one ſingle, ſhort, and eaſy Rule.

II. *To find the Logarithm of a Mixt Number, or of a Decimal, by the Table.*

Prop. 1. *To find the Logarithm of a Mixt Number, as of* 9.177.

Rule. Take the Log. of 9177, except the Index; and to that Logarithm prefix an Index according to the Number of Places in the Integral Part of the Number given, as by the little Table under the 6th Head of the firſt Sect. of this Chapter, and you have the Anſwer.

Examp. Thus the Log. of 9177 (without the Index) is 9627007; and the proper Index for 1 Place being 0, therefore the Anſwer is 0.9627007.

So alſo the Log. of 91.77 is = 1.9627007, and of 917.7 'tis = 2.9627007.

Prop. 2. *To find the Logarithm of a Decimal, as of* .5

Rule. Take the Log. of 5, except the Index, and prefix to that Log. an Index agreeable to the Decimal, as *per* the ſaid little Table under the 6th Head of the 1ſt Sect. of this Chapter.

Examples. The Log. of 5 is = 6989700, without its Index: to which Log. if you put its proper Index $\underline{1}$, (there being no Cyphers between the Point and the 5) the Anſwer is $\underline{1}.6989700$. And by the ſame Rule the Log. of .05 is $\underline{2}.6989700$; of .005 = $\underline{3}.6989700$; and of .000579 the Log. is $\underline{4}.7626785$; being the Log. of 579, with the Index of .000579 put before it.

III. *To find Mixt Numbers or Decimals to Logarithms by the Table.*

Prop. 1. *To find a Mixt Number to a Logarithm.*

Rule. Regard not the Index; but look for the Logarithm among thoſe which would have the greateſt Indexes, which having found, take the Natural Number ſtanding againſt it, and point off for Integers according to the Index given.

Example. What is the Natural Number to the Log. 1.9627007?

My Table extending to 10000, the greateſt Index is 3; among which I find my Logarithm given, and right againſt it I find 9177.

9177. Now the Index given being 1, requires that 2 of the 4 Places be Integers, ſo that the Anſwer is 91.77: And alſo the Natural Number to 2.9627007 muſt be 917.7, *&c.* as *per* the Rule for Indexes above.

Prop. 2. *To find the Decimal to a Logarithm.*

Rule. Look for your Logarithm in that part of the Table where 'tis to be found, as before in the laſt, and put down the Natural Number that ſtands right againſt it: then according to the Greatneſs of the Index given, make the Point of the Decimal.

Example. What is the Natural Number to 4.7626785?

Againſt this Logarithm I find 5790. Now becauſe my Index given is 4, I place 5 in the 4th place from the Point, and ſay therefore that .000579 is the true Decimal ſought.

IV. *To Multiply Mixt Numbers and Decimals, by adding their Logarithms.*

I have ſo fully prepared the Reader for this, that I only need to give Examples.

Example 1. A Mixt Number by a Mixt, as 9.336 by 12.09

To the Logarithm of 9.336, which is 0.9701608
Add the Log. of 12.09, which is —— 1.0824263

The Sum is the Log. of 112.9 the Product, *viz.* 2.0525871

Example 2. A Decimal by a Decimal, as .09 by .123

To the Log. of .09, which is —— 2.9542425
Add the Log. of 123, which is —— 1.0899051

The Sum is the Log. of the Product .01107, which is 2.0441476

Example 3. A Whole Number by a Decimal, as 765 by .00345

To the Log. of 765, which is 2.8836614
Add the Log. of .00345, which is 3.5378191

The Sum is the Log. of the Product 2.639, *viz.* 0.4214805

Example 4. A Mixt Number by a Decimal, as 1.23 by .00009

To the Log. of 1.23, which is —— 0.0899051
Add the Log. of .00009, which is 5.9542425

The Sum is the Log. of the Product .0001107 = 4.0441476

Note, That *whenſoever* 1 *is carried to the Index, that* 1 *is affirmative*, and muſt be added to the lower Index, and that Sum to the upper, as *per* the Rules and Examples of adding Indices at the beginning of this Section; and more particularly in the three laſt Examples.

V. *To divide Mixt Numbers, Decimals,* &c. *by Subſtracting their Logarithms.*

In the following Scheme you have all the Caſes that can happen under this Head: which will be eaſily performed, only obſerving the firſt and ſecond Examples of ſubſtracting Indices, and the Rule at the beginning of this Section.

The 9 Cases of Division, done by Logarithms.

Cases.	*Numbers given, or Dividends and Divisors, &c.*	*Their Logarithms: Substract that of the Divisor.*
1.	Divide 237.3	—— 2.3752977
	by 25.32	—— 1.4034637
	Quote = 9.376	Rests 0.9718340
2.	Divide 11.5	—— 1.0606978
	by 932	—— 2.9694159
	Quote .01234	Rests 2.0912819
3.	Divide .1234	—— 1.0913151
	by .321	—— 1.5065050
	Quote .3846	Rests 1.5848101
4.	Divide .463	—— 1.6655810
	by 3214	—— 3.5070459
	Quote = .000144	Rests 4.1585351
5.	Divide 2	—— 0.3010300
	by 3.214	—— 0.5070459
	Quote .6222	Rests 1.7939841
6.	Divide 5	—— 0.6989700
	by 365	—— 2.5622929
	Quote = .0137	Rests 2.1366771
7.	Divide 1.26	—— 0.1003705
	by .002	—— 3.3010300
	Quote = 630	Rest 2.7993405
8.	Divide .5	—— 1.6989700
	by 7.5	—— 0.8750612
	Quote = .0667	Rests 2.8239088
9.	Divide 999	—— 2.9995655
	by .00013	—— 4.1139433
	Quo. 7684615.384	Rests 6.8856222

VI. *To Square any Mixt Number or Decimal, or to Extract the Square Root thereof.*

1. *To ſquare ſuch Numbers.*

Rule. Multiply the Logarithm of the Roots given by 2; but if the Index be negative, out of the Product of that by the 2 deduct what Tens are carried from the place next toward the right hand, and put down the Remainer for the Index of the Product (or Log. of the Square.)

Examples.

I. For Squaring Decimals, &c. by Logarithms.

The Square Roots given.	*The Logarithms of thoſe Roots.*	*Double the laſt Col. or the Log. of the Squares.*	*The Squares ſought.*
987.6.	2.9945811	5.9891622	975353.76
98.76.	1.9945811	3.9891622	9753.5376
9.876.	0.9945811	1.9891622	97.5353
.9876.	$\underline{1}$.9945811	$\underline{1}$.9891622	.9753
.09876.	$\underline{2}$.9945811	$\underline{3}$.9891622	.009753
.009876.	$\underline{3}$.9945811	$\underline{5}$.9891622	.00009753
.0009876.	$\underline{4}$.9945811	$\underline{7}$.9891622	.0000009753
.00009876.	$\underline{5}$.9945811	$\underline{9}$.9891622	.000●00009753

Or, II. For the Square Roots; theſe Titles obſerve.

Squ. Roots ſought (or the Natural N° to the half Logarithms.)	*½ the Logarithms of the Squares; or thoſe of the Roots.*	*Logarithms of the Squares given.*	*Squares given, whoſe Roots are required.*

2. *Obſerve in Extracting the Square Root*, that is to ſay, in taking half of the Log. of the Square; if the Index be an odd Number, add 1 to it in your mind, and ſo proceed: As in taking half $\underline{3}$.989, &c. I ſay half of 4 is 2; of 19, = 9: of 18, = 9, &c.

VII. *To*

VII. *To Cube, or Extract the Cube Root of Decimals, &c.*

Examples.

I. For the Cubing of Decimals by Logarithms.

The Cube Roots given.	*Logarithms of those Roots.*	*Three times the Logarithms.*	*The Cube Numbers sought.*
3.1	0.4913617	1.4740851	29.791
.31	1.4913617	2.4740851	.029791
.031	2.4913617	5.4740851	.00029791
.0031	3.4913617	8.4740851	.00000029791
.00031	4.4913617	11.4740851	.00000000029791
.000031	5.4913617	14.4740851	.00000000000029791

Or, II. For the Cube Root obſerve theſe Titles of the Columns.

Cube Roots ſought.	*$\frac{1}{3}$ of thoſe Logarithms, being thoſe of the Roots.*	*Logarithms of thoſe Cube Numbers.*	*Cube Numbers given; whoſe Roots are required.*

1. You ſee by the Examples, that to Cube any Number (as thoſe next the left hand) is to multiply their Logarithms (as thoſe in the 2d Column) by 3; and the Products are thoſe in the 3d Column, whoſe Natural Numbers required are thoſe in the 4th Column. And here note, That what Tens you carry, you muſt abate out of 3 times the Index; as taught for the Square Root.

2. For the Examples of Extracting the Cube Root: The Numbers in the Column next the right hand are Examples of Cube Numbers given, the next Column are their Logarithms, and the third Column are third parts of thoſe Logarithms. Where note, That if your Index cannot be divided by 3 without a Remainer, you muſt add ſo many Units to it, as is neceſſary for that end: And ſo many Units as you borrow'd there, you muſt pay Tens to the

the next Figure, and then take a 3d of the whole Log. The Examples make all plain.

Or *for finding the Biquadrate, Surſolid, or Squared Cube Powers.*

Rule 1. Multiply the Log. of the Root given by 4, 5, or 6; obſerving the Rules given before, abating what you carry, out of the Product of the Index reſpectively.

Or *for finding the Biquadrate, Surſolid, or Squared Cube Roots.*

Rule 2. Divide the Log. of the Power by 4, 5, or 6; obſerving the Rule of adding ſo many to the Index, that you can divide it without a Remainer, and paying ſo many Tens as you added or borrow'd Units, to the next Figure.

Examples to Rule 1.	*Logarithm of the Powers.*	*Whoſe Nat. Nº are the Anſ. or Powers requir'd*
Thus *the Biquadrate* of .3; its Log. being = 1.4771212; 4 times that ——	= 3.9084848	= .0081
The *Surſolid Power* of .3; its Log. being 1.4771212; 5 times that ——	= 3.3856060	= .00243
The *Squared Cube* of .3; its Log. being 1.4771212; 6 times that ——	= 4.8627272	= .000729

And according to the laſt Rule.	*Log. of the Roots.*	*Roots ſought.*
The *Biquadrate Root* of .0081, its Log. being 3.9084848; $\frac{1}{4}$ that is ——	= 1.4771212	= .3.
The *Surſolid Root* of .00243, its Log. being 3.3856060; $\frac{1}{5}$ that is ——	= 1.4771212	= .3.
The *Squared Cube Root* of .000729, its Log. being 4.8627272; $\frac{1}{6}$ of that is	= 1.4771212	= .3.

Which latter Examples prove the Truth of the former.

VIII. *To find the Logarithms of Vulgar Fractions, and of the Operations thereby.*

Prop. To find the Log. of $\frac{3\,4\,5\,6}{4\,5\,6\,7}$.

Rule. From the Log. of the Numerator deduct the Log. of the Denominator, and the Remainer is the Log. for the Anſwer.

Exam-

Example. From the Log. of 3456, which is = 3.5385737
Take the Log. of 4567, which is = 3.6596310

Remains the Log. of $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$, which is = 1.8789427

Prop. 2. To find the Log. of a Compound Fraction.
Rule. Reduce the Compound to a Simple, and work as above.
Example. To find the Log. of $\frac{4}{5}\frac{5}{6}\frac{6}{7}$ of $\frac{5}{1}\frac{7}{1}\frac{6}{1}$.
The Compound in a Simple being $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$, the Log. is as before;
Or Log. of $\frac{4}{5}\frac{5}{6}\frac{6}{7}$ of $\frac{5}{1}\frac{7}{1}\frac{6}{1}$ = 1.8789427.

Prop. 3. To find the Logarithm of a Mixt Number.
Rule. Reduce it to an Improper Fraction, and find the Logarithm as *per* the firſt Prop. This needs no Example.

Prop. 4. To reduce a Vulgar Fraction to a Decimal.
Rule. Find the Log. of the Vulgar Fraction as *per* the firſt: then againſt that Log. in the Table you have the Decimal.
Example. To reduce $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$ to a Decimal. The Log. of this Vulgar Fraction, as before, is = 1.8789427; and the Natural Number to that, *per* the Table, is .7567 = the Anſwer.

And for reducing ſuch Fractions as this given, whoſe Terms have ſo many places, I take this to be the moſt eaſy and ſpeedy Way of reducing them to Decimals: and if your Table of Logarithms were to 100000, or where the Numerator is a place leſs than the Denominator, the Rule would in moſt Caſes be exact enough.

Prop. 5. What is the Product of one Vulgar Fraction by another, giving the Anſwer in a Decimal Fraction?
Rule. Add the Log. of the one Fraction to the Log. of the other found as *per* the 1ſt Prop. and look for the total Log. in the Table, againſt which ſtands the Decimal required.
Example. What is the Product of $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$ by $\frac{7}{9}\frac{6}{8}\frac{5}{7}\frac{6}{6}$?

$\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$ its Log. as before = 1.8789427
$\frac{7}{9}\frac{6}{8}\frac{5}{7}\frac{6}{6}$ its Log. ..... = 2.8890803

The Product for Anſwer is .05862, whoſe Log. is 2.7680230

And thus 'tis plain, that knowing by the Rules delivered how to find the Logarithms of Vulgar and Decimal Fractions, they may be multiplied or divided ſimply or promiſcuouſly. For,

6. Suppoſe it were required to divide .05862 by $\frac{9}{3}\frac{7}{7}\frac{6}{5}\frac{5}{6}$:

If

If from the Log. of .05862, which is = 2.7680230
You take the Log. $\frac{765}{9876}$, which is = 2.8890803

The Remainer is the Log. of .7567, or of $\frac{3456}{4567}$, *viz.* } = 1.8789427

Which proves the Operation above.

7. By what is above ſaid, it will follow, that the Powers or Roots of Vulgar Fractions are eaſily found by multiplying the Log. of the Vulgar Fraction aſſigned as a Root, by 2, 3, 4, 5, 6, *&c.* to obtain the Power; or by dividing the Log. of the Vulgar Fraction aſſigned as a Power, by 2, 3, *&c.* to obtain the reſpective Roots. And this I take alſo to be the eaſieſt and briefeſt Way of finding the Powers or Roots of Vulgar Fractions; eſpecially where the Terms conſiſt of 3, 4, or 5 places, and your Table of Logarithms is large enough.

SECT. IV. *The Explanation of my Table of Logarithms, whoſe Method ſomewhat differs from* Briggs, Norwood, *and ſome others.* 2dly, *Of the Arithmetical Complements of Logarithms.* 3dly, *Of Parts Proportional and Differences: And,* 4thly, *How to enlarge the Tables of Logarithms ſeveral Ways.*

1. THE eaſieſt Tables for a Learner to uſe, are thoſe that have the Indices put down, and Logarithms to the Natural Numbers which are intirely placed ſucceſſively. But I would here explain to him another kind of Table placed without the Indices, and which have not the Units place of the Natural Number with the reſt, but over the reſpective Logarithm (this is the beſt Method, and which I have therefore obſerved in my following Table of Logarithms) thus.

Numb.	0	1	2	3	4	*&c.* to 9
522	7176705	7177537	7178369	7179200	7180032	
523	7185017	7185847	7186677	7187507	7188337	
524	7193313	7194142	7194970	7195799	7196627	
525	7201593	7202420	7203247	7204074	7204901	

In

In caſe ſuch a Table as this is to be uſed; you muſt, for example to find the Logarithm of 5244, put down, 1*ſt*, 3 for your Index, as *per* the Rules in the ſixth Paragraph of the firſt Sect. of this Chapter.

2*dly*, Having found 524, (the three firſt Figures of your Natural Number given toward the left hand) caſt your eye in the Line toward the right hand, till you come under the Figure in Units place of the Number given, (here 4) where againſt 524, and under 4, you have the Log. required: for the Log. of 5244 is 3.7196627.

So the Log. of 5223 is 3.7179200; and that of 523 = 2.7185017: the Log. of the laſt Natural Number having all its places found together under *Number*; the Log. againſt 523, and under 0, is that ſought, *&c.*

Theſe Logarithms in Mr. *Briggs*'s firſt and large Table, alſo in *Norwood* and ſome others, ſtand thus; (tho they take up much room.)

N°	*Log.*	*N°*	*Log.*	*N°*	*Log.*
522	2.7176705	5221	3.7177537	5231	3.7185847
523	2.7185017	5222	3.7178369	5232	3.7186677
524	2.7193313	5223	3.7179200	5233	3.7187507
525	2.7201593	5224	3.7180032	5234	3.7188337, *&c*

II. *The Arithmetical Complement* of a Logarithm, is that which will make it 10.0000000; ſo that to find the Complement, the Logarithm given muſt be deducted from the 10.0, *&c.*

Now the beſt way to ſubſtract this (not according to the common way, but) to take every Digit of the Subtrahend from 9, except that in Units place, which take from 10: And to do this, you may begin at either end to ſubſtract.

	10.0000000
Log. given =	3.7204901
Its Compl. Arith. =	6.2795099

The Complement Arith. is often uſed in Analogies wrought by Artificial Numbers, to prevent Trouble and Confuſion: For inſtead of ſubſtracting a Logarithm from the Sum of two others, it is uſual to add the Complement Arithmetical, and from the Sum of the three Logarithms to abate Radius or 10.0 from the Index of that Sum; which gives the ſame Anſwer as tho you had deducted the Logarithm of the firſt Number.

To inſtance in a common Queſtion in the Rule of Proportion: If I give *l.* 25 : 10 : — for 48 Yards of Cloth; what will 15 coſt at that rate?

Yards 48, its Logarithm = 1.6812412

l. 25.5, its Logarithm = 1.4065402
Yards 15, its Logarithm = 1.1760912 } Add.

Sum 2.5826314

From which deducting the first Log. the Remainer is that of *l.*7.968, *viz.* —— } = 0.9013902

This by the Complement Arithmetical is done thus:

48. Compl. Arith. = 8.3187588
25.5 :: its Log. . . = 1.4065402
15, its Log. . . . . = 1.1760912

Sum abating 10 in the Index, the Log. of 7.968 the Answer as before —— } = 0.9013902

These kinds of Operations are much used in Trigonometry, with the Logarithms of Natural Sines, Tangents, *&c.*

III. *Of Differences and Proportional Parts.*

1. By the *Differences* is meant the Differences of the Logarithms of 2 Numbers immediately and numerically succeeding one another, or the Differences of the Products of such Numbers multiplied by 10.

2. By *Proportional Parts* you are to understand one or more tenth Parts of those Differences. And these are apply'd in Practice thus:

IV. *When you would find the Logarithm of any Number that is greater than what is contained in the subsequent Table of Logarithms*, and you would do this *by the help of the Table of Differences and Parts Proportional.*

Suppose you were to find the Logarithm of 51716: see the Operation.

I find in the Table the Log. of 5171 or 51710, which is —— } = 7135745
And of the next Number 5172 or 51720 = 7136585

The Difference of these Logarithms is = 840 *Diff.*
Which found in the Table of Differences (following that of the Logarithms) right against that 840 and under 6 (the Digit in Units place 504 *Add.*
7136249 *Sum.*

of

of the Number given) you find the Proportional Part 504; which added to the Logarithm of 5171, the Sum is that of 51716, which with its proper Index is 4.7136249. You may prove the Truth of this by dividing 51716 by 7, the Quote is = 7388; to the Log. of which if you add that of 7, the Sum is the Log. of 51716, as above.

2. *To prove the Truth of any Logarithms in the Table, without the Table of Proportional Parts*: as thoſe of 7000 to 7010 excluſive.

The Difference between the Logarithms of 700 and 701, or 7000 and 7010, is 6200.

Tenths of the Difference.	*Are theſe Proportional Parts.*	*Log. of 7000 repeated.*	*Sums of the laſt Logarithms and Proport. Parts reſpectively.*	*The Natural Numbers to the laſt Logarithms.*
1	620	8450981	8451601	7001
2	1240	8450981	8452221	7002
3	1860	8450981	8452841	7003
4	2480	8450981	8453461	7004
5	3100	8450981	8454081	7005
6	3720	8450981	8454701	7006
7	4340	8450981	8455321	7007
8	4960	8450981	8455941	7008
9	5580	8450981	8456561	7009

By this it appears, that the Difference between the Logarithms of 7000 and 7010 being divided into 10 parts, 1 Tenth is 620, the Part to add to the Logarithm of 7000 to give the Logarithm of 7001; 2 Tenths added to the Logarithm of 7000, gives the Logarithm of 7002; 3 Tenths of the Difference 6200, which is 1860, added to 8450981 (the Log. of 7000) giveth 8452841 = the Logarithm of 7003, *&c.*

And if you would have the Proportional Part immediately to make the Log. of 7000 that of 7008, it is only multiplying a Tenth of the Difference (which here is 620) by 8, and the Product is the Proportional Part 4960; which added to 8450981 = the Log. of 7000, the Sum is 8455941 = the Log. of 7008 required. And thus the 9 Logarithms between 7000 and 7010 excluſive, are examined and proved: And ſo may any other without a Table of Differences and Proportional Parts, or by even calculating or examining thoſe at the ſame time.

3. *To augment the Table of Logarithms*, ſo as to make that which extends only to ſhew the Log. of any Natural Number under 10000, ſubſervient to give that of any Number under 100000, *&c.* without the Table of Parts Proportional. For example;

1. To find the Log. of 64736.

The Log. of 6473 or 64730 is = 8111056
Of 6474 or 64740 is = 8111727
The Difference = 671
A Tenth of that Difference (being the Proportional Part for 1, that is to make the Log. of 64731) — 67.1
Which multiply by 6 (the Place of Units in the given Number) produceth — 6
402.6
Or becauſe the .6 is above one half, you may make the Proportional Part for 6 (in Units place of the Number given) — 403
Which added to the Log. above of 64730, *viz.* — 8111056
The Sum is the Log. of 64736 or Anſwer = 4.8111459

For as 10. 671 :: 6. 403 *ferè.*

i. e. As the Number of Cyphers added to the Natural Number in your Table with a 1 before ſuch (0) towards the left hand;
Is to the Proportional Part, or Difference of the 2 Logarithms:
So is the Figures towards the right hand exceeding the Places for which your Table is calculated,
To the Proportional Part to be added to the Log. to anſwer thoſe Figures.

2. To find (by a Table which has only the Logarithms to Numbers under 10000) what the Logarithms are for all Numbers under 1000000: For inſtance what is the Log. of 647365?

Log. of 647300 is = 8111056
of 647400 is = 8111727
Difference = 671
$\frac{1}{10}$ thereof, there being 100 difference between the Nat. N° of which theſe are Logs. — 6.71
This multiplied by 65 (the 2 places toward the right hand exceeding the Compaſs of the Table) — 65
Produceth = 436.15
Which added to Log. 647300 — 8111056
Sum = 8111492 = the Log. of 647365

For 100. 671 :: 65. 436.15 the .15 may be omitted. 3. To

3. To find (by a Table which has only the Logarithms to Natural Numbers not exceeding 10000) what the Logarithms are for all Numbers not exceeding 10000000000. For example,

I would know the Logarithm of 296682051 4.

	Nat. N°.	*Logarithms.*
As before, the Log. of the moſt places towards the left hand of the Number given, contained in my Table is of 2966, which is alſo of	2966000000 =	4721711
The Log. of the next Number is of 2967, or of	2967000000 =	4723175
The Difference of theſe Logarithms is =		1464
Which Difference divided by 1000000 that each Unit may have its Proportional Part of that Difference, that for 1 is	=	.001464
But becauſe the Number given requires 820514 of thoſe Parts for its Part Proportional, therefore I multiply the Difference (as *per* the former Rules) by	=	820514
And the Product is the Part Proport. for 820514 =		1201.232496
Which (omitting the Decimal as uſeleſs) added to the Log. of 2966000000 as above, which is		4721711
The Sum is the Logarithm ſought, *viz.*		4722912

Which having a proper Index put before it, according to the Number given, as *per* the 5th Rule of the firſt Sect. of this Chapter:

The Log. of 296682051 4 is found = 9.4722912

I have in this laſt Example made this matter ſo very plain, that I need add nothing for that end; only to obſerve,

1*ſt*, That the Log. of any Number is the ſame, tho that Number have never ſo many Cyphers added to its right hand. And therefore in

Example 1. of this Head, I call the Log. of 6473, which is 8111056, the Log. of 64730.

In *Example* 2. I call the ſame Logarithm that of 647300. And

In *Example* 3. I call the Log. of 2966 (which is 4721711) the Log. of 29660000000, making up the reſt of the Places toward the right hand (for which I have no Logarithms in my Table) with Cyphers.

2*dly*, (As in the laſt Example) whatever Number of Cyphers thoſe are, I cut off ſo many Places of the Difference of the Logarithms, *&c.*

And

And thus I have endeavoured to make this Part (which has been omitted by moſt others) very intelligible, by ſhewing how to prove the Truth of Logaritnms, or find thoſe to Numbers that are large by the Differences and Parts Proportional, as they are found in Tables for that purpoſe: And have alſo moſt eaſily and plainly ſhew'd how Logarithms of large Numbers are found without ſuch Tables of Differences and Proportional Parts. It remains next to ſhew,

1. *How to find Natural Numbers to large Logarithms by the Tables of Differences and Parts Proportional.*
2. *How to do the ſame without ſuch Tables of Differences and Parts Proportional.*

4. *To find the Natural Numbers to Logarithms beyond the Compaſs of your Table, by the Differences and Parts Proportional.*

I would know the Natural Number to the Logarithm 4.8111459 See the Operation.

The Logarithm given = 8111459

The next Log. in the Table leſs than that given is the Log. of 6473, (which are the four firſt Places towards the left hand of the Number required) *viz.* } = 8111056

The next Log. in the Table greater than the laſt (being that of 6474) is } = 8111727

The Difference of the 2 firſt Logarithms = 403

Difference of the 2 laſt Logarithms = 671

Now to find the 5th Figure of the Number ſought, or that in Units place, (the Index of the Log. given ſhewing that it hath but 5 places) look for the 2d Difference under [*Differences*] in the Table of Proportional Parts, and you'll find the next it (greater) to be 672. Then look in the ſame Line towards the right hand, and you'll find the 1ſt Difference 403; and at the head of that Column ſtands (6) which put in Units place, gives 64736 for the Number ſought. This proves the Truth of the 1ſt Example, laſt above-ſaid.

5. *To find the Natural Numbers (tho very large) to any Log. given, without the Tables of Parts Proportional.*

What is the Natural Number to the Logarithm 9.4722912?

I find the Logarithms, and take the Differences as above thus.

The

The Log. given (without the Index) is 4722912 = 1ſt Differ. 1201
The next leſs in the Table (which is the Log. of 2966) is ——— 4721711
The next Log. (greater than the laſt) is ——— 4723175 = 2d Differ. 1464

Now whereas the Index ſhews the Natural Number required to have 10 places, and there being only the firſt 4 found, there wants 6 places, which with a Unit towards the left hand is = 1000000: therefore I ſay, As the 2d Difference is to 1000000 : :

So is the 1ſt Difference to the reſt of the Nat. N° required, *i. e.*

1464. 1000000 : : 1201. 820514.

Which 820514 placed towards the right hand of the 2966 found in the Table as above, is together 29668205 14 = the Anſwer.

But the Logarithms of 7 places being too few to find ſo large a Natural Number by, makes the Differences of too few places; and conſequently the 4th Proportional in the laſt Analogy will fall ſomewhat ſhort, if done by a Table having only 7 places. However, the Example ſhews the true Method of proceeding in this Caſe, which is all here intended.

This laſt Example was deſigned to prove the Truth of the third; where .232496 being omitted, if that Decimal be annexed to the 1201, the third in the laſt Analogy, it will then be divided by the 1464, and produce the true Quotient 820514 exactly.

Sect. V. *The Uſe of Logarithms in ſolving Queſtions of Compound Intereſt.*

Queſt. 1. *WHAT is the Amount of* l. 10 in 130 Years, at 5 *per Cent?*

Rule.] Multiply the Log. of 1.05 (which is 1 *l.* and its Intereſt for a Year, which in the ſubſequent Queſtions I call the Ratio) by 130, (the N° of Years) and the Product is the Log. of the Amount of *l.*1 in 130 Years.

To this Log. add that of *l.*10 = the Principal; and the Sum is the Log. of the Anſwer, *viz.* of *l.* 5684.409 = the Amount required.

Example.

1.05, its Log. is = 0.0211893
Multiply by Years 130

	l.	
Amount of 1*l.* in 130 Years.	= 568.4, its Log.	2.7546090
To which add the Log. of the Princip. 10 *l.*		1.0
Sum or Log. of 5684.409 ———		3.7546090

Queſt.

Queſt. 2. *What is the preſent Worth* of *l.*5684 : 8 : 2 ¼ (or of *l.*5684.409) due at the end of 130 Years?

Rule.] Find the Logarithm of the Amount of *l.*1 for 130 Years, as before.

Subſtract the Log. of that Amount from the Log. of the Sum whoſe preſent Worth is required, and the Remainer is the Logarithm of the preſent Worth, as *per* Marg.

So I find *l.*5684.409 to be received 130 Years hence, is worth *l.*10 in preſent Money; which proves the 1ſt.

Example.

Ratio = 1.05, its Log. =	0.0211893
Multiply by	130
Product, or Amount of *l.*1 in 130 Years = 568.4; its Log. ——	2.7546090
Which laſt, deduct from the Log. of *l.* 5684.409	3.7546090
And the remaining Log. is that of *l.* 10, the Anſwer ——	1.0000000

Queſt 3. *What is the Amount of the Leaſe or Annuity* of *l.*25 *per ann.* in 21 Years, according to its utmoſt Improvement at 5 *per Cent?*

Rule.] Find the Amount of *l.*1 for 21 Years, as above.

2. Then from the Log. of the Amount leſs 1, deduct the Log. of the Ratio leſs 1; and the Remainer is the Amount of *l.*1 Annuity in 21 Years.

3. Add to the laſt Log. that of 25 the Annuity, and the Sum is the Anſwer, or *l.*893, the Amount of 25 *per ann.* in 21 Years.

Example.

Ratio 1.05, its Log. =	0.0211893
Multiply by Years	21
Product, or Amount of *l.*1 in 21 Years 2.786; its Log. ——	0.4449753
1.786 = the Amount leſs 1; its Log. ——	0.2518815
1.05 = Ratio, leſs 1 = .05; its Log. ——	2.6989700
Which laſt deducted, there reſts the Log. of *l.*35.72, the Amount of *l.*1 *per an.* in 21 Yrs.	1.5529115
To which adding the Log. of 25 the Annu.	1.3979400
The Sum is the Log. of *l.* 893 the Amount of the Annuity for Anſw.	2.9508515

Queſt.

Queſt. 4. *What is the preſent Worth of a Leaſe, or Annuity,* of *l.* 25 *per annum,* to continue 21 Years at 5 *per Cent.* Intereſt?

Rule.] Find the Amount of *l.*1 in 21 Years, as in the laſt Queſtion.

2*dly*, To the Log. of that Amount add the Log. of the Ratio, leſs 1.

3*dly*, Subſtract that Sum from the Log. of the Amount, leſs 1; and the Remainer is the Log. of the preſent Worth of *l.*1 Annuity to continue 21 Years.

4*thly*, To the laſt Log. add that of the Annuity given, and the Sum is the Log. of the preſent Worth required.

Example.

2.786 = the Amount of *l.*1 in 21 Years (as above) its Log.	0.4449753
.05 = Ratio leſs 1, its Log.	2.6989700
Sum = ...	1.1439453
1.786 = Amount, leſs 1, its Log.	0.2518815
Reſts the Log. of 12.821 *l.* which is the preſent Worth of *l.* 1 *per ann.* to continue 21 Years	1.1079362
To which add the Log. of *l.*25 the Annuity	1.3979400
And the Sum is the Log. of the Anſwer *l.* 320.527, which is	2.5058762

Queſt. 5. *What Annuity or Leaſe will the Sum* of *l.* 320.527, or *l.*320: 10: 6½ purchaſe, to continue 21 Years, allowing the Purchaſer 5 *per Cent?*

Rule.] Find the Amount of *l.*1 in 21 Years, as in the 1ſt, &c. Queſtions.

2*dly*, To the Log. of that Amount add the Log. of the Ratio leſs 1, as in the laſt.

3*dly*, From that Sum deduct the Logarithm of the Amount leſs 1, and the Remainer is the Log. of the Annuity that *l.*1 will purchaſe for 21 Years.

4*thly*, To the Log. of the Annuity of *l.*1 add the Log. of the Purchaſe-Money, and the Sum is the Log. of the Annuity ſought here *l.*25 *per ann.* as *per* the Work in the Margin.

Example.

2.786 the Amount of *l.*1 in 21 Years, its Log.	0.4449753
.05 = Ratio leſs 1, its Log.	2.6989700
Sum ..... =	1.1439453
1.786 = Amount, leſs 1, deduct its Log.	0.2518815
Reſts the Log. of the Annuity that *l.* 1 will purchaſe for 21 Years, *viz.* of .078, which Log. is	2.8920638
320.527 the Purchaſe-Money, its Log. add =	2.5058762
*l.*25 *per ann.* the Annuity required, whoſe Log. is the Sum of the 2 laſt Logs.	1.3979400

Queſt. 6. What is an Eſtate worth in Fee, which contains Arable Land *l.*60 *per ann.* Underwood which once in 15 Years is worth *l.* 200, and there is alſo Timber which 'tis computed after 30 Years Growth will be worth *l.*1000 ?

Rule.] The Arable Land at 20 Years Purchaſe is worth ——— } *l.* 1200 : — : —

2*dly*, *l.*200 (the firſt for the Underwood) due 15 Years hence at 6 *per Cent.* by the Logarithms (as *per* the 2d Queſtion) is found thus : 1.06 its Log. = 0.0253058

Multiply by Years 15

2.397 = Amount of *l.*1 in 15 Years, its Log. ——— } 0.3795870

Which taken from the Log. of *l.* 200 ——— } 2.3010300

The Remainer is that of the preſent Worth ——— } 1.9214430 = Log. of *l.* 83 : 9 : —

3*dly*, By the ſame Rule the preſent Value of *l.*200 due 30 Years hence is ——— } 34 : 16 : —

4*thly*, By ditto Rule the Value of the 3d 200 due 45 Years hence, is ——— } 14 : 10 : —

5*thly*, The Value of the 4th *l.*200 due 60 Years hence, is *per* ditto Rule ——— } 6 : 1 : —

6*thly*, The Value of *l.*1000 for the Timber due 30 Years hence, is in preſent ——— } 174 : 2 : —

The Sum of which ſix Articles is the Anſwer near enough (for 'tis needleſs in this Purchaſe to take notice of the 5th 200 *l.*) ——— } 1512 : 18 : —

Queſt.

Queſt. 7. *What Annuity payable Quarterly, will l.* 320.527 *purchaſe,* to continue 21 Years at 5 *per Cent?*

Rule.] Firſt take a fourth of the Log. of the Ratio.

2*dly*, Multiply that 4th by the Quarters in the Years given, and that gives the Log. of the Amount of *l.*1 for thoſe Quarters.

3*dly*, To the Log. of that Amount add that of the Rate *per* Quarter leſs 1.

4*thly*, From that Sum deduct the Log. of the Amount for the Quarters, leſs 1; and the Remainer is the Log. of the Annuity that *l.* 1 will purchaſe for 21 Years payable quarterly.

5*thly*, To the laſt Log. add that of the Purchaſe-Money, and the Sum is the Log. of the Anſwer.

Example.

1.05 the Ratio, its Log.	0.0211893
A 4th of that Log. wch is the Log. of 1.012, the Rate *per* Quarter	0.0052974
Which Log. multiply by the Quarters in 21 Yrs.	 84
And that produceth the Log. of 2.786 = the Amount of *l.*1 for 84 Quarters, *viz.*	0.4449816
To which Log. add that of .012 = Rate per Quarter, leſs 1	2.0791812
Sum	2.5241628
1.786 = the Amount, leſs 1, its Log. deduct	0.2518815
Reſts the Log. of .01872 *l* = the Annuity that *l.*1 will purchaſe, payable quarterly, *viz.*	2.2722813
To which Log. add that of *l.* 320.527 the Purch. Mony, which is	2.5058762
And the Sum is the Log. of *l.*6, the Quarterly Payment requir'd	0.7781575

By this Example it is obvious, that the *l.* 320.527 will purchaſe but *6l. per* Quarter, at the Rate of 24 *l. per ann.* Whereas by the 5th Queſtion it appears, that Sum will purchaſe *l.* 25 *per ann.* if payable yearly. So that there is *l.*1 *per ann.* difference in that Purchaſe between receiving the Rent quarterly and yearly, the Purchaſer having the Uſe of the Quarters Rents.

Queſt. 8. *What is the preſent Worth of* l. 24 per ann. *payable by equal Quarterly Payments*, to continue 21 Years?

Rule.] Firſt find the Amount of *l.*1 for 84 Quarters, as above.

2*dly*, To that Log. add the Log. of the Rate *per* Quarter (found as above) leſs 1.

3*dly*, Deduct that Sum from the Log. of the Amount leſs 1, and the Remainer is the Log. of the preſent Worth of *l* 1 *per* Quarter, to continue 84 Quarters.

4*thly*, To that Log. add that of *l.*6 *per* Quarter, (as the 4th of 24 *per ann.*) and the Sum is the Log. of the preſent Worth of the Annuity given, payable quarterly.

This proves the Truth of the laſt Queſtion.

Example.

Rate *per* Quarter found as above, is———	1.012
2.786 = the Amount of *l.*1 for 84 Qurs (found as above) its Log.—	0.4449816
.012 = the Rate *per* Quarter leſs 1, its Log. Add———	2.0791812
Sum of thoſe 2 Logs.	2.5241628
1.786 = the Amount above, leſs 1, deduct its Logarithm———	0.2518815
Reſts the Log. of *l.* 53.42 = preſent Worth of *l.* 1 *per* Quarter, to continue 21 Years——	1.7277187
6 *l. per* Quarter (as *per* the Annuity given) its Log. Add———	0.7781575
And the Sum is the Log. of 320.527 the Anſw.	2.5058762

Queſt 9. In an Account of Conſequence 'tis found that *l* 5684.409 is the Amount of 10*l.* Compound Intereſt at 5 *per Cent.* what Time was required for that Increaſe?

Rule.] From the Log. of the Amount deduct that of the Principal put out.

2*dly*, Divide the remaining Log. by the Log of the Ratio, and the Quotient is the Years required. See the Operation.

Example.

*l.*5684.409 its Log.	3.7546090
10 *l.* the Principal, its Log. Deduct——	1.
1.05, its Log. .0211893 } the Diviſor——— }	2.7546090(130
So the Anſwer is 130 Years.	635670

Queſt.

Queſt. 10. In a certain Account *l.* 5684.409 is charged as the Amount of 10 *l.* in 130 Years Compound Intereſt: But upon a Tryal the Simple Intereſt being only allowed according to the Rate agreed on, whence the ſaid Amount is computed; now it not appearing by any Bond, or Entry, what Rate or Intereſt was reckoned, the Queſtion is how that may be found out?

Rule. From the Log. of the Amount deduct that of the Principal put forth.

2*dly*, Divide that remaining Log. by the Number of Years, and the Quote is the Log. of *l.*1 and its Intereſt for a Year. Therefore ſay,

3*dly*, As *l.*1 is to *l.*1 and its Intereſt found :: So is 100 to 100 and its Intereſt. Whence taking 100, the reſt muſt be the Rate of Intereſt required.

Example.

5684.409 the Amount, its Log.	3.7546090
10 = the Principal, deduct its Log.	1.
Divide by 130) reſts ...	2.7546090
Quoteth the Log. of *l.* 1.05, which is	0.0211893

Therefore

As 1. 1.05 :: 100. 105

And 105 leſs 100 = 5 the Intereſt required *per Cent.*

Queſt. 11. A Gentleman bought a Leaſe of *l.* 25 *per ann.* to continue 21 Years for *l.* 320 527: At what Rate was Intereſt allowed him *per Cent?*

To do this by the Logarithms, take the Log. of *l.*320.527, which is	2.5058762
Whence deduct the Log. of the Annuity given 25, which is	1.3979400
And the Remainer is the Log. of 12 821 (or the preſent Worth of *l.* 1 *per ann.* to continue 21 Years; *which note.*	1.1079362

Now to find what Intereſt this is reckon'd at, I make tryal (as *per Queſt.* 4.) and find *at 6 per Cent.* that the preſent Worth of *l.*1 *per ann.* to continue 21 Years is 11.764: which being leſs than the 12.821 (the preſent Worth of *l.*1 *per ann.* as above) I know that 6 *per Cent.* is too much, (for the more the preſent Worth is, the leſs is the Rate of Intereſt, and the contrary.)

Therefore I make tryal at 4 *per Cent.* and (by the Rules under the 4th Queſtion) I find the preſent Worth of *l.*1

per

per ann. at 4 *per Cent.* is (to continue 21 Years) 14.031: which being more than the above 12.821, I know that 4 *per Cent.* is too little. So that it neither being at 6 nor 4, but finding the Rate to lie between them, I ſuppoſe it 5 *per Cent.* And trying by the Rules under the ſaid 4th Queſtion I find the Log. of the preſent Worth of *l.* 1 *per ann.* to continue 21 Years is that of 12.821, the ſame as above, *viz.* ——— 1.1079362

And conſequently I know that 5 *per Cent.* is the Rate ſought.

Or thus to find the Rate:

Having found the preſent Worth at 4 and 6 *per Cent.* as above, proceed thus.

1. Take the Difference between the Years Purchaſe given (or found by the Logarithms) which Years Purchaſe as above are ——— } 12.821

And the Years Purchaſe at 4 *per Cent.* found ——— 14.031

Which Difference is ——— = 1.210

2. The Years Purchaſe at 4 *per Cent.* are found = 14.031

Ditto at 6 *per Cent.* as above are = 11.764

2d Difference = 2.267

3. Say 2.267. 1.210 :: 2. 1

i. e. As the 2d Difference is to the firſt: ſo is 2 the Difference between the Rates *per Cent.* (ſuppoſed) as before, to 1 = the Proportion to be added to the leſſer Rate 4 to give the Rate ſought for; in which caſe the 1ſt place in the 4th Proportional is ſufficient, where (0) is next the Point in the Decimal, if the Diviſion were carry'd on farther.

Queſt. 12. *What is the preſent Value of the Reverſion of an Annuity or Leaſe* of *l.*50 *per ann.* to continue 21 Years, to commence after the Expiration of 7 Years, at 5 *per Cent. per ann.* allowed to the Purchaſer.

Rule.] Firſt add the Years (the 7 to the 21) make 28.

2*dly*, Find the preſent Worth of the Annuity to continue 28 Years by the 4th Queſtion, which you'll find ——— } *l.*745 : — : —

3*dly*, Find

3dly, Find the preſent Worth of the *l.* 50 *per ann.* (by the Rules under the ſaid 4th Queſt.) to continue 7 Years, which is——— } *l.* 283 : 6 : —

4thly, Deduct the latter from the former, and the Remainer is the preſent Worth of the Reverſion required——— } *l.* 461 : 14 : —

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13. *When it is required to purchaſe an Eſtate either in Fee or for Years mentioned, with Money due at any time to come:* as ſuppoſe I would purchaſe an Annuity to continue 14 Years, with *l.* 1000 due to me at the end of 5 Years; the Annuity to commence preſently at 5 *per Cent.*

Rule.] Firſt by the 2d Queſtion find the preſent Worth of *l.*1000 due at the End of 5 Years, which is } *l.* 783: 10 : —

2dly, By the 5th Queſtion find what Annuity to continue 14 Years *l.*783 : 10 will purchaſe; which for Anſwer you'll find at 5 *per Cent.*——— } *l.* 79 : 2 *per ann.* 14 Years.

I need not tell the Reader, that in caſe the *Purchaſe had been required in Fee, to divide the preſent Worth of* l.1000 = l.783.5 *by the Number of Years Purchaſe agreed on*; as at 20 it will be——— *l.* 39 : 3 : 6 *per ann.* for ever.

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14. *When you would Fine off Rent:* As ſuppoſe a Leaſe is *l.* 90 *per ann.* and the Tenant is willing to fine off *l.*40 *per ann.* for his Leaſe of 21 Years at 5 *per Cent.* in this Caſe there is nothing to be done but by the 4th Queſt. find the preſent Worth of *l.*40 *per ann.* to continue 21 Years; which preſent Worth *l.* 512 : 16 : 11 muſt be given the Landlord as a Fine, and *l.* 50 *per ann.* muſt be paid Annual Rent. On the contrary,

15. *When you would Rent off a Fine:* As ſuppoſe a Leaſe for 21 Years is to be lett at *l.*50 *per ann.* with a Fine of *l.* 512.846; but the Leaſee not having ſo much ready Money to ſpare, he is willing to pay an Equivalent Rent, and no Fine. To anſwer this, you need only to conſider by the Rules under the 5th Queſtion, what Annuity to continue 21 Years at 5 *per Cent.* the ſaid Fine = *l.*512.846 will

will purchaſe; which you'll find *l.* 40 *per ann.* which added to the *l.* 50, ſhews that he muſt pay *l.* 90 *per ann.* if he pays no Fine.

Theſe 15 Queſtions and Propoſitions contain the Fundamental Rules for ſolving all Queſtions of Compound Intereſt whatever; and whoſoever doth thoroughly underſtand them, may be able to give a true Anſwer to any Queſtion relating to Selling or Purchaſing Eſtates, *&c.* without the help of Tables for that purpoſe; provided he has well learnt the Logarithms.

The End of Logarithmical Arithmetic, and its Uſe.

A TABLE

A

TABLE

OF THE

LOGARITHMS

TO ALL

NUMBERS,

Not exceeding 10000, whether they be Intire, Broken, or mixt Numbers.

And the Differences of *Logarithms*, and the Parts proportional, whereby the *Logarithm* of any Number is produced to 100000.

Particularly useful in Extracting the Square, Cube, *&c.* Roots, and solving Questions in Compound Interest, *&c.*

Natural Numbers.	0	1	2	3	4
1	0000000	0413927	0791812	1139433	1461280
2	3010300	3222193	3424227	3617278	3802112
3	4771212	4913617	5051500	5185139	5314789
4	6020600	6127838	6232493	6334684	6434527
5	6989700	7075702	7160033	7242759	7323937
6	7781512	7853298	7923917	7993405	8061800
7	8450980	8512583	8573325	8633229	8692317
8	9030900	9084850	9138138	9190781	9242793
9	9542425	9590414	9637878	9684829	9731278
100	0000000	0004341	0008677	0013009	0017337
101	0043214	0047511	0051805	0056094	0060379
102	0086002	0090257	0094509	0098756	0102999
103	0128372	0132587	0136794	0141003	0145205
104	0170333	0174507	0178677	0182843	0187005
105	0211893	0216027	0220157	0224284	0228406
106	0253059	0257154	0261245	0265333	0269416
107	0293838	0297895	0301948	0305997	0310043
108	0334237	0338257	0342273	0346284	0350293
109	0374265	0378247	0382226	0386202	0390173
110	0413927	0417873	0421816	0425755	0429691
111	0453230	0457140	0461048	0464952	0468852
112	0492180	0496056	0499928	0503797	0507663
113	0530784	0534626	0538464	0542299	0546130
114	0569048	0572856	0576661	0580462	0584260
115	0606978	0610753	0614525	0618293	0622058
116	0644580	0648322	0652061	0655797	0659530
117	0681859	0685569	0689276	0692980	0696681
118	0718820	0722499	0726175	0729847	0733517
119	0755470	0759118	0762762	0766404	0770043
120	0791812	0795430	0799045	0802656	0806265
121	0827854	0831441	0835026	0838608	0842187
122	0863598	0867157	0870712	0874264	0877814
123	0899051	0902580	0906107	0909631	0913151
124	0934217	0937718	0941216	0944711	0948204
125	0969100	0972573	0976043	0979511	0982975
126	1003705	1007151	1010593	1014033	1017471
127	1038037	1041455	1044871	1048284	1051694
128	1072100	1075491	1078880	1082266	1085650
129	1105897	1109262	1112625	1115985	1119343
130	1139433	1142773	1146110	1149444	1152776
131	1172713	1176027	1179338	1182647	1185954
132	1205739	1209028	1212314	1215598	1218880
133	1238516	1241780	1245042	1248301	1251558
134	1271048	1274288	1277525	1280760	1283993

Natural Numbers.	5	6	7	8	9
1	1760912	2041200	2304489	2552725	2787536
2	3979400	4149733	4313637	4471580	4623980
3	5440680	5563025	5682017	5797836	5910646
4	6532125	6627578	6720978	6812412	6901961
5	7403627	7481880	7558748	7634280	7708520
6	8129133	8195439	8260748	8325089	8388491
7	8750613	8808136	8864907	8920946	8976271
8	9294189	9344984	9395192	9444827	9493900
9	9777236	9822712	9867717	9912261	9956352
100	0021661	0025980	0030295	0034605	0038912
101	0064660	0068937	0073209	0077478	0081742
102	0107239	0111473	0115704	0119931	0124154
103	0149403	0153597	0157787	0161973	0166155
104	0191163	0195317	0199467	0203613	0207755
105	0232524	0236639	0240750	0244857	0248960
106	0273496	0277572	0281644	0285712	0289777
107	0314085	0318123	0322157	0326188	0330214
108	0354297	0358298	0362295	0366289	0370279
109	0394141	0398105	0402066	0406023	0409977
110	0433623	0437551	0441476	0445398	0449315
111	0472749	0476642	0480532	0484418	0488301
112	0511525	0515384	0519239	0523091	0526939
113	0549958	0553783	0557605	0561423	0565237
114	0588055	0591846	0595634	0599419	0603200
115	0625820	0629578	0633334	0637085	0640834
116	0663259	0666985	0670708	0674428	0678145
117	0700379	0704073	0707765	0711453	0715138
118	0737183	0740847	0744507	0748164	0751818
119	0773679	0777312	0780941	0784568	0788192
120	0809870	0813473	0817073	0820669	0824263
121	0845763	0849336	0852906	0856473	0860037
122	0881361	0884905	0888446	0891984	0895519
123	0916669	0920185	0923696	0927206	0930712
124	0951693	0955180	0958664	0962146	0965624
125	0986437	0989896	0993353	0996806	1000257
126	1020905	1024337	1027766	1031192	1034616
127	1055102	1058506	1061909	1065308	1068705
128	1089031	1092410	1095785	1099159	1102529
129	1122698	1126050	1129400	1132746	1136091
130	1156105	1159432	1162756	1166077	1169396
131	1189257	1192559	1195858	1199154	1202448
132	1222159	1225435	1228709	1231981	1235250
133	1254813	1258064	1261314	1264561	1267806
134	1287223	1290450	1293676	1296890	1300110

Natural Numbers	0	1	2	3	4
135	1303338	1306553	1309767	1312978	1316187
136	1335389	1338581	1341771	1344958	1348144
137	1367206	1370374	1373541	1376705	1379867
138	1398791	1401937	1405080	1408222	1411361
139	1430148	1433271	1436392	1439511	1442628
140	1461280	1464381	1467480	1470577	1473671
141	1492191	1495270	1498347	1501422	1504494
142	1522883	1525941	1528996	1532049	1535100
143	1553360	1556396	1559430	1562462	1565491
144	1583625	1586640	1589653	1592663	1595672
145	1613680	1616674	1619666	1622656	1625644
146	1643528	1646502	1649474	1652443	1655411
147	1673173	1676127	1679078	1682027	1684975
148	1702617	1705550	1708482	1711411	1714339
149	1731863	1734776	1737688	1740598	1743506
150	1760913	1763807	1766699	1769590	1772478
151	1789769	1792645	1795518	1798389	1801259
152	1818436	1821292	1824146	1826999	1829850
153	1846941	1849752	1852588	1855421	1858253
154	1875207	1878026	1880844	1883659	1886473
155	1903317	1906118	1908917	1911714	1914510
156	1931246	1934029	1936810	1939590	1942367
157	1958996	1961762	1964525	1967287	1970047
158	1986571	1989319	1992065	1994809	1997552
159	2013971	2016702	2019431	2022158	2024883
160	2041200	2043913	2046625	2049335	2052044
161	2068259	2070955	2073650	2076344	2079035
162	2095150	2097830	2100508	2103185	2105860
163	2121876	2124540	2127201	2129862	2132521
164	2148438	2151085	2153732	2156376	2159018
165	2174839	2177471	2180100	2182727	2185355
166	2201081	2203696	2206310	2208922	2211533
167	2227165	2229764	2232363	2234959	2237554
168	2253093	2255677	2258260	2260841	2263421
169	2278867	2281436	2284003	2286570	2289134
170	2304489	2307043	2309596	2312146	2314696
171	2329961	2332500	2335038	2337574	2340108
172	2355284	2357809	2360331	2362853	2365373
173	2380461	2382971	2385479	2387986	2390491
174	2405492	2407988	2410481	2412974	2415465
175	2430380	2432861	2435341	2437819	2440296
176	2455127	2457593	2460059	2462523	2464986
177	2479733	2482186	2484637	2487085	2489536
178	2504200	2506639	2509077	2511513	2513948

Natural Numbers.	5	6	7	8	9
135	1319393	1322597	1325798	1328998	1332194
136	1351326	1354507	1357685	1360861	1364034
137	1383027	1386184	1389339	1392492	1395643
138	1414498	1417632	1420765	1423895	1427022
139	1445742	1448854	1451964	1455072	1458177
140	1476763	1479853	1482941	1486026	1489110
141	1507564	1510632	1513698	1516762	1519824
142	1538149	1541195	1544240	1547282	1550322
143	1568519	1571544	1574568	1577589	1580608
144	1598678	1601683	1604685	1607686	1610684
145	1628630	1631614	1634595	1637575	1640553
146	1658376	1661340	1664301	1667260	1670218
147	1687920	1690863	1693805	1696744	1699682
148	1717264	1720188	1723110	1726029	1728947
149	1746412	1749316	1752218	1755118	1758016
150	1775365	1778250	1781132	1784013	1786892
151	1804126	1806992	1809856	1812718	1815578
152	1832698	1835545	1838390	1841233	1844075
153	1861084	1863912	1866739	1869563	1872386
154	1889285	1892095	1894903	1897709	1900514
155	1917304	1920096	1922886	1925674	1928461
156	1945143	1947917	1950690	1953460	1956229
157	1972806	1975562	1978317	1981070	1983821
158	2000293	2003032	2005769	2008505	2011239
159	2027607	2030329	2033049	2035768	2038485
160	2054750	2057455	2060159	2062869	2065560
161	2081725	2084413	2087100	2089785	2092468
162	2108534	2111203	2113876	2116544	2119211
163	2135178	2137833	2140487	2143139	2145789
164	2161659	2164298	2166936	2169572	2172206
165	2187980	2190603	2193225	2195845	2198464
166	2214142	2216750	2219356	2221960	2224563
167	2240148	2242740	2245331	2247920	2250507
168	2265999	2268576	2271151	2273724	2276296
169	2291697	2294258	2296818	2299377	2301934
170	2317244	2319790	2322335	2324879	2327421
171	2342641	2345173	2347703	2350232	2352759
172	2367891	2370408	2372923	2375437	2377950
173	2392995	2395497	2397998	2400498	2402996
174	2417954	2420442	2422929	2425414	2427898
175	2442771	2445245	2447718	2450189	2452658
176	2467447	2469907	2472365	2474823	2477278
177	2491984	2494430	2496874	2499317	2501759
178	2516382	2518814	2521246	2523675	2526103

Natural Numbers	0	1	2	3	4
179	2528530	2530956	2533380	2535803	2538224
180	2552725	2555137	2557548	2559957	2562365
181	2576786	2579184	2581582	2583978	2586373
182	2600714	2603099	2605484	2607867	2610248
183	2624511	2626883	2629255	2631625	2633993
184	2648178	2650538	2652896	2655253	2657609
185	2671717	2674064	2676410	2678752	2681097
186	2695129	2697464	2699797	2702128	2704459
187	2718416	2720738	2723058	2725378	2727696
188	2741578	2743888	2746196	2748503	2750809
189	2764618	2766915	2769211	2771506	2773800
190	2787536	2789821	2792105	2794388	2796669
191	2810334	2812607	2814879	2817150	2819419
192	2833012	2835274	2837534	2839793	2842051
193	2855573	2857823	2860071	2862318	2864565
194	2878017	2880255	2882492	2884728	2886963
195	2900346	2902573	2904798	2907022	2909246
196	2922561	2924776	2926990	2929203	2931415
197	2944662	2946866	2949069	2951271	2953471
198	2966652	2968845	2971036	2973227	2975417
199	2988531	2990713	2992893	2995073	2997251
200	3010300	3012471	3014641	3016809	3018977
201	3031961	3034121	3036280	3038438	3040595
202	3053514	3055663	3057811	3059959	3062105
203	3074960	3077099	3079237	3081374	3083509
204	3096302	3098430	3100557	3102684	3104809
205	3117539	3119657	3121774	3123889	3126004
206	3138672	3140780	3142887	3144992	3147097
207	3159703	3161801	3163897	3165993	3168087
208	3180633	3182721	3184807	3186893	3188977
209	3201463	3203540	3205617	3207692	3209767
210	3222193	3224260	3226327	3228393	3230457
211	3242825	3244882	3246939	3248995	3251050
212	3263359	3265407	3267454	3269500	3271545
213	3283796	3285834	3287872	3289909	3291944
214	3304138	3306167	3308195	3310222	3312248
215	3324385	3326404	3328423	3330440	3332457
216	3344537	3346548	3348557	3350565	3352572
217	3364597	3366598	3368598	3370597	3372595
218	3384565	3386557	3388547	3390537	3392526
219	3404441	3406424	3408405	3410386	3412366
220	3424227	3426200	3428173	3430145	3432116
221	3443923	3445887	3447851	3449814	3451776
222	3463530	3465486	3467441	3469395	3471348

Natural Numbers.	5	6	7	8	9
179	2540645	2543063	2545481	2547897	2550312
180	2564772	2567177	2569581	2571984	2574386
181	2588766	2591158	2593549	2595939	2598327
182	2612629	2615008	2617385	2619762	2622137
183	2636361	2638727	2641092	2643455	2645817
184	2659964	2662317	2664669	2667020	2669369
185	2683439	2685780	2688119	2690457	2692794
186	2706788	2709116	2711443	2713769	2716093
187	2730013	2732328	2734643	2736956	2739268
188	2753113	2755417	2757719	2760020	2762320
189	2776092	2778383	2780673	2782962	2785250
190	2798950	2801229	2803507	2805784	2808059
191	2821688	2823955	2826221	2828486	2830750
192	2844307	2846563	2848817	2851070	2853322
193	2866810	2869054	2871296	2873538	2875778
194	2889196	2891428	2893659	2895889	2898118
195	2911468	2913688	2915908	2918127	2920344
196	2933626	2935835	2938044	2940251	2942457
197	2955671	2957869	2960067	2962263	2964458
198	2977605	2979792	2981979	2984164	2986348
199	2999420	3001605	3033781	3005955	3008128
200	3021144	3023309	3025474	3027637	3029799
201	3042751	3044905	3047059	3049212	3051363
202	3064250	3066394	3068537	3070679	3072820
203	3085644	3087778	3089910	3092042	3094172
204	3106933	3109056	3111178	3113299	3115420
205	3128118	3130231	3132343	3134454	3136563
206	3149200	3151303	3153405	3155505	3157605
207	3170181	3172273	3174365	3176455	3178545
208	3191061	3193143	3195224	3197305	3199384
209	3211840	3213913	3215984	3218055	3220124
210	3232521	3234584	3236645	3238706	3240766
211	3253104	3255157	3257209	3259260	3261310
212	3273589	3275633	3277675	3279716	3281757
213	3293979	3296012	3298045	3300077	3302108
214	3314273	3316297	3318320	3320343	3322364
215	3334473	3336488	3338501	3340514	3342526
216	3354579	3356585	3358589	3360593	3362596
217	3374593	3376589	3378584	3380579	3382572
218	3394514	3396501	3398488	3400473	3402458
219	3414345	3416323	3418301	3420277	3422252
220	3434086	3436055	3438023	3439991	3441957
221	3453737	3455698	3457657	3459615	3461573
222	3473320	3475252	3477202	3479152	3481101

Natural Numbers.	0	1	2	3	4
223	3483049	3484996	3486942	3488887	3490832
224	3502480	3504419	3506356	3508293	3510228
225	3521825	3523755	3525684	3527612	3529539
226	3541084	3543006	3544926	3546845	3548764
227	3560259	3562171	3564083	3565994	3567905
228	3579348	3581253	3583156	3585059	3586961
229	3598355	3600251	3602146	3604040	3605934
230	3617278	3619166	3621053	3622939	3624825
231	3636120	3638000	3639878	3641756	3643633
232	3654880	3656751	3658622	3660492	3662361
233	3673559	3675423	3677285	3679147	3681008
234	3692159	3694014	3695869	3697723	3699576
235	3710679	3712526	3714373	3716219	3718065
236	3729120	3730960	3732799	3734637	3736475
237	3747483	3749316	3751147	3752977	3754807
238	3765769	3767594	3769418	3771240	3773062
239	3783979	3785796	3787612	3789427	3791241
240	3802112	3803922	3805730	3807538	3809345
241	3820170	3821972	3823773	3825573	3827373
242	3838154	3839948	3841741	3843534	3845326
243	3856063	3857850	3859636	3861421	3863206
244	3873898	3875678	3877457	3879235	3881012
245	3891661	3893433	3895205	3896975	3898746
246	3909351	3911116	3912880	3914644	3916407
247	3926969	3928727	3930485	3932241	3933997
248	3944517	3946268	3948018	3949767	3951516
249	3961993	3963734	3965480	3967223	3968964
250	3979400	3981137	3982873	3984608	3986343
251	3996737	3998467	4000196	4001925	4003653
252	4014005	4015728	4017451	4019172	4020893
253	4031205	4032921	4034637	4036352	4038066
254	4048337	4050047	4051755	4053464	4055171
255	4065402	4067105	4068807	4070508	4072209
256	4082400	4084096	4085791	4087486	4089180
257	4099331	4101021	4102710	4104398	4106085
258	4116197	4117880	4119562	4121244	4122925
259	4132998	4134674	4136350	4138025	4139700
260	4149733	4151404	4153073	4154742	4156410
261	4166405	4168069	4169732	4171394	4173056
262	4183013	4184670	4186327	4187983	4189638
263	4199557	4201208	4202859	4204509	4206158
264	4216039	4217684	4219328	4220972	4222614
265	4232459	4234097	4235735	4237372	4239009
266	4248816	4250449	4252080	4253712	4255342

Natural Numbers	5	6	7	8	9
223	3492775	3494718	3496660	3498601	3500541
224	3512163	3514098	3516031	3517963	3519895
225	3531465	3533391	3535316	3537239	3539162
226	3550682	3552599	3554515	3556430	3558345
227	3569813	3571723	3573630	3575537	3577443
228	3588862	3590762	3592662	3594560	3596458
229	3607827	3609719	3611610	3613500	3615390
230	3626709	3628593	3630476	3632358	3634239
231	3645510	3647386	3649260	3651134	3653007
232	3664230	3666097	3667964	3669830	3671695
233	3682869	3684728	3686587	3688445	3690302
234	3701428	3703280	3705131	3706981	3708830
235	3719909	3721753	3723596	3725438	3727279
236	3738311	3740147	3741983	3743817	3745651
237	3756636	3758464	3760292	3762118	3763944
238	3774884	3776704	3778524	3780343	3782161
239	3793055	3794868	3796680	3798492	3800302
240	3811151	3812956	3814761	3816565	3818368
241	3829171	3830969	3832766	3834563	3836359
242	3847117	3848908	3850698	3852487	3854275
243	3864990	3866773	3868555	3870337	3872118
244	3882789	3884565	3886340	3888114	3889888
245	3900515	3902284	3904052	3905819	3907585
246	3918169	3919931	3921691	3923452	3925211
247	3935752	3937506	3939260	3941013	3942765
248	3953264	3955011	3956758	3958504	3960249
249	3970705	3972446	3974185	3975924	3977662
250	3988077	3989811	3991543	3993275	3995007
251	4005380	4007106	4008832	4010557	4012282
252	4022614	4024333	4026052	4027771	4029488
253	4039780	4041492	4043205	4044916	4046627
254	4056878	4058584	4060289	4061994	4063698
255	4073909	4075608	4077307	4079005	4080703
256	4090874	4092567	4094259	4095950	4097641
257	4107772	4109459	4111144	4112829	4114513
258	4124605	4126285	4127964	4129643	4131320
259	4141374	4143047	4144719	4146391	4148063
260	4158077	4159744	4161410	4163076	4164741
261	4174717	4176377	4178037	4179696	4181355
262	4191293	4192947	4194601	4196254	4197906
263	4207806	4209454	4211101	4212748	4214394
264	4224257	4225898	4227539	4229180	4230820
265	4240645	4242281	4243915	4245550	4247183
266	4256972	4258601	4260230	4261858	4263486

Natural Numbers.	0	1	2	3	4
267	4265113	4266739	4268365	4269990	4271614
268	4281348	4282968	4284588	4286207	4287825
269	4297523	4299137	4300751	4302364	4303976
270	4313638	4315246	4316853	4318460	4320067
271	4329693	4331295	4332897	4334498	4336098
272	4345689	4347285	4348881	4350476	4352071
273	4361626	4363217	4364807	4366396	4367985
274	4377506	4379090	4380674	4382258	4383841
275	4393327	4394906	4396484	4398062	4399639
276	4409091	4410664	4412237	4413809	4415380
277	4424798	4426365	4427932	4429499	4431065
278	4440448	4442010	4443571	4445132	4446692
279	4456042	4457598	4459154	4460709	4462264
280	4471580	4473131	4474681	4476231	4477780
281	4487063	4488608	4490153	4491697	4493241
282	4502491	4504031	4505570	4507109	4508647
283	4517864	4519399	4520932	4522466	4523998
284	4533183	4534712	4536241	4537769	4539296
285	4548449	4549972	4551495	4553018	4554540
286	4563660	4565179	4566696	4568213	4569731
287	4578819	4580332	4581844	4583356	4584868
288	4593925	4595433	4596940	4598446	4599953
289	4608978	4610481	4611983	4613484	4614985
290	4623980	4625477	4626974	4628470	4629966
291	4638930	4640422	4641914	4643405	4644895
292	4653828	4655316	4656802	4658288	4659774
293	4668676	4670158	4671640	4673120	4674601
294	4683473	4684950	4686427	4687903	4689378
295	4698220	4699692	4701163	4702634	4704105
296	4712917	4714384	4715850	4717317	4718782
297	4727564	4729027	4730488	4731949	4733410
298	4742163	4743620	4745076	4746533	4747988
299	4756712	4758164	4759616	4761067	4762518
300	4771212	4772660	4774107	4775553	4776999
301	4785665	4787108	4788550	4789991	4791432
302	4800069	4801507	4802945	4804381	4805818
303	4814426	4815859	4817292	4818724	4820156
304	4828736	4830164	4831592	4833019	4834446
305	4842998	4844422	4845845	4847268	4848690
306	4857214	4858633	4860052	4861470	4862888
307	4871384	4872798	4874212	4875626	4877039
308	4885507	4886917	4888326	4889735	4891144
309	4899585	4900990	4902395	4903799	4905203
310	4913617	4915018	4916418	4917818	4919217

Natural Numbers	5	6	7	8	9
267	4273238	4274861	4276484	4278106	4279727
268	4289443	4291060	4292677	4294293	4295908
269	4305588	4307199	4308809	4310419	4312029
270	4321673	4323278	4324883	4326487	4328090
271	4337698	4339298	4340896	4342494	4344092
272	4353665	4355258	4356851	4358444	4360035
273	4369573	4371161	4372748	4374334	4375920
274	4385423	4387005	4388587	4390167	4391747
275	4401216	4402792	4404368	4405943	4407517
276	4416951	4418522	4420092	4421661	4423229
277	4432630	4434195	4435759	4437322	4438885
278	4448252	4449811	4451370	4452928	4454485
279	4463818	4465372	4466925	4468477	4470029
280	4479329	4480877	4482424	4483971	4485517
281	4494784	4496326	4497868	4499410	4500951
282	4510184	4511721	4513258	4514794	4516329
283	4525531	4527062	4528593	4530124	4531654
284	4540823	4542349	4543875	4545400	4546924
285	4556061	4557582	4559102	4560622	4562142
286	4571246	4572762	4574277	4575791	4577305
287	4586378	4587889	4589399	4590908	4592417
288	4601458	4602963	4604468	4605972	4607475
289	4616486	4617986	4619485	4620984	4622482
290	4631461	4632956	4634450	4635944	4637437
291	4646386	4647875	4649364	4650853	4652341
292	4661259	4662743	4664227	4665711	4667194
293	4676081	4677560	4679039	4680518	4681996
294	4690853	4692327	4693801	4695275	4696748
295	4705575	4707044	4708513	4709982	4711450
296	4720247	4721711	4723175	4724639	4726102
297	4734870	4736329	4737788	4739247	4740705
298	4749443	4750898	4752352	4753806	4755259
299	4763968	4765418	4766867	4768316	4769765
300	4778445	4779890	4781334	4782778	4784222
301	4792873	4794313	4795754	4797192	4798631
302	4807254	4808689	4810124	4811559	4812993
303	4821587	4823018	4824448	4825878	4827307
304	4835873	4837299	4838725	4840150	4841574
305	4850112	4851533	4852954	4854375	4855795
306	4864305	4865721	4867138	4868554	4869969
307	4878451	4879863	4881275	4882686	4884097
308	4892552	4893959	4895366	4896773	4898179
309	4906607	4908009	4909412	4910814	4912216
310	4920616	4922014	4923413	4924810	4926207

Natural Numbers.	0	1	2	3	4
311	4927604	4929000	4930396	4931791	4933186
312	4941546	4942938	4944329	4945720	4947110
313	4955443	4956831	4958218	4959604	4960990
314	4969296	4970679	4972062	4973444	4974825
315	4983106	4984484	4985862	4987240	4988617
316	4996871	4998245	4999619	5000992	5002365
317	5010593	5011962	5013332	5014701	5016069
318	5024271	5025637	5027002	5028366	5029731
319	5037907	5039268	5040629	5041989	5043349
320	5051500	5052857	5054213	5055569	5056925
321	5065050	5066403	5067755	5069107	5070459
322	5078559	5079907	5081255	5082603	5083950
323	5092025	5093370	5094713	5096057	5097400
324	5105450	5106790	5108130	5109469	5110808
325	5118834	5120170	5121505	5122841	5124175
326	5132176	5133508	5134840	5136171	5137501
327	5145478	5146805	5148133	5149460	5150787
328	5158738	5160062	5161386	5162709	5164031
329	5171959	5173279	5174598	5175917	5177235
330	5185139	5186455	5187771	5189086	5190400
331	5198280	5199592	5200903	5202214	5203525
332	5211381	5212689	5213996	5215303	5216610
333	5224442	5225746	5227050	5228353	5229656
334	5237465	5238765	5240064	5241364	5242663
335	5250448	5251744	5253040	5254335	5255631
336	5263393	5264685	5265977	5267269	5268560
337	5276299	5277588	5278876	5280163	5281451
338	5289167	5290452	5291736	5293020	5294303
339	5301997	5303278	5304558	5305839	5307118
340	5314789	5316066	5317343	5318619	5319895
341	5327544	5328817	5330090	5331363	5332635
342	5340261	5341531	5342800	5344069	5345338
343	5352941	5354207	5355473	5356738	5358003
344	5365584	5366847	5368109	5369370	5370631
345	5378191	5379450	5380708	5381966	5383223
346	5390761	5392016	5393271	5394525	5395779
347	5403295	5404546	5405797	5407048	5408298
348	5415792	5417040	5418288	5419535	5420781
349	5428254	5429498	5430742	5431986	5433229
350	5440680	5441921	5443161	5444401	5445641
351	5453071	5454308	5455545	5456781	5458017
352	5465427	5466660	5467894	5469126	5470359
353	5477747	5478977	5480207	5481436	5482665
354	5490033	5491259	5492486	5493712	5494937

Natural Numbers.	5	6	7	8	9
311	4934580	4935974	4937368	4938761	4940154
312	4948500	4949890	4951279	4952667	4954056
313	4962375	4963761	4965145	4966529	4967913
314	4976206	4977587	4978967	4980347	4981727
315	4989994	4991370	4992746	4994121	4995496
316	5003737	5005109	5006481	5007852	5009222
317	5017437	5018805	5020172	5021539	5022905
318	5031094	5032458	5033821	5035183	5036545
319	5044709	5046068	5047426	5048785	5050142
320	5058280	5059635	5060990	5062344	5063697
321	5071810	5073160	5074511	5075860	5077210
322	5085297	5086644	5087990	5089335	5090680
323	5098743	5100085	5101427	5102768	5104109
324	5112147	5113485	5114823	5116160	5117497
325	5125510	5126844	5128178	5129511	5130844
326	5138832	5140162	5141491	5142820	5144149
327	5152113	5153439	5154764	5156089	5157414
328	5165354	5166676	5167997	5169318	5170639
329	5178554	5179872	5181189	5182506	5183823
330	5191715	5193028	5194342	5195655	5196968
331	5204835	5206145	5207455	5208764	5210073
332	5217916	5219222	5220528	5221833	5223138
333	5230958	5232260	5233562	5234863	5236164
334	5243961	5245259	5246557	5247854	5249151
335	5256925	5258219	5259513	5260807	5262100
336	5269851	5271141	5272431	5273721	5275010
337	5282738	5284024	5285311	5286596	5287882
338	5295587	5296869	5298152	5299434	5300716
339	5308398	5309677	5310955	5312234	5313512
340	5321171	5322446	5323721	5324996	5326270
341	5333907	5335179	5336450	5337721	5338991
342	5346606	5347874	5349141	5350408	5351675
343	5359257	5360532	5361795	5363059	5364322
344	5371892	5373153	5374413	5375672	5376932
345	5384481	5385737	5386994	5388250	5389506
346	5397032	5398286	5399538	5400791	5402043
347	5409548	5410798	5412047	5413296	5414544
348	5422028	5423274	5424519	5425765	5427010
349	5434472	5435714	5436956	5438198	5439439
350	5446880	5448119	5449358	5450596	5451834
351	5459253	5460489	5461724	5462958	5464193
352	5471591	5472823	5474055	5475286	5476517
353	5483894	5485123	5486351	5487578	5488806
354	5496162	5497387	5498612	5499836	5501060

Natural Numbers	0	1	2	3	4
355	5502283	5503507	5504730	5505952	5507174
356	5514500	5515720	5516939	5518158	5519377
357	5526682	5527898	5529114	5530330	5531545
358	5538830	5540043	5541256	5542468	5543680
359	5550944	5552154	5553362	5554572	5555781
360	5563025	5564231	5565437	5566643	5567848
361	5575072	5576275	5577477	5578680	5579881
362	5587086	5588285	5589484	5590683	5591882
363	5599066	5600262	5601458	5602654	5603849
364	5611014	5612207	5613399	5614592	5615784
365	5622929	5624118	5625308	5626497	5627685
366	5634811	5635997	5637183	5638369	5639555
367	5646661	5647844	5649027	5650209	5651392
368	5658478	5659658	5660838	5662017	5663196
369	5670264	5671440	5672617	5673793	5674969
370	5682017	5683191	5684364	5685537	5686710
371	5693739	5694910	5696080	5697249	5698419
372	5705429	5706597	5707764	5708930	5710097
373	5717088	5718252	5719416	5720580	5721743
374	5728716	5729877	5731038	5732198	5733358
375	5740313	5741471	5742628	5743786	5744943
376	5751878	5753033	5754188	5755342	5756496
377	5763413	5764565	5765717	5766868	5768019
378	5774917	5776067	5777215	5778363	5779511
379	5786392	5787538	5788683	5789828	5790973
380	5797836	5798979	5800121	5801263	5802405
381	5809250	5810389	5811529	5812668	5813807
382	5820634	5821770	5822907	5824043	5825179
383	5831988	5833122	5834255	5835388	5836521
384	5843312	5844443	5845574	5846704	5847834
385	5854617	5855735	5856863	5857990	5859117
386	5865873	5866998	5868123	5869247	5870371
387	5877110	5878232	5879353	5880475	5881596
388	5888317	5889436	5890555	5891674	5892792
389	5899496	5900612	5901728	5902844	5903959
390	5910646	5911759	5912873	5913985	5915098
391	5921768	5922878	5923988	5925098	5926208
392	5932861	5933968	5935076	5936183	5937290
393	5943925	5945030	5946135	5947239	5948344
394	5954962	5956064	5957166	5958268	5959369
395	5965971	5967070	5968169	5969268	5970367
396	5976952	5978048	5979145	5980241	5981336
397	5987905	5988999	5990092	5991186	5992279
398	5998831	5999922	6001013	6002103	6003193

Natural Numbers.	5	6	7	8	9
355	5508396	5509618	5510839	5512059	5513280
356	5520595	5521813	5523031	5524248	5525465
357	5532760	5533975	5535189	5536403	5537617
358	5544892	5546103	5547314	5548524	5549735
359	5556989	5558197	5559404	5560612	5561818
360	5569053	5570257	5571461	5572665	5573869
361	5581083	5582284	5583485	5584686	5585886
362	5593080	5594278	5595476	5596673	5597870
363	5605044	5606239	5607433	5608627	5609820
364	5616975	5618167	5619358	5620548	5621739
365	5628875	5630062	5631250	5632437	5633624
366	5640740	5641925	5643109	5644293	5645477
367	5652573	5653755	5654936	5656117	5657298
368	5664375	5665553	5666731	5667909	5669087
369	5676144	5677320	5678494	5679669	5680843
370	5687882	5689054	5690226	5691397	5692568
371	5699588	5700757	5701926	5703094	5704262
372	5711263	5712428	5713594	5714759	5715924
373	5722906	5724069	5725231	5726393	5727555
374	5734518	5735678	5736837	5737996	5739154
375	5746099	5747256	5748412	5749568	5750723
376	5757650	5758803	5759956	5761109	5762261
377	5769169	5770320	5771470	5772620	5773769
378	5780659	5781806	5782953	5784100	5785246
379	5792118	5793262	5794406	5795550	5796693
380	5803547	5804688	5805829	5806969	5808110
381	5814945	5816084	5817222	5818359	5819497
382	5826314	5827450	5828585	5829719	5830854
383	5837654	5838786	5839918	5841050	5842181
384	5848963	5850093	5851222	5852351	5853479
385	5860244	5861370	5862496	5863622	5864748
386	5871495	5872618	5873742	5874865	5875987
387	5882717	5883838	5884958	5886078	5887198
388	5893910	5895028	5896145	5897262	5898379
389	5905075	5906189	5907304	5908418	5909532
390	5916210	5917322	5918434	5919546	5920657
391	5927318	5928427	5929536	5930644	5931753
392	5938397	5939503	5940609	5941715	5942820
393	5949447	5950551	5951654	5952757	5953860
394	5960470	5961571	5962671	5963771	5964871
395	5971465	5972563	5973660	5974758	5975855
396	5982432	5983527	5984622	5985717	5986811
397	5993371	5994464	5995556	5996648	5997739
398	6004283	6005373	6006462	6007551	6008640

Natural Numbers.	0	1	2	3	4
399	6009729	6010817	6011905	6012993	6014081
400	6020600	6021685	6022771	6023856	6024941
401	6031444	6032527	6033609	6034692	6035774
402	6042261	6043341	6044421	6045500	6046580
403	6053050	6054128	6055205	6056282	6057359
404	6063814	6064888	6065963	6067037	6068111
405	6074550	6075622	6076694	6077766	6078837
406	6085260	6086330	6087399	6088468	6089537
407	6095944	6097011	6098078	6099144	6100210
408	6106602	6107666	6108730	6109794	6110857
409	6117233	6118295	6119356	6120417	6121478
410	6127839	6128898	6129957	6131015	6132073
411	6138418	6139475	6140531	6141587	6142643
412	6148972	6150026	6151080	6152133	6153187
413	6159501	6160552	6161603	6162654	6163705
414	6170003	6171052	6172101	6173149	6174197
415	6180481	6181527	6182573	6183619	6184665
416	6190933	6191977	6193021	6194064	6195107
417	6201360	6202402	6203443	6204484	6205524
418	6211763	6212802	6213840	6214879	6215917
419	6222140	6223177	6224213	6225149	6226284
420	6232493	6233527	6234560	6235594	6236627
421	6242821	6243852	6244884	6245915	6246945
422	6253124	6254153	6255182	6256211	6257239
423	6263404	6264430	6265457	6266483	6267509
424	6273659	6274683	6275707	6276730	6277754
425	6283889	6284911	6285933	6286954	6287975
426	6294096	6295115	6296134	6297153	6298172
427	6304279	6305296	6306312	6307329	6308345
428	6314438	6315452	6316467	6317481	6318495
429	6324573	6325585	6326597	6327609	6328620
430	6334685	6335694	6336704	6337713	6338723
431	6344773	6345780	6346788	6347795	6348801
432	6354837	6355843	6356848	6357852	6358857
433	6364879	6365882	6366884	6367887	6368889
434	6374897	6375898	6376898	6377898	6378898
435	6384893	6385891	6386889	6387887	6388884
436	6394865	6395861	6396857	6397852	6398847
437	6404814	6405808	6406802	6407795	6408788
438	6414741	6415733	6416724	6417715	6418705
439	6424645	6425634	6426623	6427612	6428601
440	6434527	6435514	6436500	6437487	6438473
441	6444386	6445371	6446355	6447339	6448323
442	6454223	6455205	6456187	6457169	6458151

Natural Numbers.	5	6	7	8	9
399	6015168	6016255	6017341	6018428	6019514
400	6026025	6027109	6028193	6029277	6030361
401	6036855	6037937	6039018	6040099	6041180
402	6047659	6048738	6049816	6050895	6051973
403	6058435	6059512	6060587	6061663	6062738
404	6069185	6070259	6071332	6072405	6073478
405	6079909	6080979	6082050	6083120	6084190
406	6090605	6091674	6092742	6093809	6094877
407	6101276	6102342	6103407	6104472	6105537
408	6111921	6112948	6114046	6115109	6116171
409	6122539	6123599	6124660	6125720	6126779
410	6133132	6134189	6135247	6136304	6137361
411	6143698	6144754	6145809	6146863	6147918
412	6154240	6155292	6156345	6157397	6158449
413	6164755	6165805	6166855	6167905	6168954
414	6175245	6176293	6177340	6178387	6179434
415	6185710	6186755	6187800	6188845	6189889
416	6196150	6197193	6198235	6199277	6200319
417	6206565	6207605	6208645	6209684	6210724
418	6216955	6217992	6219030	6220067	6221104
419	6227320	6228355	6229300	6230424	6231459
420	6237660	6238693	6239725	6240757	6241789
421	6247976	6249006	6250036	6251066	6252095
422	6258267	6259295	6260322	6261350	6262377
423	6268534	6269559	6270585	6271610	6272634
424	6278777	6279800	6280823	6281845	6282867
425	6288996	6290016	6291036	6292057	6293076
426	6299190	6300208	6301226	6302244	6303262
427	6309361	6310377	6311392	6312408	6313423
428	6319508	6320522	6321535	6322548	6323560
429	6329632	6330643	6331653	6332664	6333674
430	6339732	6340740	6341749	6342757	6343765
431	6349808	6350814	6351820	6352826	6353832
432	6359861	6360865	6361869	6362872	6363876
433	6369891	6370893	6371894	6372895	6373896
434	6379898	6380897	6381896	6382895	6383894
435	6389882	6390879	6391876	6392872	6393869
436	6399842	6400837	6401832	6402826	6403820
437	6409781	6410773	6411765	6412758	6413749
438	6419696	6420686	6421676	6422666	6423656
439	6429589	6430577	6431565	6432552	6433540
440	6439459	6440445	6441430	6442416	6443401
441	6449307	6450291	6451274	6452257	6453240
442	6459133	6460114	6461095	6462076	6463057

Natural Numbers.	0	1	2	3	4
443	6464037	6465017	6465997	6466977	6467957
444	6473830	6474808	6475785	6476763	6477740
445	6483600	6484576	6485552	6486527	6487502
446	6493349	6494322	6495296	6496269	6497242
447	6503075	6504047	6505018	6505989	6506960
448	6512780	6513749	6514719	6515687	6516656
449	6522463	6523430	6524397	6525364	6526331
450	6532125	6533090	6534055	6535019	6535984
451	6541765	6542728	6543691	6544653	6545616
452	6551384	6552345	6553306	6554266	6555226
453	6560982	6561941	6562899	6563857	6564815
454	6570559	6571515	6572471	6573427	6574383
455	6580114	6581068	6582023	6582976	6583930
456	6589648	6590601	6591553	6592505	6593456
457	6599162	6600112	6601062	6602012	6602962
458	6608655	6609603	6610551	6611499	6612446
459	6618127	6619073	6620019	6620964	6621910
460	6627578	6628522	6629466	6630410	6631353
461	6637009	6637951	6638893	6639835	6640776
462	6646420	6647360	6648299	6649239	6650178
463	6655810	6656748	6657685	6658623	6659560
464	6665180	6666116	6667051	6667987	6668922
465	6674530	6675463	6676397	6677331	6678264
466	6683859	6684791	6685723	6686654	6687585
467	6693169	6694099	6695028	6695958	6696887
468	6702459	6703386	6704314	6705242	6706169
469	6711728	6712654	6713580	6714506	6715431
470	6720979	6721903	6722826	6723750	6724673
471	6730209	6731131	6732053	6732974	6733896
472	6739420	6740340	6741260	6742179	6743099
473	6748611	6749529	6750447	6751365	6752283
474	6757783	6758700	6759615	6760531	6761447
475	6766936	6767850	6768764	6769678	6770592
476	6776069	6776982	6777894	6778806	6779718
477	6785184	6786094	6787004	6787914	6788824
478	6794279	6795187	6796096	6797004	6797912
479	6803355	6804262	6805168	6806074	6806980
480	6812412	6813317	6814222	6815126	6816030
481	6821451	6822354	6823256	6824159	6825061
482	6830470	6831371	6832272	6833173	6834073
483	6839471	6840370	6841269	6842168	6843066
484	6848454	6849351	6850248	6851145	6852041
485	6857417	6858313	6859208	6860103	6860998
486	6866363	6867256	6868149	6869043	6869936

Natural Numbers.	5	6	7	8	9
443	6468936	6469915	6470894	6471873	6472851
444	6478718	6479695	6480671	6481648	6482624
445	6488477	6489452	6490426	6491401	6492375
446	6498215	6499187	6500160	6501132	6502104
447	6507930	6508901	6509871	6510841	6511811
448	6517624	6518593	6519561	6520528	6521496
449	6527297	6528263	6529229	6530195	6531160
450	6536948	6537912	6538876	6539839	6540802
451	6546578	6547539	6548501	6549462	6550423
452	6556186	6557145	6558105	6559064	6560023
453	6565773	6566730	6567688	6568645	6569602
454	6575339	6576294	6577250	6578205	6579159
455	6584884	6585837	6586790	6587743	6588696
456	6594408	6595359	6596310	6597261	6598212
457	6603911	6604860	6605809	6606758	6607706
458	6613393	6614340	6615287	6616234	6617181
459	6622855	6623800	6624745	6625690	6626634
460	6632296	6633239	6634182	6635125	6636067
461	6641717	6642658	6643599	6644539	6645480
462	6651117	6652056	6652995	6653933	6654872
463	6660497	6661434	6662371	6663307	6664244
464	6669857	6670792	6671727	6672661	6673595
465	6679197	6680130	6681062	6681995	6682927
466	6688516	6689447	6690378	6691308	6692239
467	6697816	6698745	6699674	6700602	6701530
468	6707096	6708023	6708950	6709876	6710802
469	6716356	6717281	6718206	6719130	6720054
470	6725596	6726519	6727442	6728365	6729287
471	6734817	6735738	6736659	6737574	6738500
472	6744018	6744937	6745856	6746775	6747693
473	6753200	6754117	6755034	6755951	6756867
474	6762362	6763277	6764192	6765107	6766022
475	6771505	6772418	6773332	6774244	6775157
476	6780629	6781540	6782452	6783362	6784273
477	6789734	6790643	6791552	6792461	6793370
478	6798819	6799727	6800634	6801541	6802448
479	6807886	6808792	6809697	6810602	6811507
480	6816934	6817838	6818741	6819645	6820548
481	6825963	6826865	6827766	6828668	6829569
482	6834973	6835873	6836773	6837673	6838572
483	6843965	6844863	6845761	6846659	6847556
484	6852938	6853834	6854730	6855626	6856522
485	6861892	6862787	6863681	6864575	6865469
486	6870828	6871721	6872613	6873506	6874398

Natural Numbers.	0	1	2	3	4
487	6875290	6876181	6877073	6877964	6878855
488	6884198	6885088	6885978	6886867	6887757
489	6893089	6893977	6894864	6895752	6896640
490	6901961	6902847	6903733	6904616	6905505
491	6910815	6911699	6912584	6913468	6914352
492	6919651	6920534	6921416	6922298	6923180
493	6928469	6929350	6930231	6931111	6931991
494	6937269	6938148	6939027	6939906	6940785
495	6946052	6946929	6947806	6948683	6949560
496	6954817	6955692	6956568	6957443	6958318
497	6963564	6964438	6965311	6966185	6967058
498	6972293	6973165	6974037	6974909	6975780
499	6981005	6981876	6982746	6983616	6984485
500	6989700	6990569	6991437	6992305	6993173
501	6998377	6999244	7000111	7000977	7001843
502	7007037	7007902	7008767	7009632	7010496
503	7015680	7016543	7017406	7018269	7019132
504	7024305	7025167	7026028	7026890	7027751
505	7032914	7033774	7034633	7035493	7036352
506	7041505	7042363	7043221	7044079	7044937
507	7050080	7050936	7051792	7052649	7053505
508	7058637	7059492	7060347	7061201	7062055
509	7067178	7068031	7068884	7069737	7070589
510	7075702	7076553	7077405	7078256	7079107
511	7084206	7085059	7085908	7086758	7087607
512	7092700	7093548	7094396	7095244	7096091
513	7101174	7102020	7102866	7103713	7104559
514	7109631	7110476	7111321	7112165	7113010
515	7118072	7118915	7119759	7120601	7121441
516	7126497	7127339	7128180	7129021	7129862
517	7134905	7135745	7136585	7137425	7138264
518	7143298	7144136	7144974	7145812	7146650
519	7151674	7152510	7153347	7154183	7155019
520	7160033	7160869	7161703	7162538	7163373
521	7168377	7169211	7170044	7170877	7171710
522	7176705	7177537	7178369	7179200	7180032
523	7185017	7185847	7186677	7187507	7188337
524	7193313	7194142	7194970	7195799	7196627
525	7201593	7202420	7203247	7204074	7204901
526	7209857	7210683	7211508	7212334	7213159
527	7218106	7218930	7219754	7220578	7221401
528	7226339	7227162	7227984	7228806	7229628
529	7234557	7235378	7236198	7237019	7237839
530	7242759	7243578	7244397	7245216	7246035

Natural Numbers.	5	6	7	8	9
487	6879746	6880637	6881528	6882418	6883308
488	6888646	6889535	6890423	6891312	6892200
489	6897527	6898414	6899301	6900188	6901074
490	6906390	6907275	6908161	6909046	6909930
491	6915235	6916119	6917002	6917885	6918768
492	6924062	6924944	6925826	6926707	6927588
493	6932872	6933752	6934631	6935511	6936390
494	6941663	6942541	6943419	6944297	6945174
495	6950437	6951313	6952189	6953065	6953941
496	6959193	6960067	6960942	6961816	6962690
497	6967931	6968804	6969676	6970549	6971421
498	6976652	6977523	6978394	6979264	6980135
499	6985355	6986224	6987093	6987963	6988831
500	6994041	6994908	6995776	6996643	6997510
501	7002709	7003575	7004441	7005307	7006172
502	7011361	7012225	7013089	7013953	7014816
503	7019995	7020857	7021719	7022582	7023444
504	7028612	7029472	7030333	7031193	7032054
505	7037212	7038071	7038929	7039788	7040647
506	7045793	7046652	7047509	7048366	7049223
507	7054360	7055216	7056072	7056927	7057782
508	7062910	7063764	7064617	7065471	7066324
509	7071442	7072294	7073146	7073998	7074850
510	7079957	7080808	7081659	7082509	7083359
511	7088456	7089305	7090154	7091003	7091851
512	7096939	7097786	7098633	7099480	7100327
513	7105404	7106250	7107096	7107941	7108786
514	7113854	7114698	7115542	7116385	7117229
515	7122287	7123129	7123971	7124813	7125655
516	7130703	7131544	7132385	7133225	7134065
517	7139104	7139943	7140782	7141620	7142459
518	7147488	7148325	7149162	7150000	7150837
519	7155856	7156691	7157527	7158363	7159198
520	7164207	7165042	7165876	7166710	7167544
521	7172543	7173376	7174208	7175041	7175873
522	7180863	7181694	7182525	7183356	7184186
523	7189167	7189996	7190826	7191655	7192484
524	7197455	7198283	7199111	7199938	7200766
525	7205727	7206554	7207380	7208206	7209032
526	7213984	7214809	7215633	7216458	7217282
527	7222225	7223048	7223871	7224694	7225517
528	7230450	7231272	7232093	7232914	7233736
529	7238660	7239480	7240300	7241120	7241939
530	7246854	7247672	7248491	7249309	7250127

Natural Numbers.	0	1	2	3	4
531	7250945	7251763	7252581	7253398	7254215
532	7259116	7259933	7260749	7261565	7262380
533	7267272	7268087	7268901	7269716	7270531
534	7275413	7276226	7277039	7277852	7278664
535	7283538	7284349	7285161	7285972	7286784
536	7291648	7292458	7293268	7294078	7294888
537	7299743	7300551	7301360	7302168	7302977
538	7307823	7308630	7309437	7310244	7311051
539	7315888	7316693	7317499	7318304	7319109
540	7323938	7324749	7325546	7326350	7327153
541	7331973	7332775	7333578	7334380	7335182
542	7339993	7340794	7341595	7342396	7343197
543	7347998	7348798	7349598	7350397	7351196
544	7355989	7356787	7357585	7358383	7359181
545	7363965	7364762	7365558	7366355	7367151
546	7371926	7372722	7373517	7374312	7375107
547	7379873	7380667	7381461	7382254	7383048
548	7387806	7388598	7389390	7390182	7390974
549	7395723	7396514	7397305	7398096	7398886
550	7403627	7404416	7405206	7405995	7406784
551	7411516	7412304	7413092	7413880	7414668
552	7419391	7420177	7420964	7421750	7422537
553	7427251	7428037	7428822	7429607	7430392
554	7435098	7435881	7436665	7437449	7438232
555	7442930	7443712	7444495	7445277	7446059
556	7450748	7451529	7452310	7453091	7453871
557	7458552	7459332	7460111	7460890	7461670
558	7466342	7467120	7467898	7468676	7469454
559	7474118	7474895	7475672	7476448	7477225
560	7481880	7482656	7483431	7484206	7484981
561	7489629	7490403	7491177	7491950	7492724
562	7497363	7498136	7498908	7499681	7500453
563	7505087	7505855	7506626	7507398	7508168
564	7512791	7513561	7514331	7515100	7515870
565	7520484	7521253	7522022	7522790	7523558
566	7528164	7528932	7529699	7530466	7531232
567	7535831	7536596	7537362	7538128	7538893
568	7543483	7544248	7545012	7545777	7546541
569	7551123	7551886	7552649	7553412	7554178
570	7558749	7559510	7560279	7561034	7561795
571	7566361	7567122	7567882	7568642	7569402
572	7573960	7574719	7575479	7576237	7576996
573	7581546	7582304	7583062	7583819	7584577
574	7589119	7589875	7590632	7591388	7592144

Natural Numbers.	5	6	7	8	9
531	7255033	7255850	7256667	7257483	7258300
532	7263196	7264012	7264827	7265642	7266457
533	7271344	7272158	7272972	7273786	7274599
534	7279477	7280290	7281101	7281914	7282726
535	7287595	7288406	7289216	7290027	7290838
536	7295697	7296506	7297316	7298125	7298934
537	7303785	7304593	7305400	7306208	7307015
538	7311857	7312663	7313470	7314276	7315082
539	7319914	7320719	7321524	7322329	7323133
540	7327957	7328760	7329564	7330367	7331170
541	7335985	7336787	7337588	7338390	7339191
542	7343997	7344798	7345598	7346398	7347198
543	7351995	7352794	7353593	7354392	7355191
544	7359979	7360776	7361574	7362371	7363168
545	7367948	7368744	7369540	7370335	7371131
546	7375902	7376696	7377491	7378285	7379079
547	7383841	7384634	7385427	7386220	7387013
548	7391766	7392558	7393350	7394141	7394932
549	7399677	7400467	7401257	7402047	7402837
550	7407573	7408362	7409151	7409939	7410728
551	7415455	7416283	7417030	7417817	7418604
552	7423323	7424109	7424895	7425680	7426466
553	7431176	7431961	7432745	7433530	7434314
554	7439015	7439799	7440582	7441365	7442147
555	7446841	7447622	7448404	7449187	7449967
556	7454652	7455432	7456212	7456992	7457772
557	7462449	7463228	7464006	7464785	7465564
558	7470232	7471009	7471787	7472564	7473341
559	7478001	7478777	7479553	7480329	7481105
560	7485756	7486531	7487306	7488080	7488854
561	7493498	7494271	7495044	7495817	7496590
562	7501225	7501997	7502769	7503541	7504312
563	7508939	7509710	7510480	7511251	7512021
564	7516639	7517409	7518178	7518947	7519716
565	7524326	7525094	7525862	7526629	7527397
566	7531999	7532766	7533532	7534298	7535065
567	7539659	7540424	7541189	7541954	7542719
568	7547305	7548069	7548832	7549596	7550359
569	7554937	7555700	7556462	7557224	7557987
570	7562556	7563318	7564079	7564840	7565600
571	7570162	7570922	7571682	7572441	7573201
572	7577755	7578513	7579272	7580030	7580788
573	7585334	7586091	7586848	7587605	7588362
574	7592900	7593656	7594412	7595168	7595923

Natural Numbers.	0	1	2	3	4
575	7596678	7597434	7598189	7598944	7599699
576	7604225	7604979	7605733	7606486	7607240
577	7611758	7612511	7613263	7614016	7614768
578	7619278	7620030	7620781	7621532	7622283
579	7626786	7627536	7628286	7629035	7629785
580	7634280	7635029	7635777	7636526	7637274
581	7641761	7642509	7643256	7644003	7644750
582	7649230	7649976	7650722	7651468	7652214
583	7656686	7657430	7658175	7658920	7659664
584	7664128	7664872	7665616	7666359	7667102
585	7671559	7672301	7673043	7673785	7674527
586	7678976	7679717	7680458	7681199	7681940
587	7686381	7687121	7687860	7688600	7689339
588	7693773	7694512	7695250	7695988	7696727
589	7701153	7701890	7702627	7703364	7704101
590	7708520	7709256	7709992	7710728	7711463
591	7715875	7716610	7717344	7718079	7718813
592	7723217	7723951	7724684	7725417	7726150
593	7730547	7731279	7732011	7732743	7733475
594	7737864	7738596	7739326	7740057	7740788
595	7745170	7745899	7746629	7747359	7748088
596	7752463	7753191	7753920	7754648	7755376
597	7759743	7760471	7761198	7761925	7762652
598	7767012	7767738	7768464	7769190	7769916
599	7774268	7774993	7775718	7776443	7777167
600	7781513	7782236	7782960	7783683	7784407
601	7788745	7789467	7790190	7790912	7791634
602	7795965	7796686	7797408	7798129	7798850
603	7803173	7803893	7804613	7805333	7806053
604	7810369	7811088	7811807	7812526	7813245
605	7817554	7818272	7818989	7819707	7820424
606	7824726	7825443	7826159	7826876	7827592
607	7831887	7832602	7833318	7834033	7834748
608	7839036	7839750	7840464	7841178	7841892
609	7846173	7846886	7847599	7848312	7849024
610	7853298	7854010	7854722	7855434	7856145
611	7860412	7861123	7861833	7862544	7863254
612	7867514	7868224	7868933	7869643	7870352
613	7874605	7875313	7876021	7876730	7877438
614	7881684	7882391	7883098	7883805	7884512
615	7888751	7889457	7890163	7890869	7891575
616	7895807	7896512	7897217	7897922	7898626
617	7902852	7903555	7904259	7904963	7905666
618	7909885	7910587	7911293	7911992	7912695

Natural Numbers	5	6	7	8	9
575	7600453	7601208	7601962	7602717	7603471
576	7607993	7608746	7609500	7610253	7611005
577	7615520	7616272	7617024	7617775	7618527
578	7623034	7623784	7624535	7625285	7626035
579	7630534	7631284	7632033	7632782	7633531
580	7638022	7638770	7639518	7640266	7641014
581	7645497	7646244	7646991	7647737	7648484
582	7652959	7653705	7654450	7655195	7655941
583	7660409	7661153	7661897	7662641	7663385
584	7667845	7668588	7669331	7670074	7670816
585	7675269	7676011	7676752	7677494	7678235
586	7682680	7683421	7684161	7684901	7685641
587	7690079	7690818	7691557	7692296	7693035
588	7697465	7698203	7698940	7699678	7700416
589	7704838	7705575	7706311	7707048	7707784
590	7712199	7712934	7713670	7714405	7715140
591	7719547	7720282	7721016	7721750	7722483
592	7726884	7727616	7728349	7729082	7729814
593	7734207	7734939	7735670	7736402	7737133
594	7741519	7742249	7742979	7743710	7744440
595	7748818	7749547	7750276	7751005	7751734
596	7756104	7756832	7757560	7758288	7759016
597	7763379	7764106	7764833	7765559	7766286
598	7770642	7771367	7772093	7772818	7773543
599	7777892	7778616	7779340	7780065	7780789
600	7785130	7785853	7786576	7787299	7788022
601	7792356	7793078	7793800	7794522	7795243
602	7799571	7800291	7801012	7801732	7802453
603	7806773	7807492	7808212	7808931	7809650
604	7813963	7814681	7815400	7816118	7816836
605	7821141	7821859	7822576	7823293	7824010
606	7828308	7829024	7829740	7830456	7831171
607	7835463	7836178	7836892	7837607	7838321
608	7842606	7843319	7844033	7844746	7845460
609	7849737	7850450	7851162	7851874	7852586
610	7856857	7857568	7858279	7858990	7859701
611	7863965	7864675	7865385	7866095	7866805
612	7871061	7871770	7872479	7873188	7873896
613	7878146	7878853	7879561	7880269	7880976
614	7885219	7885926	7886632	7887339	7888045
615	7892281	7892986	7893691	7894397	7895102
616	7899331	7900035	7900739	7901444	7902148
617	7906370	7907073	7907776	7908479	7909182
618	7913397	7914099	7914801	7915503	7916205

Natural Numbers.	⊙	1	2	3	4
619	7916906	7917608	7918309	7919011	7919712
620	7923917	7924617	7925318	7926018	7926718
621	7930916	7931615	7932314	7933014	7933712
622	7937904	7938602	7939300	7939998	7940696
623	7944880	7945578	7946274	7946971	7947668
624	7951846	7952542	7953238	7953933	7954629
625	7958800	7959495	7960190	7960884	7961578
626	7965743	7966437	7967131	7967824	7968517
627	7972675	7973368	7974060	7974753	7975445
628	7979596	7980288	7980979	7981671	7982362
629	7986506	7987197	7987887	7988577	7989267
630	7993405	7994097	7994784	7995473	7996162
631	8000294	8000982	8001670	8002358	8003045
632	8007171	8007858	8008545	8009232	8009919
633	8014037	8014723	8015409	8016095	8016781
634	8020893	8021578	8022262	8022947	8023632
635	8027737	8028421	8029105	8029789	8030472
636	8034571	8035254	8035937	8036619	8037302
637	8041394	8042076	8042758	8043439	8044121
638	8048207	8048887	8049568	8050248	8050929
639	8055009	8055688	8056368	8057047	8057726
640	8061800	8062478	8063157	8063835	8064513
641	8068580	8069258	8069935	8070612	8071290
642	8075350	8076027	8076703	8077379	8078055
643	8082110	8082785	8083460	8084136	8084811
644	8088859	8089533	8090207	8090881	8091555
645	8095597	8096270	8096944	8097617	8098290
646	8102325	8102997	8103670	8104342	8105013
647	8109043	8109714	8110385	8111056	8111727
648	8115750	8116420	8117090	8117760	8118430
649	8122447	8123116	8123785	8124454	8125123
650	8129134	8129802	8130470	8131138	8131805
651	8135810	8136477	8137144	8137811	8138478
652	8142476	8143142	8143808	8144474	8145140
653	8149132	8149797	8150462	8151127	8151791
654	8155777	8156441	8157105	8157769	8158433
655	8162413	8163076	8163739	8164402	8165064
656	8169038	8169700	8170362	8171024	8171686
657	8175654	8176315	8176976	8177636	8178297
658	8182259	8182919	8183579	8184239	8184898
659	8188854	8189513	8190172	8190831	8191489
660	8195439	8196097	8196755	8197413	8198071
661	8202015	8202672	8203328	8203987	8204642
662	8208580	8209236	8209892	8210548	8211203

Natural Numbers.	5	6	7	8	9
619	7920413	7921114	7921815	7922516	7923216
620	7927418	7928118	7928817	7929517	7930217
621	7934411	7935110	7935809	7936507	7937206
622	7941394	7942091	7942789	7943486	7944183
623	7948365	7949061	7949757	7950454	7951150
624	7955324	7956020	7956715	7957410	7958105
625	7962273	7962967	7963662	7964356	7965050
626	7969211	7969904	7970597	7971290	7971983
627	7976137	7976829	7977521	7978213	7978905
628	7983053	7983744	7984435	7985125	7985816
629	7989957	7990647	7991337	7992027	7992716
630	7996851	7997540	7998228	7998917	7999605
631	8003734	8004421	8005109	8005796	8006484
632	8010505	8011292	8011978	8012665	8013351
633	8017466	8018152	8018837	8019522	8020208
634	8024316	8025001	8025685	8026369	8027053
635	8031156	8031839	8032522	8033205	8033888
636	8037984	8038666	8039348	8040031	8040712
637	8044802	8045483	8046164	8046845	8047526
638	8051609	8052289	8052969	8053649	8054329
639	8058405	8059085	8059763	8060442	8061121
640	8065191	8065869	8066547	8067225	8067903
641	8071967	8072643	8073320	8073997	8074674
642	8078731	8079407	8080083	8080759	8081434
643	8085485	8086160	8086835	8087510	8088184
644	8092229	8092903	8093577	8094250	8094924
645	8098962	8099635	8100308	8100980	8101653
646	8105685	8106357	8107029	8107700	8108371
647	8112398	8113068	8113739	8114409	8115080
648	8119100	8119769	8120439	8121108	8121778
649	8125792	8126460	8127129	8127797	8128465
650	8132473	8133141	8133808	8134475	8135143
651	8139144	8139811	8140477	8141144	8141810
652	8145805	8146471	8147136	8147801	8148467
653	8152456	8153120	8153785	8154449	8155113
654	8159096	8159760	8160423	8161087	8161750
655	8165727	8166389	8167052	8167714	8168376
656	8172347	8173009	8173670	8174331	8174993
657	8178958	8179618	8180278	8180939	8181599
658	8185558	8186217	8186877	8187536	8188195
659	8192146	8192806	8193465	8194123	8194781
660	8198728	8199386	8200043	8200700	8201358
661	8205298	8205955	8206611	8207268	8207924
662	8211859	8212514	8213170	8213825	8214480

Natural Numbers.	0	1	2	3	4
663	8215135	8215790	8216445	8217100	8217755
664	8221681	8222335	8222989	8223643	8224296
665	8228216	8228869	8229522	8230175	8230828
666	8234742	8235394	8236046	8236698	8237350
667	8241258	8241909	8242560	8243211	8243862
668	8247765	8248415	8249065	8249715	8250364
669	8254261	8254910	8255559	8256208	8256857
670	8260748	8261396	8262044	8262692	8263340
671	8267225	8267872	8268519	8269166	8269813
672	8273693	8274339	8274985	8275631	8276277
673	8280151	8280796	8281441	8282086	8282731
674	8286599	8287243	8287887	8288532	8289176
675	8293038	8293681	8294324	8294967	8295611
676	8299467	8300109	8300752	8301394	8302036
677	8305887	8306528	8307169	8307811	8308452
678	8312297	8312937	8313578	8314218	8314858
679	8318698	8319337	8319977	8320616	8321255
680	8325089	8325728	8326366	8327005	8327643
681	8331471	8332109	8332746	8333384	8334021
682	8337844	8338480	8339117	8339754	8340390
683	8344207	8344843	8345479	8346114	8346750
684	8350561	8351196	8351831	8352465	8353100
685	8356906	8357540	8358174	8358807	8359441
686	8363241	8363874	8364507	8365140	8365773
687	8369567	8370199	8370832	8371463	8372095
688	8375884	8376516	8377147	8377778	8378409
689	8382192	8382822	8383453	8384083	8384713
690	8388491	8389120	8389750	8390379	8391008
691	8394780	8395409	8396037	8396666	8397294
692	8401061	8401688	8402316	8402943	8403571
693	8407332	8407959	8408586	8409212	8409838
694	8413595	8414220	8414846	8415472	8416097
695	8419848	8420473	8421098	8421722	8422347
696	8426092	8426716	8427340	8427964	8428588
697	8432328	8432951	8433574	8434197	8434819
698	8438554	8439176	8439798	8440420	8441042
699	8444772	8445393	8446014	8446635	8447256
700	8450980	8451601	8452221	8452841	8453461
701	8457180	8457800	8458419	8459038	8459658
702	8463371	8463990	8464608	8465227	8465845
703	8469553	8470171	8470789	8471406	8472024
704	8475727	8476343	8476960	8477577	8478193
705	8481891	8482507	8483123	8483739	8484355
706	8488047	8488662	8489277	8489892	8490507

Natural Numbers	5	6	7	8	9
663	8218409	8219064	8219718	8220372	8221027
664	8224950	8225603	8226257	8226910	8227563
665	8231481	8232133	8232786	8233438	8234090
666	8238002	8238653	8239305	8239956	8240607
667	8244513	8245163	8245814	8246464	8247114
668	8251014	8251664	8252313	8252963	8253612
669	8257506	8258154	8258803	8259451	8260100
670	8263988	8264635	8265283	8265931	8266578
671	8270460	8271107	8271753	8272400	8273046
672	8276923	8277569	8278214	8278860	8279505
673	8283376	8284021	8284665	8285310	8285955
674	8289820	8290463	8291107	8291751	8292394
675	8296254	8296896	8297539	8298182	8298824
676	8302678	8303320	8303962	8304603	8305245
677	8309093	8309734	8310375	8311016	8311656
678	8315499	8316139	8316778	8317418	8318058
679	8321895	8322534	8323173	8323812	8324450
680	8328281	8328919	8329558	8330195	8330833
681	8334659	8335296	8335933	8336570	8337207
682	8341027	8341663	8342299	8342937	8343571
683	8347385	8348021	8348656	8349291	8349926
684	8353735	8354369	8355003	8355638	8356272
685	8360075	8360708	8361341	8361975	8362608
686	8366405	8367038	8367670	8368303	8368935
687	8372727	8373359	8373990	8374622	8375253
688	8379039	8379670	8380301	8380931	8381562
689	8385343	8385973	8386602	8387232	8387861
690	8391637	8392266	8392895	8393523	8394152
691	8397922	8398550	8399178	8399806	8400433
692	8404198	8404825	8405452	8406079	8406706
693	8410465	8411091	8411717	8412343	8412969
694	8416722	8417348	8417973	8418598	8419223
695	8422971	8423596	8424220	8424844	8425468
696	8429211	8429735	8430458	8431081	8431705
697	8435442	8436065	8436687	8437310	8437932
698	8441664	8442286	8442907	8443529	8444150
699	8447877	8448498	8449119	8449739	8450360
700	8454081	8454701	8455321	8455941	8456561
701	8460277	8460896	8461515	8462134	8462752
702	8466463	8467081	8467700	8468318	8468935
703	8472641	8473258	8473876	8474493	8475110
704	8478810	8479426	8480043	8480659	8481275
705	8484970	8485586	8486201	8486817	8487432
706	8491122	8491736	8492351	8492965	8493580

Natural Numbers.	0	1	2	3	4
707	8494194	8494808	8495423	8496037	8496651
708	8500333	8500946	8501559	8502172	8502786
709	8506462	8507075	8507687	8508300	8508912
710	8512583	8513195	8513807	8514418	8515030
711	8518696	8519307	8519917	8520528	8521139
712	8524800	8525410	8526020	8526629	8527239
713	8530895	8531504	8532113	8532722	8533331
714	8536982	8537590	8538198	8538806	8539414
715	8543060	8543668	8544275	8544882	8545489
716	8549130	8549737	8550343	8550949	8551556
717	8555192	8555797	8556403	8557008	8557614
718	8561244	8561849	8562454	8563059	8563663
719	8567289	8567893	8568497	8569101	8569704
720	8573325	8573928	8574531	8575134	8575737
721	8579353	8579955	8580557	8581159	8581761
722	8585372	8585973	8586575	8587176	8587777
723	8591383	8591984	8592584	8593185	8593785
724	8597386	8597985	8598585	8599185	8599784
725	8603380	8603979	8604578	8605177	8605776
726	8609366	8609964	8610562	8611160	8611758
727	8615344	8615941	8616539	8617136	8617733
728	8621314	8621910	8622507	8623103	8623699
729	8627275	8627871	8628467	8629062	8629658
730	8633229	8633823	8634418	8635013	8635608
731	8639174	8639768	8640362	8640956	8641550
732	8645111	8645704	8646297	8646890	8647483
733	8651040	8651632	8652225	8652817	8653409
734	8656961	8657552	8658144	8658735	8659327
735	8662873	8663464	8664055	8664646	8665236
736	8668778	8669368	8669958	8670548	8671138
737	8674675	8675264	8675853	8676442	8677031
738	8680564	8681152	8681740	8682329	8682917
739	8686444	8687032	8687620	8688207	8688794
740	8692317	8692904	8693491	8694077	8694664
741	8698182	8698768	8699354	8699940	8700526
742	8704039	8704624	8705209	8705795	8706380
743	8709888	8710473	8711057	8711641	8712226
744	8715729	8716313	8716897	8717480	8718064
745	8721563	8722146	8722728	8723311	8723894
746	8727388	8727970	8728552	8729134	8729716
747	8733206	8733788	8734369	8734950	8735531
748	8739016	8739597	8740177	8740757	8741338
749	8744818	8745398	8745978	8746557	8747137
750	8750613	8751192	8751771	8752349	8752928

Natural Numbers.	5	6	7	8	9
707	8497264	8497878	8498492	8499106	8499719
708	8503399	8504011	8504624	8505237	8505850
709	8509524	8510136	8510748	8511360	8511972
710	8515641	8516252	8516863	8517474	8518085
711	8521749	8522359	8522970	8523580	8524190
712	8527849	8528458	8529058	8529677	8530286
713	8533940	8534548	8535157	8535765	8536374
714	8540022	8540630	8541238	8541845	8542453
715	8546096	8546703	8547310	8547917	8548524
716	8552162	8552768	8553374	8553980	8554586
717	8558219	8558824	8559429	8560035	8560640
718	8564268	8564872	8565476	8566081	8566685
719	8570308	8570912	8571515	8572118	8572722
720	8576340	8575943	8577545	8578148	8578750
721	8582363	8582965	8583567	8584169	8584770
722	8588379	8588980	8589581	8590181	8590782
723	8594385	8594986	8595586	8596186	8596786
724	8600384	8600983	8601583	8602182	8602781
725	8605374	8606973	8607571	8608170	8608768
726	8612356	8612954	8613552	8614149	8614747
727	8618330	8618927	8619524	8620120	8620717
728	8624296	8624892	8625488	8626084	8626679
729	8630253	8630848	8631443	8632039	8632634
730	8635202	8636797	8637391	8637985	8638580
731	8642143	8642737	8643331	8643924	8644517
732	8648076	8648669	8649262	8649855	8650447
733	8654001	8654593	8655185	8655777	8656369
734	8659918	8660509	8661100	8661691	8662282
735	8665827	8666417	8667008	8667598	8668188
736	8671728	8672317	8672907	8673496	8674086
737	8677620	8678209	8678798	8679387	8679975
738	8683505	8684093	8684681	8685269	8685857
739	8689382	8689969	8690556	8691143	8691730
740	8695251	8695837	8696423	8697010	8697596
741	8701112	8701697	8702283	8702868	8703454
742	8705965	8707549	8708134	8708719	8709304
743	8712810	8713394	8713978	8714562	8715146
744	8718647	8719230	8719814	8720397	8720980
745	8724476	8725059	8725641	8726224	8726806
746	8730298	8730880	8731461	8732043	8732625
747	8736112	8736693	8737274	8737855	8738435
748	8741918	8742498	8743078	8743658	8744238
749	8747716	8748296	8748875	8749454	8750034
750	8753507	8754086	8754664	8755243	8755821

Natural Numbers	0	1	2	3	4
751	8756399	8756978	8757556	8758134	8758712
752	8762178	8762756	8763333	8763911	8764488
753	8767950	8768526	8769103	8769680	8770256
754	8773713	8774289	8774865	8775441	8776017
755	8779469	8780045	8780620	8781195	8781770
756	8785218	8785792	8786367	8786941	8787515
757	8790959	8791532	8792106	8792680	8793253
758	8796692	8797265	8797838	8798411	8798983
759	8802418	8802990	8803562	8804134	8804705
760	8808136	8808707	8809279	8809850	8810421
761	8813847	8814417	8814988	8815558	8816129
762	8819550	8820120	8820689	8821259	8821829
763	8825245	8825815	8826384	8826953	8827522
764	8830934	8831502	8832070	8832639	8833207
765	8836614	8837182	8837750	8838317	8838885
766	8842288	8842855	8843421	8843988	8844555
767	8847954	8848520	8849086	8849652	8850218
768	8853612	8854178	8854743	8855308	8855874
769	8859263	8859828	8860393	8860957	8861522
770	8864907	8865471	8866035	8866599	8867163
771	8870544	8871107	8871670	8872233	8872796
772	8876173	8876736	8877298	8877860	8878423
773	8881795	8882357	8882918	8883480	8884042
774	8887410	8887971	8888532	8889093	8889653
775	8893017	8893577	8894138	8894698	8895258
776	8898617	8899177	8899736	8900296	8900855
777	8904210	8904769	8905328	8905887	8906445
778	8909796	8910354	8910912	8911470	8912028
779	8915375	8915932	8916489	8917047	8917604
780	8920946	8921503	8922059	8922616	8923173
781	8926510	8927066	8927622	8928178	8928734
782	8932068	8932623	8933178	8933733	8934288
783	8937618	8938172	8938727	8939281	8939836
784	8943161	8943715	8944268	8944822	8945376
785	8948697	8949250	8949803	8950356	8950909
786	8954225	8954778	8955330	8955883	8956435
787	8959747	8960299	8960851	8961403	8961954
789	8965262	8965813	8966364	8966915	8967466
789	8970770	8971320	8971871	8972421	8972971
790	8976271	8976821	8977370	8977920	8978469
791	8981765	8982314	8982863	8983412	8983960
792	8987252	8987800	8988348	8988897	8989445
793	8992732	8993279	8993827	8994375	8994922
794	8998205	8998752	8999299	8999846	9000392

Natural Numbers.	5	6	7	8	9
751	8759290	8759868	8760445	8761023	8761601
752	8765065	8765642	8766219	8766796	8767373
753	8770833	8771409	8771985	8772561	8773137
754	8776592	8777168	8777743	8778319	8778894
755	8782345	8782919	8783494	8784069	8784643
756	8788089	8788663	8789237	8789811	8790385
757	8793826	8794400	8794973	8795546	8796119
758	8799556	8800128	8800701	8801273	8801846
759	8805278	8805850	8806421	8806993	8807564
760	8810992	8811563	8812134	8812705.	8813275
761	8816699	8817269	8817840	8818410	8818980
762	8822398	8822968	8823537	8824107	8824676
763	8828090	8828659	8829228	8829797	8830365
764	8833775	8834343	8834911	8835479	8836047
765	8839452	8840019	8840586	8841154	8841721
766	8845122	8845688	8846255	8846821	8847387
767	8850784	8851350	8851915	8852481	8853047
768	8856439	8857004	8857569	8858134	8858699
769	8862086	8862651	8863215	8863779	8864343
770	8867726	8868290	8868854	8869417	8869980
771	8873359	8873922	8874485	8875048	8875610
772	8878985	8879547	8880109	8880671	8881233
773	8884603	8885165	8885726	8886287	8886848
774	8890214	8890775	8891336	8891896	8892457
775	8895818	8896378	8896935	8897498	8898058
776	8901415	8901974	8902533	8903092	8903651
777	8907004	8907562	8908121	8908679	8909238
778	8912586	8913144	8913702	8914259	8914817
779	8918161	8918718	8919275	8919832	8920389
780	8923729	8924285	8924842	8925398	8925954
781	8929290	8929846	8930401	8930957	8931512
782	8934843	8935398	8935953	8936508	8937063
783	8940390	8940944	8941498	8942053	8942607
784	8945929	8946483	8947037	8947590	8948143
785	8951462	8952015	8952567	8953120	8953673
786	8956987	8957539	8958092	8958644	8959195
787	8962506	8963057	8963608	8964160	8964711
788	8968017	8968568	8969118	8969669	8970219
789	8973521	8974071	8974621	8975171	8975721
790	8979019	8979568	8980117	8980667	8981216
791	8984519	8985058	8985606	8986155	8985703
792	8989993	8990541	8991089	8991636	8992184
793	8995469	8996017	8996564	8997111	8997658
794	9000939	9001486	9002032	9002579	9003125

Natural Numbers.	0	1	2	3	4
795	9003671	9004218	9004764	9005310	9005856
796	9009131	9009676	9010222	9010767	9011313
797	9014583	9015128	9015673	9016218	9016762
798	9020029	9020573	9021117	9021661	9022205
799	9025468	9026011	9026555	9027098	9027641
800	9030900	9031443	9031985	9032528	9033071
801	9036325	9036867	9037409	9037951	9038493
802	9041744	9042285	9042827	9043368	9043909
803	9047155	9047696	9048237	9048778	9049318
804	9052560	9053101	9053641	9054181	9054721
805	9057960	9058498	9059038	9059577	9060116
806	9063351	9063889	9064428	9064967	9065505
807	9068735	9069273	9069812	9070350	9070887
808	9074114	9074651	9075188	9075726	9076263
809	9079485	9080022	9080559	9081095	9081632
810	9084850	9085386	9085922	9086458	9086994
811	9090209	9090744	9091279	9091815	9092350
812	9095560	9096095	9096630	9097165	9097699
813	9100905	9101440	9101974	9102508	9103042
814	9106244	9106778	9107311	9107844	9108378
815	9111576	9112109	9112642	9113174	9113707
816	9116902	9117434	9117966	9118498	9119030
817	9122220	9122752	9123234	9123815	9124346
818	9127533	9128064	9128595	9129126	9129656
819	9132839	9133369	9133899	9134430	9134960
820	9138139	9138668	9139198	9139727	9140257
821	9143432	9143961	9144489	9145018	9145547
822	9148718	9149246	9149775	9150303	9150831
823	9153998	9154526	9155054	9155581	9156109
824	9159272	9159799	9160326	9160853	9161380
825	9164539	9165066	9165592	9166118	9166645
826	9169800	9170326	9170852	9171378	9171903
827	9175055	9175580	9176105	9176630	9177155
828	9180303	9180828	9181352	9181877	9182401
829	9185545	9186069	9186593	9187117	9187640
830	9190781	9191304	9191827	9192350	9192873
831	9196010	9196533	9197055	9197578	9198100
832	9201233	9201755	9202277	9202799	9203321
833	9206450	9206971	9207493	9208014	9208535
834	9211661	9212181	9212702	9213222	9213743
835	9216865	9217385	9217905	9218425	9218945
836	9222063	9222582	9223102	9223621	9224140
837	9227255	9227773	9228292	9228811	9229330
838	9232440	9232958	9233477	9233995	9234513

Natural Numbers	5	6	7	8	9
795	9006402	9006948	9007494	9008039	9008585
796	9011858	9012403	9012948	9013493	9014038
797	9017307	9017851	9018396	9018940	9019485
798	9022749	9023293	9023837	9024381	9024924
799	9028185	9028728	9029271	9029814	9030357
800	9033613	9034156	9034698	9035241	9035783
801	9039035	9039577	9040119	9040661	9041202
802	9044450	9044992	9045533	9046073	9046615
803	9049859	9050399	9050940	9051480	9052020
804	9055261	9055800	9056340	9056880	9057419
805	9060655	9061195	9061734	9062274	9062812
806	9066044	9066582	9067121	9067659	9068197
807	9071425	9071963	9072501	9073038	9073576
808	9076800	9077337	9077874	9078411	9078948
809	9082169	9082705	9083241	9083778	9084314
810	9087530	9088066	9088602	9089137	9089673
811	9092885	9093420	9093955	9094490	9095025
812	9098234	9098768	9099303	9099837	9100371
813	9103576	9104109	9104643	9105177	9105710
814	9108911	9109444	9109977	9110510	9111043
815	9114240	9114772	9115305	9115837	9116369
816	9119562	9120094	9120626	9121157	9121689
817	9124878	9125409	9125940	9126471	9127002
818	9130187	9130717	9131248	9131778	9132309
819	9135490	9136019	9136549	9137079	9137609
820	9140786	9141315	9141844	9142373	9142903
821	9146076	9146604	9147133	9147661	9148190
822	9151359	9151887	9152415	9152943	9153471
823	9156636	9157163	9157691	9158218	9158745
824	9161907	9162433	9162960	9163487	9164013
825	9167171	9167697	9168223	9168749	9169275
826	9172429	9172954	9173479	9174005	9174530
827	9177680	9178205	9178730	9179254	9179779
828	9182925	9183449	9183973	9184497	9185021
829	9188164	9188687	9189211	9189734	9190258
830	9193396	9193919	9194442	9194965	9195488
831	9198623	9199145	9199667	9200189	9200711
832	9203842	9204364	9204886	9205407	9205929
833	9209056	9209577	9210098	9210619	9211140
834	9214263	9214784	9215304	9215824	9216345
835	9219465	9219984	9220504	9221024	9221543
836	9224659	9225179	9225698	9226217	9226736
837	9229848	9230367	9230885	9231404	9231922
838	9235031	9235549	9236066	9236584	9237102

Natural Numbers.	0	1	2	3	4
839	9237620	9238137	9238655	9239172	9239690
840	9242793	9243310	9243827	9244344	9244860
841	9247960	9248476	9248993	9249509	9250025
842	9253121	9253637	9254152	9254668	9255184
843	9258276	9258791	9259306	9259821	9260336
844	9263424	9263939	9264453	9264968	9265482
845	9268567	9269081	9269595	9270109	9270622
846	9273704	9274217	9274730	9275243	9275757
847	9278834	9279347	9279859	9280372	9280885
848	9283959	9284471	9284983	9285495	9286007
849	9289077	9289588	9290100	9290611	9291123
850	9294189	9294700	9295211	9295722	9296233
851	9299296	9299806	9300316	9300826	9301336
852	9304396	9304906	9305415	9305925	9306434
853	9309490	9309999	9310508	9311017	9311526
854	9314579	9315087	9315596	9316104	9316612
855	9319661	9320169	9320677	9321185	9321692
856	9324738	9325245	9325752	9326259	9326767
857	9329808	9330315	9330822	9331328	9331835
858	9334873	9335379	9335885	9336391	9336897
859	9339932	9340437	9340943	9341448	9341953
860	9344984	9345489	9345994	9346499	9347004
861	9350032	9350536	9351040	9351544	9352049
862	9355073	9355576	9356080	9356584	9357087
863	9360108	9360611	9361114	9361617	9362120
864	9365137	9365640	9366143	9366645	9367148
865	9370161	9370663	9371165	9371667	9372169
866	9375179	9375680	9376182	9376683	9377184
867	9380191	9380692	9381193	9381693	9382194
868	9385197	9385697	9386198	9386698	9387198
869	9390198	9390697	9391197	9391697	9392196
870	9395193	9395692	9396191	9396690	9397189
871	9400182	9400680	9401179	9401677	9402176
872	9405165	9405663	9406161	9406659	9407157
873	9410142	9410640	9411137	9411635	9412132
874	9415114	9415611	9416108	9416605	9417101
875	9420081	9420577	9421073	9421569	9422065
876	9425041	9425537	9426032	9426528	9427024
877	9429996	9430491	9430986	9431481	9431976
878	9434945	9435440	9435934	9436429	9436923
879	9439889	9440383	9440877	9441371	9441865
880	9444827	9445320	9445814	9446307	9446800
881	9449759	9450252	9450745	9451238	9451730
882	9454686	9455178	9455671	9456163	9456655

Natural Numbers.	5	6	7	8	9
839	9240208	9240724	9241246	9241759	9242276
840	9245377	9245894	9246410	9246927	9247444
841	9250541	9251057	9251573	9252689	9252605
842	9255699	9256215	9256730	9257245	9257761
843	9260851	9261366	9261880	9262395	9262910
844	9265995	9266511	9267025	9267539	9268053
845	9271136	9271650	9272163	9272677	9273190
846	9276270	9276783	9277296	9277808	9278321
847	9281397	9281909	9282422	9282934	9283446
848	9286518	9287030	9287542	9288054	9288565
849	9291634	9292145	9292656	9293167	9293678
850	9296743	9297254	9297764	9298275	9298785
851	9301847	9302357	9302866	9303376	9303886
852	9306944	9307453	9307963	9308472	9308981
853	9312035	9312544	9313053	9313561	9314070
854	9317121	9317629	9318137	9318645	9319153
855	9322200	9322708	9323215	9323723	9324230
856	9327274	9327781	9328288	9328795	9329301
857	9332341	9332848	9333354	9333860	9334367
858	9337403	9337909	9338415	9338920	9339426
859	9342459	9342964	9343469	9343974	9344479
860	9347509	9348013	9348518	9349022	9349527
861	9352553	9353057	9353561	9354065	9354569
862	9357591	9358095	9358598	9359101	9359605
863	9362623	9363126	9363629	9364132	9364635
864	9367650	9368152	9368655	9369157	9369659
865	9372671	9373172	9373674	9374176	9374677
866	9377686	9378187	9378688	9379189	9379690
867	9382695	9383195	9383696	9384196	9384697
868	9387698	9388198	9388698	9389198	9389698
869	9392696	9393195	9393695	9394194	9394693
870	9397688	9398187	9398685	9399184	9399683
871	9402674	9403172	9403670	9404169	9404667
872	9407654	9408152	9408650	9409147	9409645
873	9412629	9413126	9413623	9414120	9414617
874	9417598	9418095	9418591	9419088	9419584
875	9422561	9423058	9423554	9424049	9424545
876	9427519	9428015	9428510	9429005	9429501
877	9432471	9432966	9433461	9433956	9434450
878	9437418	9437912	9438406	9438900	9439395
879	9442358	9442852	9443346	9443840	9444333
880	9447294	9447787	9448280	9448773	9449266
881	9452223	9452716	9453208	9453701	9454193
882	9457147	9457639	9458131	9458623	9459115

Natural Numbers.	0	1	2	3	4
883	9459607	9460099	9460591	9461082	9461574
884	9464523	9465014	9465505	9465996	9466487
885	9469433	9469923	9470414	9470905	9471395
886	9474337	9474827	9475317	9475807	9476297
887	9479236	9479726	9480215	9480705	9481194
888	9484130	9484619	9485108	9485597	9486085
889	9489018	9489506	9489994	9490483	9490971
890	9493900	9494388	9494876	9495364	9495852
891	9498777	9499264	9499752	9500239	9500726
892	9503649	9504135	9504622	9505109	9505596
893	9508515	9509001	9509487	9509973	9510459
894	9513375	9513861	9514347	9514832	9515318
895	9518230	9518716	9519201	9519686	9520171
896	9523080	9523565	9524049	9524534	9525018
897	9527924	9528409	9528893	9529377	9529861
898	9532763	9533247	9533730	9534214	9534697
899	9537597	9538080	9538563	9539046	9539529
900	9542425	9542908	9543390	9543872	9544355
901	9547248	9547730	9548212	9548694	9549176
902	9552065	9552547	9553028	9553510	9553991
903	9556877	9557358	9557839	9558320	9558801
904	9561684	9562165	9562645	9563125	9563605
905	9566486	9566966	9567445	9567925	9568405
906	9571282	9571761	9572241	9572720	9573199
907	9576073	9576552	9577030	9577509	9577988
908	9580858	9581337	9581815	9582293	9582771
909	9585639	9586117	9586594	9587072	9587549
910	9590414	9590891	9591368	9591845	9592322
911	9595184	9595660	9596137	9596614	9597090
912	9599948	9600425	9600901	9601377	9601853
913	9604708	9605183	9605659	9606135	9606610
914	9609462	9609937	9610412	9610887	9611362
915	9614211	9614686	9615160	9615635	9616109
916	9618955	9619429	9619903	9620377	9620851
917	9623693	9624167	9624640	9625114	9625587
918	9628427	9628900	9629373	9629846	9630319
919	9633155	9633628	9634100	9634573	9635045
920	9637878	9638350	9638822	9639294	9639766
921	9642596	9643068	9643539	9644011	9644482
922	9647309	9647780	9648251	9648722	9649193
923	9652017	9652488	9652958	9653428	9653899
924	9656720	9657190	9657660	9658130	9658599
925	9661417	9661887	9662356	9662826	9663295
926	9666110	9667579	9667048	9667517	9667985

Natural Numbers.	5	6	7	8	9
883	9462066	9462557	9463048	9463540	9464031
884	9466978	9467469	9467960	9468451	9468942
885	9471886	9472376	9472866	9473357	9473847
886	9476787	9477277	9477767	9478257	9478747
887	9481684	9482173	9482662	9483151	9483641
888	9486574	9487063	9487552	9488040	9488529
889	9491460	9491948	9492436	9492924	9493412
890	9496330	9496827	9497314	9497802	9498290
891	9501213	9501701	9502188	9502675	9503162
892	9506082	9506569	9507055	9507542	9508028
893	9510946	9511432	9511918	9512404	9512889
894	9515803	9516289	9516774	9517260	9517745
895	9520656	9521141	9521626	9522111	9522595
896	9525503	9525987	9526472	9526956	9527440
897	9530345	9530828	9531312	9531796	9532280
898	9535181	9535664	9536147	9536631	9537114
899	9540012	9540494	9540977	9541460	9541943
900	9544837	9545319	9545802	9546284	9546766
901	9549657	9550139	9550621	9551102	9551584
902	9554472	9554953	9555434	9555915	9556397
903	9559282	9559762	9560243	9560723	9561204
904	9564086	9564566	9565046	9565526	9566006
905	9568885	9569364	9569844	9570323	9570803
906	9573678	9574157	9574636	9575115	9575594
907	9578466	9578945	9579423	9579902	9580380
908	9583249	9583727	9584205	9584683	9585161
909	9588027	9588505	9588982	9589459	9589937
910	9592799	9593276	9593754	9594230	9594707
911	9597567	9598043	9598520	9598996	9599472
912	9602329	9602805	9603280	9603756	9604232
913	9607086	9607561	9608036	9608511	9608987
914	9611837	9612312	9612787	9613261	9613736
915	9616583	9617058	9617532	9618006	9618481
916	9621325	9621799	9622272	9622746	9623220
917	9626061	9626534	9627007	9627481	9627954
918	9630792	9631264	9631737	9632210	9632683
919	9635517	9635990	9636462	9636934	9637406
920	9640238	9640710	9641181	9641653	9642125
921	9644953	9645425	9645896	9646367	9646838
922	9649664	9650134	9650605	9651076	9651546
923	9654369	9654839	9655309	9655780	9656250
924	9659069	9659539	9660009	9660478	9660948
925	9663764	9664233	9664703	9665172	9665641
926	9668454	9668923	9669392	9669860	9670329

Natural Numbers.	0	1	2	3	4
927	9670797	9671266	9671734	9672203	9672671
928	9675480	9675948	9676416	9676883	9677351
929	9680157	9680625	9681092	9681559	9682027
930	9684829	9685296	9685763	9686230	9686697
931	9689497	9689963	9690430	9690896	9691362
932	9694159	9694625	9695091	9695557	9696023
933	9698816	9699282	9699747	9700213	9700678
934	9703469	9703934	9704399	9704863	9705328
935	9708116	9708581	9709045	9709509	9709974
936	9712758	9713222	9713686	9714150	9714614
937	9717396	9717859	9718323	9718786	9719249
938	9722028	9722491	9722954	9723417	9723880
939	9726656	9727118	9727581	9728043	9728506
940	9731278	9731741	9732202	9732664	9733126
941	9735896	9736358	9736819	9737281	9737742
942	9740509	9740970	9741431	9741892	9742353
943	9745117	9745577	9746038	9746498	9746959
944	9749720	9750180	9750640	9751100	9751560
945	9754318	9754778	9755237	9755695	9756156
946	9758911	9759370	9759829	9760288	9760747
947	9763500	9763958	9764417	9764875	9765334
948	9768083	9768541	9768999	9769457	9769915
949	9772662	9773120	9773577	9774035	9774492
950	9777236	9777693	9778150	9778607	9779064
951	9781805	9782262	9782718	9783175	9783631
952	9786369	9786826	9787282	9787738	9788194
953	9790929	9791385	9791840	9792296	9792751
954	9795484	9795939	9796394	9796849	9797304
955	9800034	9800488	9800943	9801398	9801852
956	9804579	9805033	9805487	9805942	9806396
957	9809119	9809573	9810027	9810481	9810934
958	9813655	9814108	9814562	9815015	9815468
959	9818186	9818639	9819092	9819544	9819997
960	9822712	9823165	9823617	9824069	9824522
961	9827234	9827686	9828138	9828589	9829041
962	9831751	9832202	9832654	9833105	9833556
963	9836263	9836714	9837165	9837616	9838066
964	9840770	9841221	9841671	9842122	9842572
965	9845273	9845723	9846173	9846623	9847073
966	9849771	9850221	9850670	9851120	9851569
967	9854265	9854714	9855163	9855612	9856061
968	9858754	9859202	9859651	9860099	9860548
969	9863238	9863686	9864134	9864582	9865030
970	9867717	9868165	9868613	9869060	9869508

Natural Numbers.	5	6	7	8	9
927	9673139	9673607	9674076	9674544	9675012
928	9677819	9678287	9678754	9679222	9679690
929	9682494	9682961	9683428	9683895	9684362
930	9687164	9687630	9688097	9688564	9689030
931	9691829	9692295	9692761	9693227	9693693
932	9696488	9696954	9697420	9697885	9698351
933	9701143	9701608	9702074	9702539	9703004
934	9705793	9706258	9706722	9707187	9707652
935	9710438	9710902	9711366	9711830	9712294
936	9715078	9715542	9716005	9716469	9716932
937	9719713	9720176	9720639	9721102	9721565
938	9724343	9724805	9725268	9725731	9726193
939	9728968	9729430	9729892	9730354	9730816
940	9733588	9734050	9734511	9734973	9735435
941	9738203	9738664	9739126	9739587	9740048
942	9742814	9743274	9743735	9744196	9744656
943	9747419	9747879	9748340	9748800	9749260
944	9752020	9752479	9752939	9753399	9753858
945	9756615	9757075	9757534	9757993	9758452
946	9761206	9761665	9762124	9762582	9763041
947	9765792	9766251	9766709	9767167	9767625
948	9770373	9770831	9771289	9771747	9772204
949	9774950	9775407	9775864	9776322	9776779
950	9779521	9779978	9780435	9780892	9781348
951	9784088	9784544	9785001	9785457	9785913
952	9788650	9789106	9789562	9790017	9790473
953	9793207	9793662	9794118	9794573	9795028
954	9797759	9798214	9798669	9799124	9799579
955	9802307	9802761	9803216	9803670	9804125
956	9806850	9807304	9807758	9808212	9808666
957	9811388	9811841	9812295	9812748	9813202
958	9815921	9816374	9816827	9817280	9817733
959	9820450	9820902	9821355	9821807	9822260
960	9824974	9825426	9825878	9826330	9826782
961	9829493	9829945	9830396	9830848	9831299
962	9834007	9834459	9834910	9835361	9835812
963	9838517	9838968	9839419	9839869	9840320
964	9843022	9843473	9843923	9844373	9844823
965	9847523	9847973	9848422	9848872	9849322
966	9852019	9852468	9852917	9853366	9853816
967	9856510	9856959	9857407	9857856	9858305
968	9860996	9861445	9861893	9862341	9862790
969	9865478	9865926	9866374	9866822	9867270
970	9869955	9870403	9870850	9871298	9871745

Natural Numbers	0	1	2	3	4
971	9872192	9872640	9873087	9873534	9873981
972	9876663	9877109	9877556	9878003	9878449
973	9881128	9881575	9882021	9882467	9882913
974	9885590	9886035	9886481	9886927	9887373
975	9890046	9890492	9890937	9891382	9891828
976	9894498	9894943	9895388	9895833	9896278
977	9898946	9899390	9899835	9900279	9900723
978	9903389	9903833	9904277	9904721	9905164
979	9907827	9908270	9908714	9909158	9909601
980	9912261	9912704	9913147	9913590	9914033
981	9916690	9917133	9917575	9918018	9918461
982	9921115	9921557	9921999	9922441	9922884
983	9925535	9925977	9926419	9926860	9927302
984	9929951	9930392	9930834	9931275	9931716
985	9934362	9934803	9935244	9935685	9936126
986	9938769	9939210	9939650	9940090	9940531
987	9943172	9943612	9944051	9944491	9944931
988	9947569	9948009	9948448	9948888	9949327
989	9951963	9952402	9952841	9953280	9953719
990	9956352	9956791	9957229	9957668	9958106
991	9960737	9961175	9961613	9962051	9962489
992	9965117	9965554	9965992	9966430	9966868
993	9969492	9969930	9970367	9970804	9971242
994	9973864	9974301	9974738	9975174	9975611
995	9978231	9978667	9979104	9979540	9979976
996	9982593	9983029	9983465	9983901	9984337
997	9986952	9987387	9987823	9988258	9988694
998	9991305	9991740	9992176	9992611	9993046
999	9995655	9996090	9996524	9996959	9997393

Natural Numbers.	5	6	7	8	9
971	9874428	9874875	9875322	9875769	9876216
972	9878896	9879343	9879789	9880236	9880682
973	9883360	9883806	9884252	9884698	9885144
974	9887818	9888264	9888710	9889155	9889601
975	9892273	9892718	9893163	9893608	9894050
976	9896722	9897167	9897612	9898056	9898501
977	9901168	9901612	9902056	9902500	9902944
978	9905608	9906052	9906496	9906940	9907383
979	9910044	9910488	9910931	9911374	9911818
980	9914476	9914919	9915362	9915805	9916247
981	9918903	9919345	9919788	9920230	9920673
982	9923326	9923768	9924210	9924651	9925093
983	9927744	9928185	9928627	9929068	9929510
984	9932157	9932598	9933039	9933480	9933921
985	9936566	9937007	9937448	9937888	9938329
986	9940971	9941411	9941851	9942291	9942731
987	9945371	9945811	9946251	9946690	9947130
988	9949767	9950206	9950645	9951085	9951524
989	9954158	9954597	9955036	9955474	9955913
990	9958545	9958983	9959422	9959860	9960298
991	9962927	9963365	9963803	9964241	9964679
992	9967305	9967743	9968180	9968618	9969055
993	9971679	9972116	9972553	9972990	9973427
994	9976048	9976485	9976921	9977358	9977794
995	9980413	9980849	9981285	9981721	9982157
996	9984773	9985209	9985645	9986080	9986516
997	9989129	9989564	9990000	9990435	9990870
998	9993481	9993916	9994350	9994785	9995220
999	9997828	9998262	9998697	9999131	9999566

10000 it's Log.=4.0000000

The End of the Table of Logarithms.

A TABLE OF THE Proportional Parts OF THE DIFFERENCES OF LOGARITHMS.

By which the foregoing TABLE is made to extend to the Logarithm of 100000, &c.

Differences of Logarithms.	Tenth Parts of those Differences. (VIZ.)								
	1	2	3	4	5	6	7	8	9
435	44	87	130	174	218	261	305	348	392
436	44	87	131	174	218	262	305	349	392
438	44	88	131	175	219	263	307	350	394
440	44	88	132	176	220	264	308	352	396
442	44	88	133	177	221	265	309	354	398
444	44	89	133	178	222	266	311	355	400
446	45	89	134	178	223	268	312	357	401
448	45	90	134	179	224	269	314	358	403
450	45	90	135	180	225	270	315	360	405
452	45	90	136	181	226	271	316	362	407
454	45	91	136	182	227	272	318	363	409
456	46	91	137	182	228	274	319	365	410
458	46	92	137	183	229	275	321	366	412

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of thoſe Differences. 1	2	3	4	5	6	7	8	9
460	46	92	138	184	230	276	322	368	414
462	46	92	139	185	231	277	323	370	416
464	46	93	139	186	232	278	325	371	418
466	47	93	140	186	233	280	326	373	419
468	47	94	140	187	234	281	328	374	421
470	47	94	141	188	235	282	329	376	423
472	47	94	142	189	236	283	330	378	425
474	47	95	142	190	237	284	332	379	427
476	48	95	143	190	238	286	333	381	428
478	48	96	143	191	239	287	335	382	430
480	48	96	144	192	240	288	336	384	432
482	48	96	145	193	241	289	337	386	434
484	48	97	145	194	242	290	339	387	436
486	49	97	146	194	243	292	340	389	437
488	49	98	146	195	244	293	342	390	439
490	49	98	147	196	245	294	343	392	441
492	49	98	148	197	246	295	344	394	443
494	49	99	148	198	247	296	346	395	445
496	50	99	149	198	248	298	347	397	446
498	50	100	149	199	249	299	348	398	448
500	50	100	150	200	250	300	350	400	450
502	50	100	151	201	251	301	351	402	452
504	50	101	151	202	252	302	353	403	454
506	51	101	152	202	253	304	354	405	455
508	51	102	152	203	254	305	356	406	457
510	51	102	153	204	255	306	357	408	459
512	51	102	154	205	256	307	358	410	461
514	51	103	154	206	257	308	360	411	463
516	52	103	155	206	258	310	361	413	464
518	52	104	155	207	259	311	363	414	466
520	52	104	156	208	260	312	364	416	468
522	52	104	157	209	261	313	365	418	470
524	52	105	157	210	262	314	367	419	472
526	53	105	158	210	263	316	368	421	473
528	53	106	158	211	264	317	370	422	475
530	53	106	159	212	265	318	371	424	477
532	53	106	160	213	266	319	372	426	479
534	53	107	160	214	267	320	374	427	481
536	54	107	161	214	268	322	375	429	482
538	54	108	161	215	269	323	377	430	484

A TABLE of Proportional Parts.

Tenth Parts of thoſe Differences.

Diff. of Logar.	1	2	3	4	5	6	7	8	9
540	54	108	162	216	270	324	378	432	486
542	54	108	163	217	271	325	379	434	488
544	54	109	163	218	272	326	381	435	490
546	55	109	164	218	273	328	382	437	491
548	55	110	164	219	274	329	384	438	493
550	55	110	165	220	275	330	385	440	495
552	55	110	166	221	276	331	386	442	497
554	55	111	166	222	277	332	388	443	499
556	56	111	167	222	278	334	389	445	500
558	56	112	167	223	279	335	391	446	502
560	56	112	168	224	280	336	392	448	504
562	56	112	169	225	281	337	393	450	506
564	56	113	169	226	282	338	395	451	508
566	57	113	170	226	283	340	396	453	509
568	57	114	170	227	284	341	398	454	511
570	57	114	171	228	285	342	399	456	513
572	57	114	172	229	286	343	400	458	515
574	57	115	172	230	287	344	401	459	516
576	58	115	173	230	288	346	403	461	518
578	58	116	173	231	289	347	405	462	520
580	58	116	174	232	290	348	406	464	522
582	58	116	175	233	291	349	407	466	524
584	58	117	175	234	292	350	409	467	526
586	59	117	176	234	293	352	410	469	527
588	59	118	176	235	294	353	412	470	529
590	59	118	177	236	295	354	413	472	531
592	59	118	178	237	296	355	414	474	533
594	59	119	178	238	297	356	416	475	535
596	60	119	179	238	298	358	417	477	536
598	60	120	179	239	299	359	419	478	538
600	60	120	180	240	300	360	420	480	540
602	60	120	181	241	301	361	421	482	542
604	60	121	181	242	302	362	423	483	544
606	61	121	182	242	303	364	424	485	545
608	61	122	182	243	304	365	426	486	547
610	61	122	183	244	305	366	427	488	549
612	61	122	184	245	306	367	428	490	551
614	61	123	184	246	307	368	430	492	553
616	62	123	185	246	308	370	431	493	554
618	62	124	185	247	309	371	433	494	556

A TABLE of Proportional Parts.

Diff of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
620	62	124	186	248	310	372	434	496	558
622	62	124	187	249	311	373	435	498	560
624	62	125	187	250	312	374	437	499	562
626	63	125	188	250	313	376	438	501	563
628	63	126	188	251	314	377	440	502	565
630	63	126	189	252	315	378	441	504	567
632	63	126	190	253	316	379	442	506	569
634	63	127	190	254	317	380	444	507	571
636	64	127	191	254	318	382	445	509	572
638	64	128	191	255	319	383	447	510	574
640	64	128	192	256	320	384	448	512	576
642	64	128	193	257	321	385	449	514	578
644	64	129	193	258	322	386	451	515	580
646	65	129	194	258	323	388	452	517	581
648	65	130	194	259	324	389	454	518	583
650	65	130	195	260	325	390	455	520	585
652	65	130	196	261	326	391	456	522	587
654	65	131	196	262	327	392	458	523	589
656	66	131	197	262	328	394	459	525	590
658	66	132	197	263	329	395	461	526	592
660	66	132	198	264	330	396	462	528	594
662	66	132	199	265	331	397	463	530	596
664	66	133	199	266	332	398	465	531	598
666	67	133	200	266	333	400	466	533	599
668	67	134	200	267	334	401	468	534	601
670	67	134	201	268	335	402	469	536	603
672	67	134	202	269	336	403	470	538	605
674	67	135	202	270	337	404	472	539	607
676	68	135	203	270	338	406	473	541	608
678	68	136	203	271	339	407	475	542	610
680	68	136	204	272	340	408	476	544	612
982	68	136	205	273	341	409	477	546	614
684	68	137	205	274	342	410	479	547	616
686	69	137	206	274	343	412	480	549	617
688	69	138	206	275	344	413	482	550	619
690	69	138	207	276	345	414	483	552	621
692	69	138	208	277	346	415	484	554	623
694	69	139	208	278	347	416	486	555	625
696	70	139	209	278	348	417	487	557	626
698	70	140	209	279	349	419	489	558	628

A TABLE of Proportional Parts.

Tenth Parts of thoſe Differences.

Diff. of Logar.	1	2	3	4	5	6	7	8	9
700	70	140	210	280	350	420	490	560	630
702	70	140	211	281	351	421	491	562	632
704	70	141	211	282	352	422	492	563	634
706	71	141	212	282	353	424	494	565	635
708	71	142	212	283	354	425	496	566	637
710	71	142	213	284	355	426	497	568	639
712	71	142	214	285	356	427	498	570	641
714	71	143	214	286	357	428	500	571	643
716	72	143	215	286	358	430	501	573	644
718	72	144	215	287	359	431	503	574	646
720	72	144	216	288	360	432	504	576	648
722	72	144	217	289	361	433	505	578	650
724	72	145	217	290	362	434	507	579	652
726	73	145	218	290	363	436	508	581	653
728	73	146	218	291	364	437	510	582	655
730	73	146	219	292	365	438	511	584	657
732	73	146	220	293	366	439	512	586	659
734	73	147	220	294	367	440	514	587	661
736	74	147	221	294	368	442	515	589	662
738	74	148	221	295	369	443	517	590	664
740	74	148	222	296	370	444	518	592	666
742	74	148	223	297	371	445	519	594	668
744	74	149	223	298	372	446	521	595	670
746	75	149	224	298	373	448	522	597	671
748	75	150	224	299	374	449	524	598	673
750	75	150	225	300	375	450	525	600	675
752	75	150	226	301	376	451	526	602	677
754	75	151	226	302	377	452	528	603	679
756	76	151	227	302	378	454	529	605	680
758	76	152	227	303	379	455	531	606	682
760	76	152	228	304	380	456	532	608	684
762	76	152	229	305	381	457	533	610	686
764	76	153	229	306	382	458	535	611	688
766	77	153	230	306	383	460	536	613	689
768	77	154	230	307	384	461	538	614	691
770	77	154	231	308	385	462	539	616	693
772	77	154	232	309	386	463	540	618	695
774	77	155	232	310	387	464	542	619	697
776	78	155	233	310	388	466	543	621	699
778	78	156	233	311	389	467	545	622	700

A TABLE of Proportional Parts.

Diff of Logar.	Tenth Parts of thoſe Differences.								
	1	2	3	4	5	6	7	8	9
780	78	156	234	312	390	468	546	624	702
782	78	156	235	313	391	469	547	626	704
784	78	157	235	314	392	470	549	627	706
786	79	157	236	314	393	472	550	629	707
788	79	158	236	315	394	473	552	630	709
790	79	158	237	316	395	474	553	632	711
792	79	158	238	317	396	475	554	634	713
794	79	159	238	318	397	477	556	635	715
796	80	159	239	318	398	478	557	637	717
798	80	160	239	319	399	479	559	638	718
800	80	160	240	320	400	480	560	640	720
802	80	160	241	321	401	481	561	642	722
804	80	161	241	322	402	482	563	643	724
806	81	161	242	322	403	484	564	645	725
808	81	162	242	323	404	485	566	645	727
810	81	162	243	324	405	486	567	648	729
812	81	162	244	325	406	487	568	650	731
814	81	163	244	326	407	488	570	651	733
816	82	163	245	326	408	490	571	653	734
818	82	164	245	327	409	491	572	654	736
820	82	164	246	328	410	492	574	656	738
822	82	164	247	329	411	493	575	658	740
824	82	165	247	330	412	494	577	659	742
826	83	165	248	330	413	496	578	661	743
828	83	166	248	331	414	497	580	662	745
830	83	166	249	332	415	498	581	664	747
832	83	166	250	333	416	499	582	666	749
834	83	167	250	334	417	500	584	667	751
836	84	167	251	334	418	502	585	669	753
838	84	168	251	335	419	503	587	670	754
840	84	168	252	336	420	504	588	672	756
842	84	168	253	337	421	505	589	674	758
844	84	169	253	338	422	506	591	675	760
846	85	169	254	338	423	508	592	677	761
848	85	170	254	339	424	509	594	678	763
850	85	170	255	340	425	510	595	680	765
852	85	170	256	341	426	511	596	682	767
854	85	171	256	342	427	512	598	683	769
856	86	171	257	342	428	514	599	685	771
858	86	172	257	343	429	515	601	686	773

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of thoſe Differences.								
	1	2	3	4	5	6	7	8	9
860	86	172	258	344	430	516	602	688	774
862	86	172	259	345	431	517	603	690	776
864	86	173	259	346	432	518	605	691	778
866	87	173	260	346	433	520	606	693	779
868	87	174	260	347	434	521	608	694	781
870	87	174	261	348	435	522	609	696	783
872	87	174	262	349	436	523	611	698	785
874	87	175	262	350	437	524	612	699	787
876	88	175	263	350	438	526	613	701	789
878	88	176	263	351	439	527	615	702	791
880	88	176	264	352	440	528	616	704	793
882	88	176	265	353	441	529	617	706	795
884	88	177	265	354	442	530	619	707	796
886	89	177	266	354	443	532	620	709	797
888	89	178	266	355	444	533	622	710	799
890	89	178	267	356	445	534	623	712	801
892	89	178	268	357	446	535	624	714	803
894	89	179	268	358	447	536	626	715	805
896	90	179	269	358	448	538	627	717	806
898	90	180	269	359	449	539	629	718	808
900	90	180	270	360	450	540	630	720	810
903	90	181	271	361	451	542	632	722	813
906	91	181	272	363	453	544	634	725	815
909	91	182	273	364	454	545	636	727	818
912	91	182	274	365	456	547	638	730	821
915	91	183	274	366	457	549	640	732	823
918	92	184	275	367	459	551	643	734	826
921	92	184	276	368	460	553	645	737	829
924	92	185	277	370	462	554	647	739	832
927	93	185	278	371	463	556	649	742	834
930	93	186	279	372	465	558	651	744	837
933	93	187	280	373	466	560	653	746	839
936	94	187	281	374	468	562	655	749	842
939	94	188	282	376	469	563	657	751	845
942	94	188	283	377	471	565	659	754	848
945	94	189	283	378	472	567	661	756	850
948	95	190	284	379	474	569	664	758	853
951	95	190	285	380	475	571	666	761	856
954	95	191	286	382	477	572	668	763	859
957	96	191	287	383	478	574	670	766	861

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of thoſe Differences. 1	2	3	4	5	6	7	8	9
960	96	192	288	384	480	576	672	768	864
963	96	193	289	385	481	578	674	770	867
966	97	193	290	386	483	580	676	773	869
969	97	194	291	388	484	581	678	775	872
972	97	194	292	389	486	583	680	778	875
975	98	195	292	390	487	585	682	780	878
978	98	196	293	391	489	587	685	782	880
981	98	196	294	392	490	589	687	785	883
984	98	197	295	394	492	590	689	787	886
987	99	197	296	395	493	592	691	790	888
990	99	198	297	396	495	594	693	792	891
993	99	199	298	397	496	596	695	794	894
996	100	199	299	398	498	598	697	797	897
999	100	200	300	400	499	599	699	799	899
1005	101	201	301	402	502	603	703	804	904
1009	101	202	303	404	504	605	706	807	908
1013	101	203	304	405	506	608	709	810	912
1017	102	203	305	407	508	610	712	814	915
1021	102	204	306	408	510	613	715	817	919
1025	103	205	307	410	512	615	717	820	923
1029	103	206	309	412	514	617	720	823	926
1033	103	207	310	413	516	620	723	826	930
1037	104	207	311	415	518	622	726	830	933
1041	104	208	312	416	520	625	729	833	937
1045	105	209	313	418	522	627	731	836	941
1049	105	210	315	420	524	629	734	839	944
1053	105	211	316	421	526	632	737	842	948
1057	106	211	317	423	528	634	740	846	951
1061	106	212	318	424	530	637	743	849	955
1065	107	213	319	426	532	639	745	852	959
1069	107	214	321	428	534	641	748	855	962
1073	107	215	322	429	536	644	751	858	966
1077	108	215	323	431	538	646	754	862	969
1081	108	216	324	432	540	649	757	865	973
1085	109	217	325	434	542	651	759	868	977
1089	109	218	327	436	544	653	762	871	980
1093	110	219	328	437	546	656	765	874	984
1097	110	219	329	439	548	658	768	878	987
1101	110	220	330	440	550	661	771	881	991
1105	111	221	331	442	552	663	773	884	994

A TABLE of Proportional Parts.

Diff of Logar.	Tenth Parts of thoſe Differences.								
	1	2	3	4	5	6	7	8	9
1109	111	222	333	444	554	665	776	887	998
1113	111	223	334	445	556	668	779	890	1002
1117	112	223	335	447	558	670	782	894	1005
1121	112	224	336	448	560	673	785	897	1009
1125	113	225	337	450	562	675	787	900	1013
1129	113	226	339	452	564	677	790	903	1016
1133	113	227	340	453	566	680	793	906	1020
1137	114	227	341	455	568	682	796	910	1023
1141	114	228	342	456	570	685	799	913	1027
1145	115	229	343	458	572	687	801	916	1031
1149	115	230	345	460	574	689	804	919	1034
1153	115	231	346	461	576	692	807	922	1038
1157	116	231	347	463	578	694	810	926	1041
1161	116	232	348	464	580	697	813	929	1045
1165	117	233	349	466	582	699	815	932	1049
1169	117	234	351	468	584	701	818	935	1052
1173	117	235	352	469	586	704	821	938	1056
1177	118	235	353	471	588	706	824	942	1059
1181	118	236	354	472	590	709	827	945	1063
1185	118	237	355	474	592	711	829	948	1067
1189	119	238	357	476	594	713	832	951	1070
1193	119	239	358	477	596	716	835	954	1074
1197	120	239	359	479	598	718	838	958	1077
1201	120	240	360	480	600	721	841	961	1081
1205	120	241	361	482	602	723	843	964	1084
1209	121	242	363	484	604	725	846	967	1088
1213	121	243	364	485	606	728	849	970	1092
1217	122	243	365	487	608	730	852	974	1095
1221	122	244	366	488	610	732	855	977	1099
1225	122	245	367	490	612	735	857	980	1102
1229	123	246	369	492	614	737	860	983	1106
1233	123	247	370	493	616	740	863	986	1110
1237	124	247	371	495	618	742	866	990	1113
1241	124	248	372	496	620	745	869	993	1117
1245	124	249	373	498	622	747	871	996	1120
1249	125	250	375	500	624	749	874	999	1124
1253	125	251	376	501	626	752	877	1002	1128
1257	126	251	377	503	628	754	880	1006	1131
1260	126	252	378	504	630	756	882	1008	1134
1263	126	253	379	505	631	758	884	1010	1137

A TABLE of Proportional Parts.

Diff of Logar.	Tenth Parts of thoſe Differences.								
	1	2	3	4	5	6	7	8	9
1267	127	253	380	507	633	760	887	1014	1140
1271	127	254	381	508	635	763	890	1017	1144
1275	127	255	382	510	637	765	892	1020	1147
1279	128	256	384	512	639	767	895	1023	1151
1283	128	257	385	513	641	770	898	1026	1155
1287	129	257	386	515	643	772	901	1030	1158
1291	129	258	387	516	645	775	904	1033	1162
1295	129	259	388	518	647	777	905	1036	1165
1299	130	260	389	520	649	779	909	1039	1169
1303	130	261	391	521	651	782	912	1042	1173
1307	131	261	392	523	653	784	915	1046	1176
1311	131	262	393	524	655	787	918	1049	1180
1315	131	263	394	526	657	789	920	1052	1183
1319	132	264	396	528	659	791	923	1055	1187
1323	132	265	397	529	661	794	926	1058	1191
1327	133	265	398	531	663	796	929	1062	1194
1331	133	266	399	532	665	799	932	1065	1198
1335	133	267	400	534	667	801	934	1068	1201
1339	134	268	402	536	669	803	937	1071	1205
1343	134	269	403	537	671	806	940	1074	1209
1347	135	269	404	539	673	808	943	1078	1212
1351	135	270	405	540	675	811	946	1081	1216
1355	135	271	406	542	677	813	948	1084	1219
1359	136	272	408	544	679	815	951	1087	1223
1363	136	273	409	545	681	818	954	1090	1227
1367	137	273	410	547	683	820	957	1094	1230
1371	137	274	411	548	685	823	960	1097	1234
1375	137	275	412	550	687	825	962	1100	1237
1379	138	276	414	552	689	827	965	1103	1241
1383	138	277	415	553	691	830	968	1106	1245
1387	139	277	416	555	693	832	971	1110	1248
1391	139	278	417	556	695	835	974	1113	1252
1395	140	279	418	558	697	837	976	1116	1255
1399	140	280	420	559	699	839	979	1119	1259
1403	140	281	421	561	701	842	982	1122	1263
1407	141	281	422	563	703	844	985	1126	1266
1411	141	282	423	564	705	847	988	1129	1270
1415	141	283	424	566	707	849	990	1132	1273
1420	142	284	426	568	710	852	994	1136	1278
1425	142	285	427	570	712	855	997	1140	1282

A TABLE of Proportional Parts.

Tenth Parts of thoſe Differences.

Diff. of Logar.	1	2	3	4	5	6	7	8	9
1430	143	286	429	572	715	858	1001	1144	1287
1435	143	287	430	574	717	861	1004	1148	1291
1440	144	288	432	576	720	864	1008	1152	1296
1445	144	289	433	578	722	867	1011	1156	1300
1450	145	290	435	580	725	870	1015	1160	1305
1455	145	291	436	582	727	873	1018	1164	1309
1460	146	292	438	584	730	876	1022	1168	1314
1465	146	293	439	586	732	879	1025	1172	1318
1470	147	294	441	588	735	882	1029	1176	1323
1475	147	295	442	590	737	885	1032	1180	1327
1480	148	296	444	592	740	888	1036	1184	1332
1485	148	297	445	594	742	891	1039	1188	1336
1490	149	298	447	596	745	894	1043	1192	1341
1495	149	299	448	598	747	897	1046	1196	1345
1500	150	300	450	600	750	900	1050	1200	1350
1505	150	301	451	602	752	903	1053	1204	1354
1510	151	302	453	604	755	906	1057	1208	1359
1515	151	303	454	606	757	909	1060	1212	1363
1520	152	304	456	608	760	912	1064	1216	1368
1525	152	305	457	610	762	915	1067	1220	1372
1530	153	306	459	612	765	918	1071	1224	1377
1535	153	307	460	614	767	921	1074	1228	1381
1540	154	308	462	616	770	924	1078	1232	1386
1545	154	309	463	618	772	927	1081	1236	1390
1550	155	310	465	620	775	930	1085	1240	1395
1555	155	311	466	622	777	933	1088	1244	1399
1560	156	312	468	624	780	936	1092	1248	1404
1565	156	313	469	626	782	939	1095	1252	1408
1570	157	314	471	628	785	942	1099	1256	1413
1575	157	315	472	630	787	945	1102	1260	1417
1580	158	316	474	632	790	948	1106	1264	1422
1585	158	317	475	634	792	951	1109	1268	1426
1590	159	318	477	636	795	954	1113	1272	1431
1595	159	319	478	638	797	957	1116	1276	1435
1600	160	320	480	640	800	960	1120	1280	1440
1605	160	321	481	642	802	963	1123	1284	1444
1610	161	322	483	644	805	966	1127	1288	1449
1615	161	323	484	646	807	969	1130	1292	1453
1620	162	324	486	648	810	972	1134	1296	1458
1625	162	325	487	650	812	975	1137	1300	1462

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of thoſe Differences. 1	2	3	4	5	6	7	8	9
1630	163	326	489	652	815	978	1141	1304	1467
1635	163	327	490	654	817	981	1144	1308	1471
1640	164	328	492	656	820	984	1148	1312	1476
1645	164	329	493	658	822	987	1151	1316	1480
1650	165	330	495	660	825	990	1155	1320	1485
1655	165	331	496	662	827	993	1158	1324	1489
1660	166	332	498	664	830	996	1162	1328	1494
1665	167	333	499	666	832	999	1166	1332	1498
1670	167	334	501	668	835	1002	1169	1336	1503
1675	167	335	502	670	837	1005	1172	1340	1507
1680	168	336	504	672	840	1008	1176	1344	1512
1685	168	337	505	674	842	1011	1179	1348	1516
1690	169	338	507	676	845	1014	1183	1352	1521
1695	169	339	508	678	847	1017	1186	1356	1525
1700	170	340	510	680	850	1020	1190	1360	1530
1705	170	341	511	682	852	1023	1193	1364	1534
1710	171	342	513	684	855	1026	1197	1368	1539
1715	171	343	514	686	857	1029	1200	1372	1543
1720	172	344	516	688	860	1032	1204	1376	1548
1725	172	345	517	690	862	1035	1207	1380	1552
1730	173	346	519	692	865	1038	1211	1384	1557
1735	173	347	520	694	867	1041	1214	1388	1561
1740	174	348	522	696	870	1044	1218	1392	1566
1745	174	349	523	698	872	1047	1221	1396	1570
1750	175	350	525	700	875	1050	1225	1400	1575
1755	175	351	526	702	877	1053	1228	1404	1579
1760	176	352	528	704	880	1056	1232	1408	1584
1765	176	353	529	706	882	1059	1235	1412	1588
1770	177	354	531	708	885	1062	1239	1416	1593
1775	178	355	532	710	887	1065	1242	1420	1597
1780	178	356	534	712	890	1068	1246	1424	1602
1785	179	357	535	714	892	1071	1249	1428	1606
1790	179	358	537	716	895	1074	1253	1432	1611
1795	180	359	538	718	897	1077	1256	1436	1615
1800	180	360	540	720	900	1080	1260	1440	1620
1805	181	361	541	722	902	1083	1263	1444	1624
1810	181	362	543	724	905	1086	1267	1448	1629
1815	182	363	544	726	907	1089	1270	1452	1633
1820	182	364	546	728	910	1092	1274	1456	1638
1825	183	365	547	730	912	1095	1277	1460	1642

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
1830	183	366	549	732	915	1098	1281	1464	1647
1835	184	367	550	734	917	1101	1284	1468	1651
1840	184	368	552	736	920	1104	1288	1472	1656
1845	185	369	553	738	922	1107	1291	1476	1660
1850	185	370	555	740	925	1110	1295	1480	1665
1855	186	371	556	742	927	1113	1298	1484	1669
1860	186	372	558	744	930	1116	1302	1488	1674
1865	187	373	559	746	932	1119	1305	1492	1678
1870	187	374	561	748	935	1122	1309	1496	1683
1875	188	375	562	750	937	1125	1312	1500	1687
1880	188	376	564	752	940	1128	1316	1504	1692
1885	189	377	565	754	942	1131	1319	1508	1696
1890	189	378	567	756	945	1134	1323	1512	1701
1895	190	379	568	758	947	1137	1326	1516	1705
1900	190	380	570	760	950	1140	1330	1520	1710
1905	191	381	571	762	952	1143	1333	1524	1714
1910	191	382	573	764	955	1146	1337	1528	1719
1915	192	383	574	766	957	1149	1340	1532	1723
1920	192	384	576	768	960	1152	1344	1536	1728
1925	193	385	577	770	962	1155	1347	1540	1732
1930	193	386	579	772	965	1158	1351	1544	1737
1935	194	387	580	774	967	1161	1354	1548	1742
1940	194	388	582	776	970	1164	1358	1552	1746
1945	195	389	583	778	972	1167	1361	1556	1750
1950	195	390	585	780	975	1170	1365	1560	1755
1955	196	391	586	782	977	1173	1368	1564	1759
1960	196	392	588	784	980	1176	1372	1568	1764
1965	197	393	589	786	982	1179	1375	1572	1768
1970	197	394	591	788	985	1182	1379	1576	1773
1975	198	395	592	790	987	1185	1382	1580	1777
1980	198	396	594	792	990	1188	1386	1584	1782
1985	199	397	595	794	992	1191	1389	1588	1786
1990	199	398	597	796	995	1194	1393	1592	1791
1995	200	399	598	798	997	1197	1396	1596	1795
2000	200	400	600	800	1000	1200	1400	1600	1800
2010	201	402	603	804	1005	1206	1407	1608	1809
2020	202	404	606	808	1010	1212	1414	1616	1818
2030	203	406	609	812	1015	1218	1421	1624	1827
2040	204	408	612	816	1020	1224	1428	1632	1836
2050	205	410	615	820	1025	1230	1435	1640	1845

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
2060	206	412	618	824	1030	1236	1442	1648	1854
2070	207	414	621	828	1035	1242	1449	1656	1863
2080	208	416	625	832	1040	1248	1456	1664	1872
2090	209	418	627	836	1045	1254	1463	1672	1881
2100	210	420	630	840	1050	1260	1470	1680	1890
2110	211	422	633	844	1055	1266	1477	1688	1899
2120	212	424	636	848	1060	1272	1484	1696	1908
2130	213	426	639	852	1065	1278	1491	1704	1917
2140	214	428	642	856	1070	1284	1498	1712	1926
2150	215	430	645	860	1075	1290	1505	1720	1935
2160	216	432	648	864	1080	1296	1512	1728	1944
2170	217	434	651	868	1085	1302	1519	1736	1953
2180	218	436	654	872	1090	1308	1526	1744	1962
2190	219	438	657	876	1095	1314	1533	1752	1971
2200	220	440	660	880	1100	1320	1540	1760	1980
2210	221	442	663	884	1105	1326	1547	1768	1989
2220	222	444	666	888	1110	1332	1554	1776	1998
2230	223	446	669	892	1115	1338	1561	1784	2007
2240	224	448	672	896	1120	1344	1568	1792	2016
2250	225	450	675	900	1125	1350	1575	1800	2025
2260	226	452	678	904	1130	1356	1582	1808	2034
2270	227	454	681	908	1135	1362	1589	1816	2043
2280	228	456	684	912	1140	1368	1596	1824	2052
2290	229	458	687	916	1145	1374	1603	1832	2061
2300	230	460	690	920	1150	1380	1610	1840	2070
2310	231	462	693	924	1155	1386	1617	1848	2079
2320	232	464	696	928	1160	1392	1624	1856	2088
2330	233	466	699	932	1165	1398	1631	1864	2097
2340	234	468	702	936	1170	1404	1638	1872	2106
2350	235	470	705	940	1175	1410	1645	1880	2115
2360	236	472	708	944	1180	1416	1652	1888	2124
2370	237	474	711	948	1185	1422	1659	1896	2133
2380	238	476	714	952	1190	1428	1666	1904	2142
2390	239	478	717	956	1195	1434	1673	1912	2151
2400	240	480	720	960	1200	1440	1680	1920	2160
2410	241	482	723	964	1205	1446	1687	1928	2169
2420	242	484	726	968	1210	1452	1694	1936	2178
2430	243	486	729	972	1215	1458	1701	1944	2187
2440	244	488	732	976	1220	1464	1708	1952	2196
2450	245	490	735	980	1225	1470	1715	1960	2205

A TABLE of Proportional Parts.

Tenth Parts of thoſe Differences.

Diff. of Logar.	1	2	3	4	5	6	7	8	9
2460	246	492	738	984	1230	1476	1722	1968	2214
2470	247	494	741	988	1235	1482	1729	1976	2223
2480	248	496	744	992	1240	1488	1736	1984	2232
2490	249	498	747	996	1245	1494	1743	1992	2241
2500	250	500	750	1000	1250	1500	1750	2000	2250
2510	251	502	753	1004	1255	1506	1757	2008	2259
2520	252	504	756	1008	1260	1512	1764	2016	2268
2530	253	506	759	1012	1265	1518	1771	2024	2277
2540	254	508	762	1016	1270	1524	1778	2032	2286
2550	255	510	765	1020	1275	1530	1785	2040	2295
2560	256	512	768	1024	1280	1536	1792	2048	2304
2570	257	514	771	1028	1285	1542	1799	2056	2313
2580	258	516	774	1032	1290	1548	1806	2064	2322
2590	259	518	777	1036	1295	1554	1813	2072	2331
2600	260	520	780	1040	1300	1560	1820	2080	2340
2610	261	522	783	1044	1305	1566	1827	2088	2349
2620	262	524	786	1048	1310	1572	1834	2096	2358
2630	263	526	789	1052	1315	1578	1841	2104	2367
2640	264	528	792	1056	1320	1584	1848	2112	2376
2650	265	530	795	1060	1325	1590	1855	2120	2385
2660	266	532	798	1064	1330	1596	1862	2128	2394
2670	267	534	801	1068	1335	1602	1869	2136	2403
2680	268	536	804	1072	1340	1608	1876	2144	2412
2690	269	538	807	1076	1345	1614	1883	2152	2421
2700	270	540	810	1080	1350	1620	1890	2160	2430
2710	271	542	813	1084	1355	1626	1897	2168	2430
2720	272	544	816	1088	1360	1632	1904	2176	2448
2730	273	546	819	1092	1365	1638	1911	2184	2457
2740	274	548	822	1096	1370	1644	1918	2192	2466
2750	275	550	825	1100	1375	1650	1925	2200	2475
2760	276	552	828	1104	1380	1656	1932	2208	2484
2770	277	554	831	1108	1385	1662	1939	2216	2493
2780	278	556	834	1112	1390	1668	1946	2224	2502
2790	279	558	837	1116	1395	1674	1953	2232	2511
2800	280	560	840	1120	1400	1680	1960	2240	2520
2810	281	562	843	1124	1405	1686	1967	2248	2529
2820	282	564	846	1128	1410	1692	1974	2256	2538
2830	283	566	849	1132	1415	1698	1981	2264	2547
2840	284	568	852	1136	1420	1704	1988	2272	2556
2850	285	570	855	1140	1425	1710	1995	2280	2565

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of thoſe Differences. 1	2	3	4	5	6	7	8	9
2860	286	572	858	1144	1430	1716	2002	2288	2574
2870	287	574	861	1148	1435	1722	2009	2296	2583
2880	288	576	864	1152	1440	1728	2016	2304	2592
2890	289	578	867	1156	1445	1734	2023	2312	2601
2900	290	580	870	1160	1450	1740	2030	2320	2510
2910	291	582	873	1164	1455	1746	2037	2328	2619
2920	292	584	876	1168	1460	1752	2044	2336	2628
2930	293	586	879	1172	1465	1758	2051	2344	2637
2940	294	588	882	1176	1470	1764	2058	2352	2646
2950	295	590	885	1180	1475	1770	2065	2360	2655
2960	296	592	888	1184	1480	1776	2072	2368	2664
2970	297	594	891	1188	1485	1782	2079	2376	2673
2980	298	596	894	1192	1490	1788	2086	2384	2682
2990	299	598	897	1196	1495	1794	2093	2392	2691
3000	300	600	900	1200	1500	1800	2100	2400	2700
3010	301	602	903	1204	1505	1806	2107	2408	2709
3020	302	604	906	1208	1510	1812	2114	2416	2718
3030	303	606	909	1212	1515	1818	2121	2424	2727
3040	304	608	912	1216	1520	1824	2128	2432	2736
3050	305	610	915	1220	1525	1830	2135	2440	2745
3060	306	612	918	1224	1530	1836	2142	2448	2754
3070	307	614	921	1228	1535	1842	2149	2456	2763
3080	308	616	924	1232	1540	1848	2156	2464	2772
3090	309	618	927	1236	1545	1854	2163	2472	2781
3100	310	620	930	1240	1550	1860	2170	2480	2790
3110	311	622	933	1244	1555	1866	2177	2488	2799
3120	312	624	936	1248	1560	1872	2184	2496	2808
3130	313	626	939	1252	1565	1878	2191	2504	2817
3140	314	628	942	1256	1570	1884	2198	2512	2826
3150	315	630	945	1260	1575	1890	2205	2520	2835
3160	316	632	948	1264	1580	1896	2212	2528	2844
3170	317	634	951	1268	1585	1902	2219	2536	2853
3180	318	636	954	1272	1590	1908	2226	2544	2862
3190	319	638	957	1276	1595	1914	2233	2552	2871
3200	320	640	960	1280	1600	1920	2240	2560	2880
3210	321	642	963	1284	1605	1926	2247	2568	2889
3220	322	644	966	1288	1610	1932	2254	2576	2898
3230	323	646	969	1292	1615	1938	2261	2584	2907
3240	324	648	972	1296	1620	1944	2268	2592	2916
3250	325	650	975	1300	1625	1950	2275	2600	2925

A TABLE of Proportional Parts.

Tenth Parts of thoſe Differences.

Diff. of Logar.	1	2	3	4	5	6	7	8	9
3260	326	652	978	1304	1630	1956	2282	2608	2934
3270	327	654	981	1308	1635	1962	2289	2616	2943
3280	328	656	984	1312	1640	1968	2296	2624	2952
3290	329	658	987	1316	1645	1974	2303	2632	2961
3300	330	660	990	1320	1650	1980	2310	2640	2970
3310	331	662	993	1324	1655	1986	2317	2648	2979
3320	332	664	996	1328	1660	1992	2324	2656	2988
3330	333	666	999	1332	1665	1998	2331	2664	2997
3340	334	668	1002	1336	1670	2004	2338	2672	3006
3350	335	670	1005	1340	1675	2010	2345	2680	3015
3360	336	672	1008	1344	1680	2016	2352	2688	3024
3370	337	674	1011	1348	1685	2022	2359	2696	3033
3380	338	676	1014	1352	1690	2028	2366	2704	3042
3390	339	678	1017	1356	1695	2034	2373	2712	3051
3400	340	680	1020	1360	1700	2040	2380	2720	3060
3410	341	682	1023	1364	1705	2046	2387	2728	3069
3420	342	684	1026	1368	1710	2052	2394	2736	3078
3430	343	686	1029	1372	1715	2058	2401	2744	3087
3440	344	688	1032	1376	1720	2064	2408	2752	3096
3450	345	690	1035	1380	1725	2070	2415	2760	3105
3460	346	692	1038	1384	1730	2076	2422	2768	3114
3470	347	694	1041	1388	1735	2082	2429	2776	3123
3480	348	696	1044	1392	1740	2088	2436	2784	3132
3490	349	698	1047	1396	1745	2094	2443	2792	3141
3500	350	700	1050	1400	1750	2100	2450	2800	3150
3510	351	702	1053	1404	1755	2106	2457	2808	3159
3520	352	704	1056	1408	1760	2112	2464	2816	3168
3530	353	706	1059	1412	1765	2118	2471	2824	3177
3540	354	708	1062	1416	1770	2124	2478	2832	3186
3550	355	710	1065	1420	1775	2130	2485	2840	3195
3560	356	712	1068	1424	1780	2136	2492	2848	3204
3570	357	714	1071	1428	1785	2142	2499	2856	3213
3580	358	716	1074	1432	1790	2148	2506	2864	3222
3590	359	718	1077	1436	1795	2154	2513	2872	3231
3600	360	720	1080	1440	1800	2160	2520	2880	3240
3610	361	722	1083	1444	1805	2166	2527	2888	3249
3620	362	724	1086	1448	1810	2172	2534	2896	3258
3630	363	726	1089	1452	1815	2178	2541	2904	3267
3640	364	728	1092	1456	1820	2184	2548	2912	3276
3650	365	730	1095	1460	1825	2190	2555	2920	3285

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of thoſe Differences. 1	2	3	4	5	6	7	8	9
3660	366	732	1098	1464	1830	2196	2562	2928	3294
3670	367	734	1101	1468	1835	2202	2569	2936	3303
3680	368	736	1104	1472	1840	2208	2576	2944	3312
3690	369	738	1107	1476	1845	2214	2583	2952	3321
3700	370	740	1110	1480	1850	2220	2590	2960	3330
3710	371	742	1113	1484	1855	2226	2597	2968	3339
3720	372	744	1116	1488	1860	2232	2604	2976	3348
3730	373	746	1119	1492	1865	2238	2611	2984	3357
3740	374	748	1122	1496	1870	2244	2618	2992	3366
3750	375	750	1125	1500	1875	2250	2625	3000	3375
3760	376	752	1128	1504	1880	2256	2632	3008	3384
3770	377	754	1131	1508	1885	2262	2639	3016	3393
3780	378	756	1134	1512	1890	2268	2646	3024	3402
3790	379	758	1137	1516	1895	2274	2653	3032	3411
3800	380	760	1140	1520	1900	2280	2660	3040	3420
3810	381	762	1143	1524	1905	2286	2667	3048	3429
3820	382	764	1146	1528	1910	2292	2674	3056	3438
3830	383	766	1149	1532	1915	2298	2681	3064	3447
3840	384	768	1152	1536	1920	2304	2688	3072	3456
3850	385	770	1155	1540	1925	2310	2695	3080	3465
3860	386	772	1158	1544	1930	2316	2702	3088	3474
3870	387	774	1161	1548	1935	2322	2709	3096	3483
3880	388	776	1164	1552	1940	2328	2716	3104	3492
3890	389	778	1167	1556	1945	2334	2723	3112	3501
3900	390	780	1170	1560	1950	2340	2730	3120	3510
3910	391	782	1173	1564	1955	2346	2737	3128	3519
3920	392	784	1176	1568	1960	2352	2744	3136	3528
3930	393	786	1179	1572	1965	2358	2751	3144	3537
3940	394	788	1182	1576	1970	2364	2758	3152	3546
3950	395	790	1185	1580	1975	2370	2765	3160	3555
3960	396	792	1188	1584	1980	2376	2772	3168	3564
3970	397	794	1191	1588	1985	2382	2779	3176	3573
3980	398	796	1194	1592	1990	2388	2786	3184	3582
3990	399	798	1197	1596	1995	2394	2793	3192	3591
4000	400	800	1200	1600	2000	2400	2800	3200	3600
4010	401	802	1203	1604	2005	2406	2807	3208	3609
4020	402	804	1206	1608	2010	2412	2814	3216	3618
4030	403	806	1209	1612	2015	2418	2821	3224	3627
4040	404	808	1212	1616	2020	2424	2828	3232	3636
4050	405	810	1215	1620	2025	2430	2835	3240	3645

A TABLE of Proportional Parts.

Tenth Parts of thoſe Differences.

Diff. of Logar.	1	2	3	4	5	6	7	8	9
4060	406	812	1218	1624	2030	2436	2842	3248	3654
4070	407	814	1221	1628	2035	2442	2849	3256	3663
4080	408	816	1224	1632	2040	2448	2856	3264	3672
4090	409	818	1227	1636	2045	2454	2863	3272	3681
4100	410	820	1230	1640	2050	2460	2870	3280	3690
4110	411	822	1233	1644	2055	2466	2877	3288	3699
4120	412	824	1236	1648	2060	2472	2884	3296	3708
4130	413	826	1239	1652	2065	2478	2891	3304	3717
4140	414	828	1242	1656	2070	2484	2898	3312	3726
4150	415	830	1245	1660	2075	2490	2905	3320	3735
4160	416	832	1248	1664	2080	2496	2912	3328	3744
4170	417	834	1251	1668	2085	2502	2919	3336	3753
4180	418	836	1254	1672	2090	2508	2926	3344	3762
4190	419	838	1257	1676	2095	2514	2933	3352	3771
4200	420	840	1260	1680	2100	2520	2940	3360	3780
4210	421	842	1263	1684	2105	2526	2947	3368	3789
4220	422	844	1266	1688	2110	2532	2954	3376	3798
4230	423	846	1269	1692	2115	2538	2961	3384	3807
4240	424	848	1272	1696	2120	2544	2968	3392	3816
4250	425	850	1275	1700	2125	2550	2975	3400	3825
4260	426	852	1278	1704	2130	2556	2982	3408	3834
4270	427	854	1281	1708	2135	2562	2989	3416	3843
4280	428	856	1284	1712	2140	2568	2996	3424	3852
4290	429	858	1287	1716	2145	2574	3003	3432	3861
4300	430	860	1290	1720	2150	2580	3010	3440	3870
4310	431	862	1293	1724	2155	2586	3017	3448	3879
4320	432	864	1296	1728	2160	2592	3024	3456	3888

The End of the Table of Parts Proportional.

CHAP. VIII.

Lineal Arithmetick. (See Plate *A.*)

THIS fourth Species of Arithmetick is performed by Right Lines, ſo that Magnitude being here conſidered, it may be called *Geometrical Arithmetick.*

2. A Right Line is properly defined to be the neareſt Diſtance between two Points, as the Line (*a c b*, *Fig.* 1.) is the neareſt Diſtance between the Points *a* and *b*.



3. The Diſpoſition of the two principal Lines, by which this kind of Arithmetick is performed, are either in a Right Angle or an Oblique.

4. A Right Angle is when one Line falls Perpendicular on another, as the Line *a b* (*Fig.* 2.) does on *b c*; but when two Lines meet not ſo, they form an Angle called *Oblique*, as the Angle *d e f* (*Fig.* 3.) by both which, Queſtions are anſwer'd.

5. As in Logarithms thoſe Artificial Numbers are not employed about the moſt eaſy and ſelf-evident Rules of *Addition* and *Subſtraction* of Natural Numbers: So here by Lines I ſhall only ſhew how *Multiplication*, *Diviſion*, the *Rule of Proportion*, with *Extraction of Roots*, are performed thereby.

SECT. II. *Multiplication by Lines, three Ways.*

1. What is the Product of $17\frac{1}{2}$ by $12\frac{1}{2}$? See *Figure* 4. in the folded Sheet (*A.*)

To anſwer this, draw a Line at pleaſure, and ſet off $12\frac{1}{2}$ equal Parts as from *a* to *b*, from the Line of equal Parts.

Then draw a perpendicular Line at pleaſure as *b c*, and ſet off from *b* to *c* $17\frac{1}{2}$.

Laſtly, Draw Lines between each equal Part, and the $\frac{1}{2}$, Parellel to (*a b*) and to (*b c*), then telling the ſeveral little Squares each of which is a Unit, you'll find 204 intire, and 17 and 12 Halves, or $14\frac{1}{2}$ which together, and the $\frac{1}{4}$ at the Angle (*c*) make the Product $218\frac{3}{4}$.

If you would multiply 175 by 125, that is, if you ſuppoſe every Unit in the Line (*a b*) to be 10, then $12\frac{1}{2}$ will be 125, and by

the ſame Rule the Line *b c* will be 175, and by ſuppoſing each little Square 100 (or 10 times 10, the Lines *a b* and *b c* being 10 times as many as before) you'll find 20400 little Squares or Units, and 17 and 12 Fiftys (or 1450) and 25 the 4th of 100 at the Angle (*c*), which added together, makes 21875 = the Product required.

A ſecond Way to multiply by Lines.

But before I proceed therein, it will be neceſſary to ſhew how to draw a Line Parallel to another, and through any Point aſſigned: So to draw a Line through the Point (*a*) (See *Fig.* 5. *Plate A.*) and Parallel to (*m n*), ſet the Foot of your Compaſſes in the Point, and extend the other to the Line *m n*, ſo as to deſcribe the Ark *c c* juſt to touch the Line *m n*; and with that Diſtance of the Feet ſet one in the Point (*e*) (or thereabout) and deſcribe the Ark (*d d*); then lay a Ruler from that Ark to the Point *a*, and draw the Line *p q*, which is the Anſwer.

To multiply by 2 *Lines, including an Oblike Angle as that* (*e d n*); *Fig.* 6.

What is the Product of 3.5 by 2.2? For anſwer, having made an Angle at pleaſure, not too Acute.

Rule.] Take a Line of equal Parts, and ſet off 1 from the Angle *d* to 1. Then ſet the Multiplicand from *d* to 3.5 on the other Leg (*d n*). 3*dly*, Set the Multiplier from 1 to 2.2. 4*thly*, Draw the Line 1 to 3.5, produced to about (*m*), for the better drawing the Line (2.2; 7.7) Parallel to (1, *m*); ſo is the diſtance between 3.5 and 7.7 the Anſwer, which meaſured upon the ſame Line of equal Parts, is found 7.7 = the Product.

2. Or if you ſet the Multiplicand from 1 to *b*; and the Multiplier from *d* to *a*, a Line (*b c*) drawn Parallel to (1, *a*) cutteth, the Product (*a c*) 7.7 as before.

3. Or if you ſuppoſe the Factors 10 times as much, the Anſwer will be by the ſame Method 770.

A third Way to multiply by Lines.

This ſhews the *Uſe of a Line of Numbers*, or as 'tis often called *Gunter*'s Line; being a Line of *Logarithms*, as will appear by the *Uſe*, *Addition performing Multiplication*, *Subſtraction*, the *Work of Diviſion*; *Diviſion*, the *Extraction of Roots*, &c.

This

This uſeful Line was the Invention of Mr. *Edmund Gunter*, Profeſſor of *Aſtronomy* at *Greſham-College* about the Year 1620; and it is a Line divided into 10 unequal Parts, and thoſe 10 Parts repeated are called 20, 30, 40, *&c.* when the former Parts are called 1, 2, 3, *&c.*

If the 1ſt 10 are Tens, The 2d ten Parts are 100's.
If the 1ſt 10 are Hundreds, The 2d Radius are ſo many (Thouſands.
If you multiply 1 place by 2 places, The Product is ſo many 10's (as the Figure denotes.

As your own Experience will eaſily diſcover to you. See the making of this Line near the End of *Sect.* 4th following.

To multiply any 2 Numbers, as thoſe in the laſt Caſe 2.2 by 3.5. See *Fig.* 7. *Plate A.*

Rule.] Extend the Foot of a Pair of Compaſſes (or other Meaſure) from 1 to 2.2, and that Extent reaches from 3.5 to 7.7;

2*dly*, Or the Extent from 1 to 3.5, reaches from 2.2 to 7.7.

3*dly*, Or if you multiply 35 by 22: In this caſe, what is called 1, 2, *&c.* in the 1ſt part, muſt be ſuppoſed 10, 20, *&c.* and what are 10th parts muſt be called Units, and if both Factors be 10 or more each, the Product is Hundreds. So the Extent as before from 10 to 22, reaches from 35 to 770; for here 35 being multiplied by a Number which is above 10, is ſo many 100's and 10's as the Meaſure extends to; for your Inſpection ſhews, that 35 by 10 is 350, which is 3 places; and if for any Number of Tens, the like: And if the 2d Extent be beyond 10 on the Line, it is ſtill ſo many Hundreds, as 30 by 50 is 1500, *&c.*

And *Note*, That whatever you call the larger Diviſions, the ſmaller in the ſame Radius are a 10th thereof, as is before hinted, and the contrary.

Sect. III. *Diviſion by Lines, three Ways.*

As I have ſhewed *Multiplication* by three different Lines, and by each two ways; ſo I intend to ſhew how to divide.

Caſe 1.] What is the Quotient of $218\frac{3}{4}$ by $17\frac{1}{2}$? See *Fig.* 4.

The Dividend is repreſented by the ſeveral ſmall Squares and Parts in the Superficies *a c*, which if you divide by the ſide (*a* $17\frac{1}{2}$) the Quotient will be one of the ſmaller ſides *a b* or ($17\frac{1}{2}$ *c*).

For having divided a Line (*a*, $17\frac{1}{2}$) into $17\frac{1}{2}$, parts, your Divisor, and made a little Square upon each part, and half of one for the half; if you add to those Squares at Right Angles so many as will make them up $218\frac{3}{4}$, and as many in one Row of Squares as another; you will then find that the Line including a Right Angle with (*a* $17\frac{1}{2}$) will be (*a b*) which will contain $12\frac{1}{2}$ of those small Squares, and is consequently the Quotient, because that Quote multiplied in the Divisor (*a*, $17\frac{1}{2}$) produces the Dividend or Number of small Squares (*a c*)=$218\frac{3}{4}$. And the same may be said of other like Superficies and their Sides, whatever they be for Numbers.

A second Way to divide by Lines.

What is the Quote of 7.7 divided by 3.5? (*Fig.* 6. for I make use of the same Numbers as in *Multiplication*, to prevent Confusion by a Multitude of prickt Lines.)

1*st*. Take 1 from any Line of equal Parts, and set off (*d* 1).

2*dly*. Set the Divisor (*d*, 3.5) off from the same Line of Parts.

3*dly*. Draw the Line (1, 3.5) and produce it to *m*, &c.

4*thly*. Set your Dividend (7.7) from $3\frac{1}{2}$ towards *n*, which extends to 7.7.

5*thly*. Through that Point draw a Line parallel to (1 *m*) and it will pass through the Point 2.2 in the Line (*d e*).

6*thly*. Take the Distance from 1 to 2.2, and apply it to your equal Parts, and you'll find it 2.2=the Quote.

Or if you were to divide 7.7 by 2.2, the Divisor set from *d* to *a*, and the Dividend from *a* to *c*: If a Line be drawn (1 *a*), and another Parallel thereto through the Point *c*, that Line will pass through *b* in the Line *d e*, and the distance (1 *b*) measured on the same Line of equal Parts, will be 3.5=the Quote.

Or if you divide 770 by 35, the Method is the same as the first of these, and 77 by 22 the same as the second or last, *viz.* 35. And the like of other Numbers.

A third Way to divide by Lines.

Suppose by the Line of Numbers, *Fig.* 7. *Plate A.* you would divide 770 by 22. Extend the Compasses from 10 to 22 (calling those 10, *&c.* in the first Radius, which are 1, 2, *&c.* and those 100's in the second, which are 10, 20, 30, *&c.*) and the same

ſame Extent reaches from 770 in the ſecond Radius, backward to 35.

Or to divide 770 by 35, the Extent from 10 to 35, extends backward from 770 to 22. And the like of other Numbers.

Sect. IV. *The direct Rule of Proportion by Lines.*

If 10 lb of Sugar coſt 22 *s.* what will 35 lb coſt?

1*ſt.* By the Lines, *Fig.* 6. ſet 10 from the Angle to 1 (which ſuppoſe 10, as if the 1 was not there;) then ſet the 22 *s.* from 1 to 2.2, that is, from 10 to 22. 3*dly*, Set the third Number from the Angle at *d* to 3.5, or to 35. Then draw a Line from 10 to 35=the third Number given, and a Line parallel to that through the Point 2.2 (here 22 will paſs through, or cut the Line *d n* in 7.7, ſo the Diſtance 7.7 to 3.5 (meaſured upon the Line whereby *d* 1 is 10) is 77 *s.*=the value of the 35 lb of Sugar. I have done this (as I ſaid before) by the ſame parallel Lines, to prevent Confuſion by a Multiplicity.

2*dly. By the Line of Numbers*, Fig. 7. If 12 C. of Sugar coſt 25 *l.* 4*s.* what will 1 Tun or 20 C. coſt?

Extend the Compaſſes from 12 C. to 25.2, and the ſame Extent reaches from 20 C. to 42 *l.* the Anſwer.

Q. 2. If you would know the Intereſt of 90 *l.* for a Year at 6 *per Cent.* extend the Compaſſes from 100 backward to 6; and the ſame Extent reaches from 90 to 5.4 *l.* or *l.* 5 : 8 : =the Anſwer.

3*dly.* To Extract the Square Root, take half the Diſtance between the Square Number given and 1. And 1 third of the Diſtance from a Cube Number to 1 is the Cube Root, taking the third next to 1.

And now I might end this part of Arithmetick, but before I do I ſhall ſhew the making of a Line of Chords of Numbers, and a Line of half Tangents, with ſomething of their Uſe, and that of equal Parts, which will be a Preparative for thoſe who would know how to perform Trigonometry Geometrically.

The Line of Chords is 90 Degrees (or equal Parts in the fourth Part of a Circle) ſet off upon a ſtraight Line, as *Fig.* 8. *Plate A.* the Degrees (*e m*) are ſet upon the Diameter (*e c n*) where (*e* 60) of the Circle, is equal to Radius or the Semi-diameter of that Circle, and *e m* equal to *e n*, &c.

Now to meaſure any Angle by this Line, extend the Compaſſes from *e* to 60 in the Line (*e n*), and with that (ſuppoſe in *Fig.* 9.) ſet

ſet one Foot in *h*, and deſcribe the Ark (*r o*), then take the Diſtance (*r o*) in your Compaſſes, and apply it to your Line of Chords, and you'll find it 33 Degrees: That is to ſay, the Line or Leg (*h f*) is diſtant from that *g h* 33 Degrees or equal Parts of the Circle (*d m e*, &c.) as thoſe are transferred upon the Line (*e d*); ſo the Angle (*h f g*) or at *f*, is found 58 Degrees, and the Sum of thoſe two Angles deducted from 180 Leaves the Angle (*f g h*, or) at *g*. And the Sides of this Triangle are meaſured by applying them between the Compaſſes to *the Line of equal Parts* (*y z*), *Fig.* 10.

The way to make a Line of Numbers or Logarithms (commonly called Gunter's *Line.)*

The Line (1, 10, *Plate A.*) is thus made: *Firſt*, Make a Line of equal Parts according to the Length you intend your Line of Numbers, as here (0, 1000) is divided actually into 100 equal Parts, and each Part ſuppoſed to be divided into 10, makes 1000; and if your Line were deſigned to be much longer, you might divide your equal Parts into 10000.

2*dly*. Lay before you a Table of Logarithms, and from that take the Logarithms (omitting the Index) of 2, 3, 4, *&c.* to 10, the largeſt Diviſions of your Line: As, for 2 I find the Logarithm .3010300, but I take Notice only of the three next places to the Point agreeable to my Line of equal Parts, as 301 taken from that Line, and ſet off upon the Line (1, 10,) from 1 to 2 gives the firſt of the firſt 9 unequal Parts of the Line of Numbers.

Then I find the Logarithms of	3	to be .477	Which Logarithms taken from the Line of equal Parts, and ſet on that of Numbers from	1 to 3	Gives you the other unequal Parts of the Line of Numbers.
	4	.602		1 to 4		
	5	.698		1 to 5		
	6	.778		1 to 6		
	7	.845		1 to 7		
	8	.903		1 to 8		
	9	.954		1 to 9		

Then for the Sub-diviſions or ſmaller unequal Parts; I proceed in the very ſame manner 'till I have ſet them all off from 1 (the beginning of the Line of Numbers:) As

The

The Logarithm:	.041		1 to 1.1	
	.079		1 to 1.2	
	.113		1 to 1.3	
	.146	Taken from the	1 to 1.4	On the Line of
	.176	Line of equal	1 to 1.5	Numbers.
	.204	Parts, extends	1 to 1.6	
	.230	from	1 to 1.7	
	.255		1 to 1.8	
	.278		1 to 1.9	
	.322		1 to 2.1	
	.342, *&c.*		1 to 2.2, *&c.*	

I have been more particular in this Affair than has been ſhewed before, becauſe I find moſt Perſons, and ſome even the Inſtrument-makers ignorant, how this very uſeful Line is made originally to any Radius, which depends on the Magnitude of the equal Parts.

But *Note*, That the Line of Numbers conſiſteth of two ſuch Lines, as (1, 10; on *Plate A. Fig.* 10.) next the *Fig.* 7. (or Numbers) and ſo from 10 is numbered 20, 30, *&c.* to 100 on the 2d Part, which is made 1 Line with the firſt, and muſt ſo be made by the Artiſt.

To make a Line of equal Parts.

To make the Line (0, 1000, *Fig.* 10.) imaginarily divided into 1000 Parts, or actually into 100

1*ſt.* I divide it into 5 with my Compaſſes.

2*dly.* I divide each 5th into 2, which makes the whole into 10.

3*dly.* I divide 1 tenth Part into 10 (as 0,100).

4*thly.* I take (0,100) between my Compaſſes, and ſetting 1 Foot in (*i*), the other extends to (*i*), or a 10th of the next 10th, and ſo forward divides the whole Line into 100, and each 100 muſt be ſuppoſed 10, in taking off the 1000th Parts to make the Line of Numbers by, or in other Uſes.

The Line of half Tangents, is the Line (*c m*, *Fig.* 8.) made by laying a Ruler to (*d*), and the ſeveral Degrees in the Quadrant (*e m*), and is uſed in meaſuring ſuch ſtraight Lines as paſs through the Center of a Circle, and do repreſent Circles, though they appear Right Lines, as (*c m*), (*c e*), (*c d*), &c. The Uſe of the Line of equal Parts has been fully ſhewn before in meaſuring Right Lines,

and

and there is nothing more falls within the Compass of Lineal or Geometrical Arithmetick; which will be of great Use to such as intend to Study Geometry, to whom I recommend the reading of *Euclid*, lib. 2. I shall therefore proceed to

CHAP. IX.

Instrumental Arithmetick.

INstrumental Arithmetick is that Part which is performed by Instruments, accommodated with Figures or Numbers for that purpose. Of this kind, that of *Neper*'s Bones justly claims the first Consideration, as being the most plain and easy Instrument for all Operations, whether in Multiplication, Division, or Extraction of Roots.

A Description of Neper's *Bones*, (See *Fig.* 1. *Plate B.*) As to their Matter, they are sometimes made of Ivory, and then called Bones, sometimes of Box-wood, and then called Virgulæ, *or little Rods*; for as to *their Magnitude and Form*, they may be made as large as you please: But the common Sort are each Rod about 2.5 Inches long, and .32 Inch broad, and are in Figure what Geometricians call a Parallelopipedon, which is a long Square solid, bounded with 6 Superficies, (or like 7, 8 *e c* Dyes joined exactly one to another.)

The Number of these Rods in a Set are 10, that is, the 9 Digits and Cypher, besides the Index-bone which is fixed to the Box, and hath only one Face divided into 9 equal Parts, whereon the 9 Digits are placed as you see *Fig.* 1. *Plate B.* But the other 10 have 4 Faces each, whereon are placed the 9 Digits at the top, and an Arithmetical Progression of those Digits added as 2, 4, 6; or 3, 9, 12, *&c.* And whatever the first Figure of any Rod is, that of the opposite Side is the Complement that makes it up 9. And the 10 times 4 Faces or Sides being 40, contain 4 of each of the 9 Digits, and their Series's Arithmetical, and 4 more Faces are of Cyphers.

How to add the Figures on the Rods. See *Fig.* 12. *Plate A.*

To reduce any Line of Figures on the Rods which stand against any Digit on the Index-face to a true legible Numbers, you must

muſt begin at the Right-hand: Thus to add the Digits ſtanding againſt 6 in the ſet, *Fig.* 1. *Plate B.* You ſee there is nothing to do but to add the two Digits together, which ſtand between the ſame Slant or Diagonal Lines, as 4 is 4, then 5 and 8 is 13; 1 and 4 is 5, and 2 is 7; 4 and 6 is 10; 1 and 3 is 4; 3 and 4 is 7; 2 and 8 is 10; 1 and 1 is 2, and 2 is 4; 1 and 6 is 7: So that ſix times 123456789, is found 740740734.

SECT. II. *To multiply by* Neper's *Rods or Bones.*

Queſt. 1. What is the Product of 34729 by 23795; See *Fig.* 13. *Plate A.*

Firſt tabulate the Multiplicand, (or either Factor, but the larger is moſt uſual;) that is, take ſo many Rods out of your Sett, that the firſt Figures (or thoſe at the upper End of each Rod) will compoſe your Multiplicand in order, as you ſee done, *Fig.* 13. of *Plate A.*

Then looking for the Figures of your Multiplier ſeverally in the Index-rod next the Left-hand, you find againſt each its reſpective Product in the Multiplicand, as *Fig.* 13. aforeſaid.

Queſt. 2. What is the Product of 3040999 by 2341?

In this Example there being 3 Nines and 2 Cyphers on the oppoſite Sides, whereof are 2 Nines makes 5 Nines, which is one more than is in a Sett of theſe Rods; therefore, if you have not another Sett, you may eaſily ſupply the Defect by obſerving the Product of each Figure (as you go on) in the Multiplier; and thoſe other Nines, and ſo making one of thoſe extraordinary: Thus in Multiplying by the 4 there (as *Fig.* 14.) and if there were another Rod for the third Nine in the Multiplier, I can eaſily ſee there would be more (as *Fig.* 15. *Plate A.*) which therefore, I ſuppoſe, and ſo put down 6, and then two Nines inſtead of the one Nine in the Rods; ſo when I multiply by the 3, I find in the Tabulet toward the Right-hand againſt 3, (ſee *Fig.* 16. *Plate A.*) But I know if I had another Rod of the

3040999=the Multiplicand.

Multiply'd by 1=3040999
by 4=12163996
by 3=9122997
by 2=6081998

71189786 59=Product.

ſame Digit 9, there would have been another, as *Fig.* 17. *Plate A.* which ſuppoſing there, it would make 2997, that is, a 9 more than I find in the Tabulet, and ſo the reſt.

SECT. III. *Diviſion by* Neper's *Bones or Rods.*

What is the Quote of 23734.691084 divided by 253.24?

An Example of the whole Proceſs, and of what help you have in dividing from the Rods. See *Figure* 18. *Plate A.*

Hence it appears, that the Rods are only uſeful in exhibiting the Products of the Diviſor by each Figure put in the Quotient, which knowing, you proceed in all other Reſpects as in Diviſion of Integers, *Sect.* 5. *Chap.* 1. *&c*, firſt Tabulating the Diviſor, and then whatſoever the Quote be, you have each Figure multiply'd in the Diviſor, although I have only placed in the Index-rod the Figures that fall in the Quote above, and that in the Order they happen therein, for the more orderly ſhewing the Nature of the Work, and how it agrees with common Diviſion; ſo that in doing this Queſtion above, you finding that 227916 is the next Number on the Rods to, and leſs than 237346 (the firſt Part of the Dividend) looking againſt that in the Index-rod you find 9; therefore put 9 in the Quote, and deduct as in the Example, and the Remainder (with 9 brought down to it) is 94309; which found in the Rods, or what is next leſs, I obſerve 3 to ſtand againſt it, which I put in the Quote, and ſo proceed with the reſt as in the Example, which is ſo plain, that it will ſerve for all. I have put the Figures of the Quote againſt their reſpective Dividuals, which added and placed as above in its proper place, is the general Quotient required 93.7241.

SECT. IV. *The Uſe of* Neper's *Bones or Rods in Extraction of the Square Root.*

What is the Square Root of 54756? (See *Fig.* 3. *Plate B.*)

Rule.] Having pointed the Square Number given, as taught in intire Numbers; by the three Points, it appears, there will be Hundreds, Tens, and Units in the Root.

2dly. Having put the Rod of the Square of the 9 Digits next towards the Right-hand of the Index-rod, you'll find that againſt 4, the

Square.	*Sq. Root.*
54756	(234
4	
147	
129	
1856	
1856	
0 Reſts.	

next

next Square to 5 (the firſt pointed towards the Left-hand) there ſtands its Root upon the ſaid fixed Rod, *viz.* 2; therefore put 2 in the Root, and deduct the ſaid Square 4 from 5, and there reſts 1.

3*dly*. Having brought as uſual the next Branch down, it makes 147. Then take a Rod, whoſe uppermoſt Number is the double of the Figure laſt put in that Root (here 4;) and place the Rod of Squares to the Right-hand, that (as you ſee *Fig.* 3. *Plate B.* in the middle Part over 30) then look in the two Rods next the Right-hand for 147, and the next leſs is 129, right againſt which on the Index-rod ſtands 3; therefore put 3 in the Root, and deduct 129 from 147, and the Remainder is 18.

4*thly*. Having placed the next two Figures (or laſt Binary of the Square given) toward the Right-hand of 18, they make 1856. Then place two Rods (between the Rods of the Index, and that of Squares) whoſe uppermoſt Digits are the double of the 2 in the Root (as you ſee done in *Fig.* 3. aforeſaid next the Right-hand) and look in the 3 Rods next that Hand for your 1856, which you find juſt, and towards the Left-hand thereof, on the Index-rod ſtands 4; therefore put 4 in the Root, and deducting, you ſee nothing remains, and that 234 is the Root required. *This I have made much more plain than has been done before, by* Fig. 3. *which ſhews the Poſition of the Rods upon each Digit put in the Root, and doubt not but it is ſufficiently Intelligible.*

But *Note*, That if the double of the Figure put in the Root be more than 9, you muſt put two Rods next the Index, which begin with the Digits of that double; and three Rods put when the double of two places is above 99. Thus, if 9 is the firſt in the Root, the double is 18; ſo I put two Rods between the Index and the Rod of Squares, beginning with 1 and 8. So alſo, if 99 were the two firſt in the Root, the double is 198; I put therefore three Rods, beginning with 1, 9, and 8, between the Index-rod and that of Squares, *&c.*

For a farther Exemplification; extract the Square Root by the Bones of 97535376. See the Work in the Margin.

1*ſt*. Here you find 81 the next leſs Square to 97, and againſt it is 9 in the fixed Rod; ſo you put 9 in the Root, and deducting 81, there reſts 16, and the next Branch or Period is 1653.

2*dly*. Doubling 9 is 18, which two Rods placed next that fixed, and that of Squares next the Right-hand; the top Digits will ſtand in this Order, ſee *Fig*. 19. *Plate A*. and in the Progreſſion lower you'll find againſt 1504, the next leſs to 1653, ſtands 8, which put in the Root, and deduct 1504, there reſts 149; which with the next Period makes 14953.

	Square.	*Square*
Example 2=	97535376	(9876=Root
	81	
	1653	
	1504	
	14953	
	13769	
	118476	
	118476	
	0	

3*dly*. Doubling 98 is 196; which three Rods placed next that fixed, and that of Squares towards the Right, the firſt Figures on the tabulated Rods will ſtand thus, ſee *Fig*. 20. *Plate A*. And in the Progreſſion thereof lower you'll find 13769; againſt which is 7, which put in the Root, and the 13769 deducted from 14953, there Reſts 1184; to which bringing down the next Branch, makes 118476.

4*thly*. Doubling 987 in the Root, makes 1974; therefore taking 6 out of the laſt Tablet, and putting the Rods beginning with 7 and 4, the top of the ſix Rods will ſtand thus, ſee *Fig*. 21. And in the Progreſſion lower you'll find on the 5 Rods next the Right-hand 118476, againſt which towards the Left-hand upon the fixed Rod is 6: So there Reſts (0), and the Operation is finiſhed.

Sect. V. *The Uſe of* Neper's *Bones in the Extraction of the Cube Root.*

The Rods make this (which is reckoned the moſt difficult Rule) very eaſy, ſo ſoon as you know how to place your Virgulæ, which I ſhall therefore illuſtrate by an Example of their Poſition at every Figure put in the Root, as I have done in the preceding Parts of Arithmetick by the Rods.

The

The Cube Number given muſt be prepared for Extraction by pointing over the Units, and every third afterwards, as is taught in extracting Roots of Vulgar intire Numbers, *&c.* for Example:

What is the Cube Root of 182284628? See *Figures* 22, 23, 24. *Plate A.*

1ſt. The firſt Figure in the Root is found by placing your Cube-rod next the Index-rod; for looking on the former for 182 (the firſt Point next the left) you find the next leſs to be 125.

```
182284628 (567
125
 57
```

And right againſt that 125, ſtands 5 on the fixed Rod, which 5 I therefore put in the Root, and ſubſtracting 125 from 182, there Reſts 57. To which bringing down the next Point or Branch, it makes 57284.

```
45216
  540
50616
 6668628
```

Then for a Diviſor to divide the 57284: Take treble the Square of the 5 (in the Root) which is 75, ſee *Fig.* 23. *Plate A.* And having tabulated that, and the Rod of Cubes, as you ſee, I find on thoſe three Rods 45216 the next leſs to be taken, againſt which in the Index is 6, which I put in the Root: To which 45216 I add 540 (which is the Square of 6 in the Root multiplied in three times the 5 there) and the Sum is 50616; which ſubſtracted from the Dividend 57284, there Reſts 6668, to which I bring down 628=the third Branch.

```
9408)  6668628
       6585943
          8232
       6668263
           365
```

And it makes the laſt Dividend 6668628; and treble the Square of the Root (56) make 9408 (a Diviſor) which Tabulated with the Rod of Cubes, as you ſee *Fig.* 24. *Plate A.* I look upon the Rods, and find 6585943 to be the next leſs Number than the 6668628;

6668628; and againſt it is 7 in the Index, which 7 I put in the Root: Then for the 8232 to be added to the Number next above it, I multiply 3 times 56 (in the Root) by the Square of 7 there; and the Sum is 6668263, which I take from the laſt Dividend 6668628, and the Remainder is 365; to which, if you deſire Decimal Places, you may put three Cyphers towards the Right, and ſo proceed as before. I have made this Example ſo demonſtrative, that I ſhall need to give no more, for by the very ſame Method you may do any other.

Note, That in the Diviſion (by 75) the next of all on the Rods, and leſs than the Dividend 57284 is 52843, againſt which ſtands 7 on the Index, but then had you put 7 in the Root inſtead of 6, you would have found a Number too great to be deducted from the ſaid Dividend or Reſolvend 57284; and therefore I take ſtill the next leſs Number which ſtands againſt 6.

2*d Note*, That the 540 and the 8232 are placed 1 Degree towards the Left-hand, becauſe they would fall ſo if put under the Tens Places of the Cube of the laſt Figure put in the Root; as is done in the common way of Extraction.

Note alſo, That if you have not the Rods of Squares and Cubes, you may do without, either by placing in the Room a Rod, whoſe firſt Number is the ſame with that which falls in the Root (which you may know by comparing the Figures on the Rods toward the Left-hand with thoſe of the Dividend) or by making them (for preſent Uſe) of Paper, according to the many Examples above given of them, till you get the Rods of Squares and Cubes.

SECT. VI. *Notation and Numeration of the Diagonal Circular Inſtrument*, (See *Plate* B.)

This Inſtrument was by me contrived chiefly for Reduction of Money, Weight, or Meaſure into Decimals; or for finding with Eaſe and Speed the Value of thoſe Fractions, and for Multiplication, Diviſion, and Evolution. It is marked on *Plate* B. with *Fig.* 5. and conſiſteth of ſeven Concentrick Circles (beſides the two outermoſt, which are Lines of Numbers) as

1. The outward or largeſt Circle (ſave two) is of equal Parts, to which all the reſt that are leſſer Circles refer in their Uſe; it is firſt divided into 10 Parts, each of which being ſubdivided into 10, makes 100 Parts; and each 100 Part again divided into 10 by the

the Diagonal Line (1000, *a*) &c. divides the whole Circle into 1000 equal Parts. So that for Example: To find in that Circle 767 thousandth Parts, you have 760 in the Arch of that Circle, and 7 in the Diagonal Line, telling the Parts upwards from 760.

2. The second Circle is of English Coin, the whole representing 1 *l.* divided into 20 *s.* each Shilling in 12 Pence, and each Penny by the Diagonal into 4 Farthings. So that to set off 13 *s.* 7 *d.* $\frac{1}{4}$ *q.* look for 13 *s.* 7 *d.* in the Ambit of the Circle, and then from 7 *d.* telling upwards, you see 1 Farthing in the Diagonal.

3. The next inner Circle (which is the third from those of Numbers) is of Beer Measure, being 1 Barrel divided into 4 Firkins, each Firkin into 9 Gallons, and each Gallon into Quarts and Pints as you see.

4. The next inward is a Barrel of Ale divided into 4 Firkins, those into each 8 Gallons, and those again into Quarts and Pints.

5. The fifth Circle inward from those of Logarithms is the Pound Averdupois divided into 16 Ounces, and those each Ounce into 16 Drams.

6. The next inward Circle is the Hundred Averdupois divided into four Quarters, and each Quarter into 28 ℔. And

7. The innermost Circle is Foot Measure, being 1 Foot divided into 12 Inches, and each Inch into 4 Quarters.

And all these Divisions and Subdivisions are actually done, and not imagined only as some Scales are, which are of little or no Use. And any Quantity in the seven inner Circles is easily pointed to by the same Rule as given for the two first; from those of Numbers, only in these there are no Diagonals, there being no need of them, the Parts being sufficiently expressed without.

8. The two outermost Circles are Lines of Numbers, whose Uses follow after 10th.

Reduction of Decimals by the Circular Instrument.

Quest. 1.] What Decimal of a Pound is 13 *s.* 6 *d.* $\frac{1}{4}$ *q*?

Rule.] Lay a Ruler from the Center to 13 *s.* 6 *d.* $\frac{1}{4}$ *q.* in the second Circle, and that will cut in the Diagonal Circle .676, which is the Answer.

Quest. 2.] What is the Value of .876543 of a Pound Sterling?

Rule.] Lay a Ruler to the Center, and to 876, and half in the Diagonal Circle (or equal Parts) and you'll find it to cut in the Circle of Money 17 *s.* 6 *d.* $\frac{1}{2}$ *q.*

Quest. 3.]

Queſt. 3.] What Decimal of a Barrel of Beer is 2 Firkins, 7 Gallons, 3 Quarts, and a Pint?

Rule.] Lay the Ruler to the Center, and the Quantity given in the third Circle from the Line of Numbers, and it will ſhew in the firſt Circle the Decimal required to be .7187, or apparently more than .718 and $\frac{1}{4}$.

Queſt. 4.] What is the Value of .876543 of a Barrèl of Ale?

Rule.] Lay a Ruler to the Center, and the Decimal given in the Diagonal Circle (I mean the four firſt Figures next the Point) and it will cut in the fourth Circle 3 Firkins, 4 Gallons, and about half a Pint.

Queſt. 5.] In 3 Firkins, 5 Gallons, 3 Quarts of Ale, how much Beer?

Rule.] Lay a Ruler from the Center to the Quantity given in the Circle of Ale Meaſure, and it will give in the Circle of Beer Meaſure 3 Firkins, 6 Gallons, 2 Quarts Beer. And in like manner Beer is reduced to Ale Meaſure.

Queſt. 6.] What Decimal of a Pound is 13 Ounces 11 Drams?

Rule.] Lay a Ruler from the Center to 13 Ounces and 11 Drams, and it will cut in the Diagonal Circle .855.

Queſt. 7.] What is the Value of .876543 of a Pound Averdupois?

Rule.] By laying a Ruler to the Center, and the Decimal given, it will ſhew the Value in the fifth Circle from thoſe of Numbers to be 14 Ounces and near half a Dram.

Queſt. 8.] What Decimal of a hundred Weight is 2 Quarters 21 Pound?

Rule.] Lay a Ruler to the Center, and the given Quantity in the ſixth Circle, and it will cut in the Diagonal Circle .687, or .6875.

Queſt. 9.] What is the Value of .876543 of an hundred Weight?

Rule.] Laying a Ruler as in the 7th Queſtion, it will cut in the 6th Circle 3 Quarters 14 Pound.

Queſt. 10.] What Decimal of a Foot is 10 Inches 3 Quarters?

Rule.] Laying a Ruler to the Center, and the Quantity given in the 7th Circle, it will cut in the equal Parts .8958, or viſibly .895 and more.

And a Line of Numbers being delineated without the largeſt or diagonal Circle (as by Rules given for drawing that Line, ſetting the Parts off from the Center) and another to turn round within it; you may by them do moſt Queſtions in Multiplication, Diviſion, and Extraction of Roots very accurately, as by Rules in *Chap.* 8. Or thus; Suppoſe I would do the ſecond Queſtion in

Sect. 4.

Sect. 4. of *Chap.* 8. i. e. If 12 *C.* of Sugar coſt *l* 25 : 4 :— what will 1 Tun coſt?

Turn 12 in the inner Circle, to 20 in the outer Circle; and then againſt 25.2 in the inner, will ſtand 42 in the outer.

Or in Caſe you have only a Cut of this Inſtrument paſted on a ſtrong Board that will not warp; you may do theſe Queſtions by the Compaſſes, and one of the Lines (or the outmoſt Circle) as if 20 require 30; what will 50 require? In the ſecond half Circle, the Extent of a Pair of Compaſſes from 20 to 30, will extend from 50 to 75, the fourth Proportional or Anſwer.

The compound Intereſt of any Sum for Years, is found by the two Lines of Numbers: As that of *l.*650 for five Years at 5 *per Cent.* the fifth Power of 1.05 is the Amount of *l.* 1 for five Years, which multiplied by *l.*650 gives *l.* 829 : 11 :—the Anſwer: For

Turn 1 in the inner Circle, to 1.05 in the outer Circle, then againſt { 1.05, 1.1, 1.16, 1.22 } In the inner Circle ſtands { 1.1, 1.16, 1.22, 1.28 } In the outer Circles. = the Amount of 1 *l.* for 5 Years.

And turning 1 in the inner Circle, to 1.28 in the innermoſt Circle; againſt 650 (the Principal) in the inner Circle, you will find 829½ in the outer Circle, the Amount required.

Many other Queſtions might be propoſed, and Uſes ſhewn of this Inſtrument; but I leave it to the Reader to find out, of his own accord, to exerciſe his Parts.

The End of Inſtrumental Arithmetick.

CHAP. X.

Algebraical Arithmetick.

I. ALGEBRA being a Word derived from the *Arabick Algiabr*, makes it probable, that the *Arabs* were the moſt ancient Profeſſors, and great Proficients in this Art. Although ſome have aſcribed the firſt Invention to the *Chineſe*, *Indians*, or *Perſians*; ſome to the *Græcians*, and that it was firſt propagated by *Plato*, though it was kept a Secret from the Principle, that

the Divulging a Truth was profaning of it. The firſt Treatiſe of this Art is ſaid to be wrote by *Diophantus*, extant in *Latin* and *Greek*; but the firſt *European* Author, was moſt probably, *Lucas de Burgo*, a *Fryar*, who publiſh'd a Book of this Subject in *Italian*, printed at *Venice*, *Anno* 1494. yet *Scipio*, *Cardan*, and others about their time firſt ſhewed the Solution of Cubical Equations. The famous *Vieta* was the Inventor of *Specious Algebra* about 1590. which was improved by *Oughtred*; and the Methods of ſolving Quadraticks, by Changing the ſecond Term, and by Compleating of the Square, were with many other excellent Improvements, introduced by the celebrated *Harriot*, who was contemporary with *Oughtred*, and flouriſhed about the Year 1630. And ſo much may ſuffice for the Invention and Improvement of this great Art.

II. Some define *Algebra* to be, *The Art of Reſtitution and Compariſon.* It is ſometimes called, *The Analitick, or the Art of Enquiring into the fundamental Nature and Reaſon of Propoſitions, and of compoſing and ſolving Equations, and diſcovering Canons and Theorems accordingly.*

III. *Algebra* is either Numeral or Literal; the forme', when a Queſtion is ſolved by the Numbers themſelves; and Literal, when done by Symbols, Letters, or Species, repreſenting the Numbers in a Queſtion.

What I have propoſed for this firſt Section, I ſhall give the Reader in this Method.

1. Shew him the Characters and Signs uſed in *Algebra*.
2. Say what is neceſſary of the Symbols or Letters, repreſenting Numbers in this Caſe.
3. Of the Notation, Numeration, or way of reading Compound *e c* Quantity.
4. Of the Definitions and Explanations of Terms peculiar to, and uſed in *Algebra*.

IV. The Learned *Algebriſts*, for the more commodious and expeditious Way of Proceſs in ſolving Problems, have very wiſely agreed upon *certain Characters*, whereby Quantities, or the Powers of them are expreſſed and connected, and the Mind of the Artiſt is briefly and methodically explained, which would appear inconveniently confuſed, ſhould Words at Length be uſed: Thus,

Signs

Signs or Characters.	Their Signification.	Literal Examples; as,	Numeral Examples: or,
+	More.	$b+a$.	$3+2=5$.
−	Less.	$b-a$.	$3-2=1$.
=	Equal to	$b+a=c$.	$3+2=5$.
⊹	Transposed to the Right.	$a+b=c$, is $a=c-b$.	$2+3=5$, is $2=5-3$.
×	Multiply'd by	$b\times a=ba$.	$3\times 2=6$.
⊹	Transposed to the Left.	$b=c-a$, is $a+b=c$.	$3=5-2$; is $2+3=5$.
÷	Divided by	$b\div a=\frac{b}{a}$	$3\div 2=\frac{3}{2}=1.5$.
∽	The Difference between	$b \backsim a$	$3 \backsim 2=1$.
∠ (or ⊐)	Lesser than	$a \angle b$.	$2 \sqsupset 3$.
∇ (or ⊏)	Greater than	$b \bigtriangledown a$.	$3 \sqsubset 2$.
∴	*So is* (when in the middle of 4 Geometrical Proportionals.)	$b.\ a::c.\ \frac{ac}{b}$	$3.\ 2::5.\ \frac{10}{3}$.
÷∴	Continual Geometrical Proportionals.	$a.\ b.\ d.\ h.\ r.$ ÷∴	$1.\ 3.\ 9.\ 27.\ 81$ ÷∴
√	The Square Root of	$\sqrt{bb}=b$.	$\sqrt{9}=3$.
$\sqrt[3]{}$	The Cube Root of	$\sqrt[3]{aaa}=a$.	$\sqrt[3]{8}=2$.
$\sqrt[4]{}$	The Biquadrat Root of	$\sqrt[4]{aaaa}=a$.	$\sqrt[4]{16}=2$.
$\sqrt[5]{}$	The Root of the 5th Power, or surfolid Root.	$\sqrt[5]{bbbbb}=b$.	$\sqrt[5]{243}=3$. for
aa (or a^2)	The Square of a	$a^2=aa$.	$2^2=4$.
aaa (or a^3)	The Cube of a	$a^3=aaa$.	$2^3=8$.
$aaaa$ (or a^4)	The Biquadrat of a	$a^4=aaaa$.	$2^4=16$.
$aaaaa$ (or a^5)	The 5th Power, or surfolid of a	$a^5=aaaaa$.	$2^5=32$.

Other Characters less used are; $a\sqrt{b}$ used in Surds, signifies a is Multiplied in the Square Root of b.

(∟) Right Angle. (▭) Square. (∠) Angle. (△) Triangle. (⌶) Equilateral. (⊙) Involution, obviating that such a Quantity is squared; and (ω) Evolution (or that the Root is Extracted.)

V. *The Species or Symbols* uſed in *Algebra*, are Letters, which are placed in an Operation to repreſent the Numbers given in any Queſtion; and theſe are commonly Conſonants, as *b*, *c*, *d*, &c. But the Letters put in an Operation by *Algebra* to ſtand for the unknown Number (or thoſe ſought for) are uſually Vowels. And I ſhall obſerve this way in the following Work, but with thoſe Limitations which I think are very neceſſary.

1. I ſhall put ſuch Conſonants to ſtand for the known Numbers as are moſt expreſſive of the Things to which thoſe Numbers relate; and the like for unknown, or if there is but one unknown or Number ſought in a Queſtion, I ſhall always put (*u*) for it.

2. I ſhall not as has been uſual put the Letters in a Product promiſcuouſly, and without Order, but ſhall place them as they have Order or Priority in the Alphabet; whereby one and the ſame Quantity is eaſier ſeen in contracting manifold compound Quantities, and ſeveral other good Uſes may be made thereof.

VI. *Of the Notation, Numeration,* or way of reading Algebraical Quantities, it may be obſerved from the foregoing Signs.

1. Such as are connected with the Affirmative Sign +, as + $a+b+c+d=p$, are to be Read, a more *b*, more *c*, more *d*, equal to *p*.

Note, Which *p*, though it hath no Sign before, is (and the like is always to be obſerved) ſuppoſed to have +, as being always Affirmative.

2. Such Quantities as are connected with the Negative Sign (−) as $a-b-c$, are to be Read thus; *a*, leſs *b*, leſs *c*. And as in the firſt Example, the Numbers ſignified by *b*, *c*, and *d*, are to − be added together to make the Number repreſented by *p*: So in this ſecond Example, either the Sum of the Numbers repreſented by *b* and *c*, are to be deducted from *a*, or elſe *b* and *c* taken from *a* ſinglely, *b* from *a*, and then *c* from the Remainder.

Or ſuppoſe $b+c-d=$ ∽ *p*; that is, the Number repreſented by *d*, being deducted from the Sum of thoſe repreſented by *b* and *c*; the Remainder is equal to the Difference between the Numbers repreſented by *p*, and that unknown Number by *a*.

Note, That ſuch Quantities as have no Numbers ſtanding to the Left-hand, are ſuppoſed to have a Unit or 1; thus *b* is 1*b*, *c* is 1*c*, &c.

In $c-\overline{b+d}-p$ ∽ *a*: Here the Difference between *p* and *a*, is to be deducted from the Remainder, when *b* more *d* is taken from *c*; for 'tis Read, *c* leſs *b* more (*d*) leſs the Difference between *p* and *a*; for

for (*a*) being unknown, it is ſo likewiſe whether the Exceſs lieth in *a* or *p*, therefore the (ᔕ) to repreſent the Difference is neceſſary.

3. Such Quantities as have the Sign × between them, are commonly expreſſed without it, and the Letters put as in a Word; thus $a \times b$, is, *a* multiplied in *b*, or *ab*; $c \times b \times p$, is *c* multiplied in *b*, and that Product in *p*, but is moſtly put thus (*bcp*); and *b*× in *b* is *bb*, $c \times c \times d \times d$ is *ccdd*, or *c* ſquared in *d* ſquared. ×

Or a ᔕ $c + b \times p \times r$ is a ᔕ $c + bpr$, that is the Product of *b*, *p*, and *r* added to the Difference between *a* and *c*.

4. So all Quantities having the Sign (÷) may be, and are moſt commonly expreſſed by way of Fraction, the Quantity to be divided being placed over the other thus, $a - b \div c + p$ will ſtand $\frac{a-b}{c+p}$; *i. e. a* leſs (*b*,) divided by *c* more *p*. ÷

5. Surd Quantities are expreſſed with their proper Marks before them, as $\sqrt{g-a}$ is the Square Root of the Remainder, when *a* is deducted from *g*. So alſo $\sqrt[3]{p+q-r}$ is the Cube Root of the Remainder, when *r* is deducted from the Sum of *p* and *q*. Laſtly, $\sqrt[4]{b - c + \frac{d}{q}}$, which is the Biquadrat Root of the Sum when (*c*) is taken from *b*, and that Remainder added to the Quote of *d*, divided by *q*. $\sqrt{}$ $\sqrt[3]{}$ $\sqrt[4]{}$

But ſome Examples admit of a double Entendre.

As under the ſecond of this 6th Head, wherein $c - \overline{b+d}$: If it were not for the Line over *b* and *d*, inſtead of deducting $b+d$ from *c*, *b* would only be ſhewed to be deducted from the Sum of *c* and *d*.

So alſo in the 4th above, if there were a Line over $b \div c + p$, it would ſtand thus $a - \overline{b \div c + p}$, which without the Line (ſuppoſing $a = 40$, $b = 20$, $c = 6$, and $p = 4$) would be but 2; whereas with the Line 'tis = 38, for

$a - b = 20$		But (with the Line) $a = 40$
is = 2		Leſs (*b*) $= 20$
And that divided by $c + p = 10$		Divided by $c + p$ $= 2$
		Which deducted is = 38,

ſo there is 36 difference. According to the 5th Example above, there is nothing more frequent in ſolving Quadratick Equations, than to have Occaſion in the Canon or Anſwer to uſe Lines to diſtinguiſh

ſtinguiſh what Quantities are to have their Roots extracted, and which are not, as $u=\sqrt{bc+p}-d$; that is, (u) is equal to the Remainder, when (d) is deducted from the Square Root of the Sum of (p), and (b) multiplied in (c), as if b be 16.

The Numbers to which Equation diſcover, from the Letters, the Value of (u), and ſo you muſt unfold any Canon of Species.

But had not the Line been drawn over $bc+p$, it would have ſhewed that the Root is to be extracted of $bc+p-d$, which would give too much.

$c=3$
Product $=48$
More $p=16$
Sum $=54$
Square Root of which is 8
Leſs $d=3$, ſo ($u=5$.)

7. In the new Notation $\frac{1}{a}$ is often expreſſed a^{-1}; $\frac{1}{a^2}$ is a^{-2}; $\frac{1}{a^3}$ is a^{-3}; $\frac{1}{a^4}=a^{-4}$, &c. And is Read one divided by a; 1 divided by a Squared; 1 by a Cubed, &c.

Alſo a^7+b^8, &c. is the 7th Power of a, plus, the 8th Power of b. And $\frac{1}{a+b}$ is wrote $\overline{a+b}^{-1}$, &c. $a^7+b^7\times\frac{1}{a+b}$ is $a^7+b^7\times\overline{a+b}^{-1}$ that is, 1 divided by a more b; and the 7th Power of a more the 7th Power of b multiplied in 1 divided by a more b.

Likewiſe $\frac{1}{a^2}b$, $\frac{1}{a^3}b^2$, &c. is in the new Notation $a^{-2}b$; $a^{-3}b^2$, &c. that is, 1 divided by the Square of a multiplied in b; one divided by the Cube of a multiplied in b ſquared.

And $\frac{a^5+b^5}{a+b}=a^5+b^5\times\frac{1}{a+b}=a^5+b^5\times\overline{a+b}^{-1}$; *i. e.* the ſurſolid of a, more that of b, divided by a more b; is expreſſed alſo $a^5+b^5\times\frac{1}{a+b}$. Or rather $a^5+b^5\times\overline{a+b}^{-1}$.

VII. An Alphabetical Explanation and Definition of moſt Terms uſed in *Algebra*, which the Reader may have Recourſe to as he finds Occaſion; for having well underſtood what is above in this Section, he may paſs this Head, and proceed to *Addition*, &c.

Abſolute Quantity, or *Abſolute Number*.] Is ſuch a known one as poſſeſſeth one Side of an Equation, as $u+cu=b$, here b is the Abſolute Quantity, whoſe Value is known, and is free from any Dependance or Power of any other. Or it is one marked with the Sign +, contrary to Negative marked −.

Affirmative

Affirmative Quantities.] Such as have either no Sign before them, or are marked with +; ſo $a+b$ are both Affirmative Quantities, as by the laſt.

Algoriſm.] Is the Operations in the ſeveral Parts of *Algebra* practically handled.

Algorithm.] The Parts of ſingle Arithmetick, as *Numeration*, *Addition*, *Subſtraction*, &c.

Arithmetick of Infinites.] The ſame as infinite Series, or converging Series.

Adfected Equations.] When in a Quadratick Equation, (which ſee *Sect.* 10. in this Chap.) the unknown Quantity is in 1 Term, and its Square in another.

Analogies.] The ſame as Proportions, as the Analogy of theſe four Quantities, *a. b* : : *c. d*, &c. and becauſe the Rectangle of the two Extreams is equal to that of the means; therefore $ad=bc$, the Analogy being converted to an Equation.

Approximation.] See *Converging Series.*

Arithmetick of *Negatives.*] See *Negative* under *N.*

Binomial Quantities or *Roots.*] Thoſe conſiſting of two Names, Terms, or Members, connected together by the Sign +, as $u+d$, &c.

Coefficients.] Thoſe Numbers or known Quantities in a Quadratick, *&c.* Equation, which are (in the ſecond Term commonly) multiplied in the Root of the firſt or unknown Quantity, as $uu+2bu$, here $2b$ is the Coefficient: And *Note*, That where there is no Number nor Quantity multiplied in the Root in the ſecond Term, then 1 or a Unit is the Coefficient, as $uu+u$ is $=uu+1u$. See *Compleating the Square.*

Compound Quantities.] Are ſuch as are either the Product, Sum, or Difference of two or more ſingle Quantities: Thus (up) is a Rectangle compounded of u and p, $u+d$, or $u-d$, the Sum or Difference of thoſe two Quantities u and d.

Compleating a Square.] This is to Square half the Coefficient: As ſuppoſe two Members of a Square $uu+bu$ be given; to find what Quantity muſt be put to compleat the Square, is to ſquare half the Coefficient or $\frac{b}{2}$, which Square is $\frac{bb}{4}$, ſo that $uu+bu+\frac{bb}{4}$ is a compleat Square: For Proof of which you may multiply $u+\frac{b}{2}$ (which is the Root of that Square) in itſelf. And if this were a Quadratick Equation, this Square of half the Coefficient muſt

muſt be added to both Parts of the Equation, as admit $uu+bu=c$; half the Coefficient being ſquared and added, will make it $uu+bu+\frac{bb}{4}=c+\frac{bb}{4}$.

Compoſing an Equation.] Is the way of proceeding (to reſolve Problems) 'till we make ſome Quantities in the Queſtion equal to ſome others; (ſee *Reſolution of Equations.*) As if I were asked, what Number that is which being multiplied by 7, and that Product divided by 3, the Quotient will make 1000, abating half the Number ſought? Here, to compoſe this Equation, I put u for the unknown Number, or that ſought; then 7 times that is $7u$; that divided by 3, makes $\frac{7u}{3}$. Then putting b repreſent 1000, this Equation is compoſed from the Nature of the Queſtion, *viz.* $\frac{7u}{3}=b-\frac{u}{2}$. (See this Equation reſolved under the word *Reſolution.*) And where the Quantities in the Propoſition or Problem can be reduced naturally into Analogy, then that Analogy is reduced, or an Equation compoſed from it: See *Analogy*.

Conſtruction of an Equation.] Is to demonſtrate Geometrically the Truth of the Equation, and Canon, from Lines or Geometrical Figures laid down for that purpoſe, as Simple and Quadratick from the Sides of Triangles and Arks of Circles; and Cubical from the Parabola and Circle, *&c.*

Converſion of Equations.] Is properly the converting of an Equation into that which is not ſo, *i. e.* into Analogy, as if $ad=bc$; then it follows, that as the Quantity a is to c or b, ſo is the other of them to d, *viz.* $a.b::c.d$. See *Analogy*, of which Example this is the Converſe, and they ought both to be remembred. But ſome call this following, and the like a Converſion, (but I call it Reduction by Multiplication, of which more under *Reduction of Equations*,) viz. $\frac{u}{r}+cd=p$, this by multiplying each Member by the Denominator (r), gives $u+cdr=pr$, &c. of any other.

Converging Series.] Lines converge, when two right ones proceed nearer and nearer to each other, 'till they meet in a Point where they make an Angle: So in Numbers, it is the orderly approaching nearer and nearer to the Truth (as to the Root of a ſurd Number or Quantity) by the common Rules. Examples hereof, ſee *Section* the 13th following: *Note*, Sometimes this Method of Proceſs is called alſo *Arithmetick of Infinites, Infinite Series, and Approximiation. Contraction.*]

Contraction.] This in *Algebra* is to leave out all repeated or superfluous Terms, or ſingle Quantities; as $4ab-3c+5c-3ab$, this contracted is $ab+2c$; for $4ab$, leſs $3ab$, leaves ab; and $5c$, leſs $3c$, leaveth $2c$. So alſo $7uu+3pu-du-3pu$ is $7u-d$; for firſt, (u) being in each Member $1u$, may be left quite out, and then 'tis $7u+3p-d-3p$; ſecondly, $+3p$ and $-3p$ deſtroy each other: So there reſts but $7u-d$ of the Total, and the like may be obſerved in Contraction of other Quantities.

Calculus.] A Term in Fluxions: It is either *Calculus Integralis*, which is the Method of finding a flowing Quantity to any Fluxion given; or elſe *Calculus Differentialis*; which is the way of finding a Fluxion of any flowing Quantity.

Canon.] As it relates to *Trigonometry*, is the Rule or Proportion by which an Angle or Side of a $\triangle$ is found; the Tables of Artificial Numbers (Logarithms, Signs, and Tangents) are ſometimes called *Canons*; but in *Algebra* it is a reſolved Equation, being the Reſult of an Operation, and is the Value of ſome unknown Quantity in known Species, or Numbers, which being wrote down in Words, contains a plain General Rule for the Solution of any Queſtion of like Nature; which is one of the great Excellencies of *Algebra*.

Coſſick Powers.] Algebraick Powers of Number or Quantity; for the firſt, ſee *Chap.* 1. *Sect.* 1. of this Treatiſe; whence it follows, that the Powers of any Quantity will ſtand thus, $u=$ a Root; $u^2=$ the Square or ſecond Power; $u^3=$ the Cube or third Power; $u^4=$ the Biquadrat or fourth Power of u, &c. See Extraction of Roots in this Chapter, *Sect.* 6.

Cubick Equations.] Vide *Sect.* 12. of this Chapter.

Depreſſion.] This Word is uſed in the Solution of Cubick *e c* Equations, where the Equation is depreſſed or reduced from one of the third Dimenſion to a Quadratick; that being one of the ways of reſolving Cubical Equations: Or Depreſſion is the Reducing of an Equation by Diviſion; as $puu+duu=cu-du$ by dividing by $p+d$, that Equation is reduced to $u=\frac{c-d}{p+d}$, firſt expunging or caſting out (u), becauſe found in each Member or Term of the Equation: See this Chapter, *Sect.* 5.

Diſcrete Proportionals.] Are Numbers or Quantities, where there is the ſame Proportion between the third and fourth, as between the firſt and ſecond; but not the ſame between the ſecond and third, as in $u.\ b::b.\ k$, there is the ſame Analogy between b and k,

as between *u* and *b*, but not the like between *b* and *h*, ſo that 'tis alſo called *Disjunct Proportion*; in Number 3. 7 : : 12, 28.

Dimenſions.] In Equations, where the firſt Term is *uuuu*, that is one of 4 Dimenſions; *uuu* is an Equation of 3 Dimenſions; *uu* of 2, *&c.* *u* being unknown.

Equation] An Equation in *Algebra* is, when one or ſeveral known or unknown Quantities are made equal to other Quantities upon a due and regular Proceſs of *Compoſing*, (ſee that Word,) as if $u+c-d=rs+tu$, that is, *u* more *c* leſs *d* is equal to *r* multiplied in $s+tu$, which Equation is reſolved as under *Reſolution*; ſee alſo Simple, Quadratick, Cubical, and Lateral Equations in this part.

Evolution.] Is marked ω, (or the Extraction of a Root,) and tells us, that the Number or Quantity before which it ſtands, is the Square Root of ſome other Step referred to in an Algebraical Operation.

Exponents.] Are thoſe ſmall Figures placed almoſt over any Quantity, to ſhew (or expound) what Power of the ſaid Quantity is expreſſed. See *Coſſick Powers* above. And

Exponential Quantities.] Is a Term in Fluxions, theſe being ſuch Quantities, whoſe Exponents are variable or flowing Quantities.

Fluxions.] Are defined to be the Doctrine, or Algorithm of the infinitely ſmall Increment or Decrement of variable and indeterminate Quantities: For the beſt Notion we have of Generating a Line, is by the Motion of a Point; of a Surface by the Motion of a Line; and of a Solid by the Motion of a Superficies: Now as this Increaſe of a Figure by Local Motion, is termed a *flowing Quantity*, ſo the Velocity of that Increaſe of the flowing Quantity is called *a Fluxion*; and is diſtinguiſh'd from the Character or Symbol of the flowing Quantity by a Point over it, *&c.* which is a kind of Artificial *Algebra*, but is wrought by Rules peculiar to it ſelf, being a late Invention and a great Improvement.

Flowing Quantities.] See *Fluxions* above.

Generating.] In Geometry Points Lines and Superficies generate as under the word *Fluxion*; or in Arithmetick, what is produced by Multiplication or Involution is a *generated Number or Quantity* in *Algebra.*

Homogeneal.] Is *of the ſame kind:* So ſuch ſurd Roots as have the ſame radical Sign are *Homogeneal*, as $\surd ab$, $\surd ac$ *ec.*; $\sqrt[3]{cd}$, $\sqrt[3]{brke}$; $\sqrt[4]{lp}$, $\sqrt[4]{g}$, &c. See this Word in the *Introduction*, Sect. 1.

Homologous.]

Homologous.] In a Proportion of four Quantities. If *a. b* : : *c. d.* here the two Antecedents *a* and *c* are *Homologous*, as are the two Conſequents *b* and *d*, &c. and *a* is *Homologous* to *c*, as *b* to *d*.

Heterogencal.] (*Of different kind*;) So Surds are *Heterogeneal* when their radical Signs are not the ſame, as $\sqrt[3]{bc}$; $\sqrt[4]{tu}$, $\sqrt[5]{rst}$, &c.

Infinite Series.] See *Converging Series* above, and *Chap.* 10. following, *Sect.* 13.

Involution.] Is marked ⊙, and ſhews that the Quantity before which it ſtands is the Square of ſome other which is referred to; the Contrary to *Evolution.*

Ineffable Quantities.] Surds, whoſe Roots can only be expreſſed by the proper radical Signs.

Irrational Quantities.] The ſame as *Ineffable*, laſt above.

Lateral Equations.] Simple Equations, which have but one Root.

Like Signs.] Thoſe Quantities in *Algebra* which have the ſame Sign, as $+b+c+d$, or $-b-c-d$, &c.

Like Quantities.] Such as have the ſame Symbols or Species, as $23bcd$, $14bcd$, $2bcd$, &c. there being the ſame Number of each Letter.

Maximis and Minimis.] Is a Term uſed in *Fluxions.* It is a Method whereby Problems are reſolved, which require the greateſt or leaſt Quantity attainable in that Caſe; it is a ſtable Quantity naturally; and therefore to determine to a *Maximum* or *Minimum* in any flowing Quantity, is to make it a permanent Quantity.

Members of an Equation.] Thoſe Quantities contained between every Sign, as $ab+du-cp=r$; here ab, du, cp, and r, are the four Members or Terms of this Equation.

Moments.] A Term in Fluxions according to ſome; being ſuch Parts of Quantity as are in a continual Flux, either decreaſing or increaſing: Or rather, Moments are the beginning (of the generating Principle) of Magnitude, and are themſelves no Magnitude conceiveable, becauſe infinitely ſmall.

Multinomial.] See *Polynomial.*

Negative Signs.] See under the laſt Article (or Head) among the Characters uſed in *Algebra* for (—).

Negative, or the Arithmetick of Negatives.] Is that kind wherein Negative Numbers or Quantities are employed, which are known by this mark (—) before them.

The Numeration is no more than only adding the Words [leſs than nothing] to what is wrote down; as $-a$, or -7 is read, *a* leſs than nothing, or 7 leſs than nothing, and $-.75$ *ec* is that Decimal leſs than nothing.

Addition.] Shews the adding of Negative, to Negative or Affirmative, as by the Rule of adding the Quantities of like Signs together, and taking the Difference for the Sum; as the Sum of $-7+-3$ is -10; $-7+3$ is -4; and $12+-13=-1$.

Subſtraction.] Is done by changing the Sign of the Subtrahend, and then adding the Quantities; as from -7 take 3, reſts -4; from $--$ take 3, reſts -10; and if from 12 you take -13, there reſts 25, *&c.*

Multiplication.] Is nothing difficult, for you only obſerve, that if Quantities of like Signs be multiplied, the Sign of the Product is Affirmative or $+$; But if the Factors have unlike Signs, the Sign of the Product is Negative or $-$: Thus -7 by -3 is 21, and ſo $+7$ by $+3$; but 7 by -3 is -21, and -12 by 13 is -156, *&c.* And the ſame Rule ſerveth for

Diviſion of Negatives.] As to prove thoſe in Multiplication, $21 \div -7$ quoteth -3; $-21 \div -3$ quoteth 7; and $-156 \div 13 = -12$. All which you'll find farther Illuſtrated, with Variety of Examples, *Sect.* 3. of *Chap.* 7. alſo *Sections* 2, 3, 4, and 5 of this *Chap.* 10. and the laſt of my Examples in *Converging Series* near the end of the Book.

Nome.] Whence Binomial, Trinomial, and Multi- (or Polynomial,) *i. e.* Roots of 2, 3, or more Names; ſee thoſe Words.

Numeral Algebra.] When the Work is performed Algebraically with Figures or Numbers, and not Letters.

Periodical Degrees, or Parodical.] Of the Terms of a Cubical *ec* Equation, are when the Exponents of the Powers therein riſe or fall gradually in an Arithmetical Ratio, as $u^4-59u^3+1154u^2-8080u=-12000$, &c. are regular Biquadraticks, having all their Terms in a gradual Order.

Permanent Quantities.] In Fluxions, ſuch Quantities as are invariable without Motion, or at Reſt, and conſequently have no Fluxions to them.

Powers of Quantities.] See that of Numbers, *Chap.* 1. *Sect.* 1. or Exponents here below; as u or c, &c. are Roots or firſt Powers, uu or cc are Squares or ſecond Powers, uuu or ccc the Cubes or third Powers.

Power;

Power; to raiſe the Powers of Numbers to that of any other.] As ſuppoſe I raiſe the Root (*b*) to the ſame Power as *cccc* or c^4, this is to multiply *b* in it ſelf *ec*, to bring it to the fourth Power or Biquadrat: Or ſuppoſe $\sqrt[3]{82}$ were to be multiplied by 2, here 2 muſt be raiſed up to the ſame Power or Cube with the 82, and it will ſtand either $2\sqrt[3]{82}$, or $\sqrt[3]{8} \times \sqrt[3]{82} = \sqrt[3]{656}$; this will be of Uſe to obſerve, when you come to the Doctrine of ſurd Quantities, *Sect.* 7. of this Chapter.

Polynomial Roots or Quantities.] Thoſe which conſiſt of above three Names or Numbers; as $u+s+t+n$, &c.

Poſitive Quantities.] The ſame with Affirmative, which are thoſe that are marked with or follow the Sign +, or elſe ſuch as have no Sign before them.

Quantity.] In *Algebra*, either that ſought for, or thoſe given in a Queſtion, or they are all thoſe Symbols which repreſent any Number. See *Simple*, *Compound*, *Surd*, and *Variable Quantity*.

Quadratick Equations.] Are ſuch (or reduceable to ſuch) as have the Square of the unknown Quantity in the firſt Term, and the Root (or that multiplied in ſome known Quantity) in the ſecond Term, as $nn+bu+c=rs$. See two Sorts, *Section* the 10th of this Chapter.

Rational Quantities.] Either ſuch whoſe Roots can be extracted accurately; or ſuch as are already the Root of ſome Quantities, or which want no Extraction.

Raiſe the Powers of Number or Quantity.] See *Powers*.

Radical Sign.] Are $\surd$ for the Square Root; $\sqrt[3]{}$ for the Cube; $\sqrt[4]{}$ for the Biquadrat Root, *&c.* ſee *Roots* and the *Powers* in the Article preceding, of Algebraical Characters: And alſo

Roots of Quantities.] Are either (in *Algebra*) put actually down (if of Rational Quantities) or if of Surd Quantities the Root is expreſſed by putting the proper Radical Sign before the Quantity; thus in the firſt Caſe, the Square Root of *bb* is *b*, of *cc* is *c*, &c. the Cube Root of *uuu* is *u*, &c. and in the ſecond Caſe, the Square Root of $u+s$ is $\sqrt{u+s}$; the Cube Root of $u+s$ is $\sqrt[3]{u+s}$; the Biquadrat Root of it is $\sqrt[4]{u+s}$, and the like of any other Surd Quantity. See *Binomial*, *Trinomial*, *Polynomial*, and *Reſidual*. *Note*, That the Surd $c\sqrt{d}$, *ec*, is *c* multiplied in the Square Root of *d*, *ec*.

Reſolution

Reſolution of (or to ſolve) Equations.] After an *Algebriſt* has done with the compoſing an Equation as under that Word, he reſolves it by ſeveral Rules requiſite thereto, as by *Multiplication*, *Diviſion*, *Addition*, *Subſtraction*, *Evolution*, &c. ſo the Equation compoſed under the word *Compoſing* as above, which is $\frac{7u}{3}=b-\frac{u}{2}$; being reſolved by multiplying by the Denominators of the Fraction, and adding $3u$ to each Part of the Equation, dividing by 14 and 3 (or 17) gives the Solution of the Queſtion, or the Cannon $u=\frac{6b}{17}$. Now (b) we find is = 1000 in the Queſtion, ſo (u) the unknown Quantity is eaſily found by dividing 6000 by 17. And the Equation under that Word, *viz.* $x+c-d=rs+tx$ being reſolved, is $x=\frac{d-c+rs}{t-1}$. Particular Rules for reſolving Equations, ſee *Section* the 9th of this Chapter; where it appears, that an Equation is reſolved when the unknown Quantity alone poſſeſſeth one Side of the Equation *a*, here *u* doth.

Reſidual Roots or Quantities.] Such as have a Negative Sign between them, as $bc-d$, &c. or $40-10=30$ the Reſidue.

Simple Quantities.] In a ſtrict Senſe are thoſe ſingle Letters, which repreſent the ſeveral Numbers in a Problem, given or required: Or ſome take them in a larger Senſe, for ſuch Quantities as are not connected by + or − to any other, though they be multiplied in other; of the firſt ſort are *b*, or *c*, or *d*, &c. of the ſecond are $4b$, bc, drs, &c.

Signs.] See the *Table of Characters*, Article IV. of this Section; alſo *Affirmative*, *Negative*, and *Radical*.

Simple Equations.] Such as are free from the Involution or Powers of the unknown Number or Quantity.

Simple Quadratick Equations.] Such as have (or are reduceable to have) the Square of the unknown Number or Quantity in only one of the Members or Terms, as $uu=pq$, that is, $u=\sqrt{pq}$, or $xxu+ub+udc$, which by the Seclufion of *u* in each., is $x=\sqrt{b+dc}$.

Series.] A Rank, Row, or Column of Numbers or Quantities orderly placed. See *Progreſſion and Converging Series*.

Species.] In *Algebra*, are the Letters which repreſent the Numbers in a Queſtion, they are alſo called *Symbols*; ſo ſpecious Arithmetick is not Numeral, but Literal *Algebra*.

Subſtitution.] Is the putting one Symbol or Quantity into the Proceſs of Algebraical Operations, in the room of ſome other: As in the Solution of a Quadratick Equation, by changing the ſecond Term, and as in Fluxions.

Surd Quantities.] Such as cannot have the Root expreſſed, and are therefore marked with their proper *Radical Sign*, which Term ſee above, and alſo *Roots of Quantities.*

Terms.] *Of an Equation*, are commonly ſaid to be the ſeveral Members thereof; (ſee that Word above;) but if in a Quadratick, as $uu+ub+ud=k+c$, the Terms by ſome are called but three Terms, as uu the firſt; $ub+ud$ the ſecond; and $k+c$ the third Term.

Tranſpoſition.] Is the changing the Places of the Members of an Equation, in order to make the unknown Quantities to poſſeſs one Side, and the known the other Side of the Equation; whereby the Knowledge is gained of what Value the Number ſought in a Queſtion, really is, by finding what it is equal to. See *Sect.* 9. of this Chapter, for Rules to perform the ſame.

Trinomial Roots or Quantities.] Are ſuch as have three Nomes or Names, as $u+rs+tu$, &c.

Vaniſh, or be Wanting.] A Quantity is ſo, when by Reduction or Contraction it is expunged or thrown out of an Equation.

Variable Quantities.] Are ſuch as are ſuppoſed in continual Motion, whereby they generate Lines, Superficies, and Solids; the ſame as *Flowing Quantities.*

Unciæ.] Are the Numbers prefixed to the Quantities, which are the Powers of ſome Binomial Roots; as in the Square of $a+b=aa+2ab+bb$, the *Unciæ* is 1, 2, 1, for aa and bb are $1aa$ and $1bb$; ſo in the Cube of $a+b$, viz. $a^3+3aab+3abb+bbb$, the *Unciæ* is 1, 3, 3, 1, *&c.* for according to Sir *Iſaac Newton*'s Rule, the *Unciæ* of any Powers are found (as ſuppoſe the third Power or Cube) by this Rule; $1\times\frac{3-0}{1}$ $(3\times\frac{3-1}{2}$ $(3\times\frac{3-2}{3}$ (1, which in Words is read thus: One multiplied in 3 the Power, leſs 0, divided by 1 is 3= the ſecond of the *Unciæ*; 3, the ſecond multiplied in 3 the Power, leſs 1, divided by 2, gives 3= the third of the *Unciæ*; and 3, the third multiplied in the Power 3, leſs 2, divided by 3, gives 1= the forth of the *Unciæ.*

And by the ſame Rule, the *Unciæ* of the ſurſolid is 1, 5, 10, 10, 5, 1, for every Quote and the firſt 1, are the *Unciæ* ar ſung.

Thus

Thus $1x^{\frac{5-0}{1}}$ ($5x^{\frac{5-1}{2}}$ ($10x^{\frac{5-2}{3}}$ ($10x^{\frac{5-3}{4}}$ ($5x^{\frac{5-4}{5}}$(1 ; see also *Sect.* 6.

Vinculum.] Is a Compound Surd Quantity multiplied by a Fluxion ; in which this is a Term.

Unfold.] To unfold an Aigebraick Canon, expressed Literally ; is to compare the known Quantities with the Numbers which they represent, and working with them as the Canon directs, we discover the Value of the unknown Number, or that sought.

Wanting.] See *Vanish.*

Having in this Section laid a plain and copious Foundation ; I shall be something briefer in the subsequent Rules, but no farther than is consistent with evident Demonstration.

SECT. II. *Addition of Algebra.*

Note $b=70$ — Example 1.

Literal	$9b=630$	Numeral
	$10b=700$	Add
	$b=70$	Add
Sum	$20b=1400$	Sum or Proof.

Note $c=35$ — Example 2.

Literal	$-12c=-420$	Numeral
	$-7c=-245$	
	$20c=700$	
Sum	$=c=35$	Sum or Proof.

Compound Quantities. *Note*, $d=4$; $g=16$; $h=6$.

Example 3.

$3ddh+$	hhg	$=$	$288+$	576
$ddh-$	$2hhg$	$=$	$96-$	1152
$-5ddh+$	hhg	$=$	$-480+$	576
Sum $-ddh$		0	$=-96+$	0 Sum for Proof.

The General Rule for *Addition.*

That the Quantities of like Signs are to be added together, and where there are different Signs, as in the second and third Examples, you must subtract the Sum under one Sign from that under another, and put the Difference down marked with the same Sign as that Sum, wherein was the Excess : As I deduct $19c$ from $20c$ (in the second Example) and the Difference is c for the Sum required.

And in the third Example, (*Colum.* 1.) the Excess lies in the Negative Sign, for taking $4ddh$ from $-5ddh$, the Remainder is $-ddh$ (or $-1ddh$.)

And

And in the ſecond Series of Quantities in that third Example, the Signs making the Quantities to Balance; I therefore put (o) down, all which is proved by the reſpective Numbers.

And whereas it may be thought ſtrange, that a Sum Total ſhould be *ddh*, or 96 leſs than nothing; it is no more than what falls out in many Caſes: For if a Merchant draws on his Factor 480 *l.* (as the Number is in *Example* 3.) and that Factor has but 288 *l.* and 96 *l.*=*viz.* 384 *l.* to pay the Bills in his Hands; 'tis plain, when the Factor has paid what is drawn on him, the Merchant will have 96 *l.* leſs than o in the Factor's Hands, *&c.* And this is alſo plain from the Reaſon given for adding Affirmative and Negative Indices of Logarithms, *Chap.* 4. foregoing; which they who have conſidered, the Addition and Subſtraction of *Algebra* will to them appear evident and reaſonable.

And when Numbers are to be added that have different Quantities annexed, there is nothing to do but to put them down in one Line with their Signs between them; as to add 5*ab* to 3*ad*, is =5*ab*+3*ad*; and *ubc* to *rst* is —*rst*+*ubc*, &c. And the Quantities in the foregoing Examples are truly added by the ſame way, being afterward Contracted as under the word *Contraction* in the laſt Section foregoing.

Sect. III. *Subſtraction of* Algebra.

The General Rule, &c. *ſee after the* Examples.

Example 1.

Note, $b=70$

From	$20b=1400$	
Literal =		= Numeral
Take	$11b=770$	
Reſts	$9b=630$	

Example 2.

From $c=$	35
Take $-19c=$	-665
Reſts $20c=$	700

Note (as in Addition of Compound Quantities) $d=4$; $g=16$, and $h=6$.

Example 3.

From —	ddh	$=-96$
Take	$-5ddh-2ghh=$	$-480-1152$
Reſts	$4ddh+2ghh=$	$384+1152$

And as a fourth Example; if from $4ddh+ghh+bb$, I take $bh+gg-hh-d$. The Remainder will ſtand thus:

$$4ddh+ghh+bb\ ;\ -bh-gg+hh+d.$$

For Proof of which. By the laſt Section it appears that $4ddh = 384$
From this Sum 5860 taking the $ghh = 576$
Sum of the four Quantities in the $bb = 4900$
Subtrahend (added according to the Sum 5860
Rules of Addition,) which is 636, *viz.* $+bb = 420$, $+gg = 256$ } 676; $-bb = 36$, $-d = 4$ } -40 } $= 636$
the Remainder is 5224 as by the Margin 5224
And ſo it will be by this Rule.

The General Rule for Subſtraction, *i. e.*

Change the Sign of the Quantities to be ſubſtracted, and then proceed as in Addition.

So in the firſt Example, $20b - 11b = 9b$, for $1400 - 770 = 630$; which is $9b$, or 9 times 70.

And in the ſecond Example, $c + 19c = 20c$, or $35 + 665 = 700$; which is 20 times 35.

In the third Example; I change the Sign of $2ghh$, (or 1152), and add the ſame to (0), makes $+2ghh = +1152$; and $-5ddh$, or -480 having the Sign changed and added to $-ddh$, and -96 gives $4ddh$, or $384 =$ the Remainder as you ſee. And

In the fourth Example; If the four Signs of the Subtrahend be changed and made as in the Remainder above, that Remainder will prove true and agreeable to the Deduction foregoing, altho' it be done differently and according to the General Rule for Subſtraction of Algebraical Integers, as by the Margin appears, where $5860 - 636 = 5224$.

Note, Theſe Examples prove the Truth of thoſe in Addition.

$4ddh = 384$
$ghh = 576$
$bb = 4900$
Sum $= 5860$
And $-bb = -420$, $-gg = -256$ } -676; $+bb = +36$, $+d = +4$ } $+40$ } $= -636$
Reſts $= 5224$

The Reaſon of the General Rule for Subſtraction.

1*ſt*. Either the Sign of the Subtrahend is $+$ or $-$, if $+$ as to take 2 from 6, or $\left.\begin{matrix}+6\\+2\end{matrix}\right\}$ to take 2 from 6 is (ſpeaking Algebraically) to ſay $6-2$ (or 6 leſs 2) whereby the Sign of the 2 is of Courſe changed.

2*dly*. And

2*dly.* And if the Sign of 2 (*ec*) as given to be ſubſtracted were (−), then by what is ſaid of Subſtraction of Indices of Logarithms, as well as from the Reaſon of the Thing it ſelf; if I take (0) from 6, there will remain 6; but if −2 (2 leſs than 0) from 6, 'tis the ſame as to add 2 to 6, ſo there muſt remain 8; for by how much the leſs any Number is that is deducted, by ſo much the greater is the Remainder; and by how much ſoever the Number ſubſtracted is leſs than 0, by ſo much does it make the Remainder greater than the Sum ſubſtracted from.

Sect. IV. *Multiplication in* Algebra.

Note, $u=25$; $b=12$; $c=20$; $d=4$; $g=5$; $h=10$.

A General Rule, ſee after the Examples.

Prop. 1. To multiply ſimple Quantities by Simple.

Case 1. Multiply $-u=25$ by $-b=12$ — Product $bu=300$

Case 2. Multiply $d=4$ by $g=5$ — Product $dg=20$

Case 3. Multiply $-c=-20$ by $h=10$ — Product $-ch=-200$

Case 4. Multiply $u=25$ by $-c=-20$ — Product $-cu=-500$

Prop. 2. To multiply compound Quantities by Simple.

Case 1. Multiply $2hu+d=500+4$ by $c=20$ — Product $=2chu+cd=10000+80=10080$

Case 2. Multiply $-cd-h=-80-10$ by $-c=-20$ — Product $+ccd+ch=+1600+200=1800$

Case 3. Multiply $2hu+d=500+4$ by $-c=-20$ — Product $-2chu-cd=-10000-80=-10080$

Prop. 3. **To multiply compound Quantities by Compound.**

Case 1. Multiply $bb+g = 100+5 = 105$
by $gg+b = 25+10 = 35$

$gg\,bb+ggg = 525$
$+bbb+gb = 315$

Product $ggbb+ggg+bbb+gb = 3675$ Product.

Case 2. Multiply $-bb-g = -100-5 = -105$
by $-gg-b = -25-10 = -35$

$gg\,bb+ggg = 525$
$+bbb+gb = 315$

Product $ggbb+ggg+bbb+gb = 3675$ Product.

Case 3. Multiply $bb+g = 100+5 = 105$
by $-gg-b = -25-10 = -35$

$-ggbb-ggg = -525$
$-bbb-gb = -315$

Product $= -ggbb-ggg-bbb-gb = -3675$ Product.

To multiply *Coſſick Powers*, ſee at the end of *Diviſion* of *Algebra*.

The General Rule for Multiplication.

If the Signs of both the Factors are like, then the Sign of the Product is Affirmative: But if the Signs of the Factors are different, the Sign of the Product is Negative.

This Rule will appear to be obſerved in the nine Caſes of *Multiplication* as above, which I have demonſtrated by Numbers, which agree to the Quantities of the Factors and Products: For Inſtance, the Product of the firſt Caſe of *Prop.* 3. is as in the Margin, where the Value of the Product is equal to the Rectangle of thoſe Numbers 105 and 35 (the Values of the Factors) being multiplied together.

Quantities.	Values.
$ggbb$ =	2500
ggg =	125
bbb =	1000
gb =	50
Sum =	3675

The

The Reason of the General Rule for Multiplication.

1*st*. *Multiplication* being nothing but the Work of many Additions; it follows, that $-ab$ being added to $-ab$, is $-2ab$, which is the same as multiplying $-ab$ by 2; and so $-gh-2gh-3gh$ added, is $=-6gh$, which is the same as $-gh\times 6$; which shews, that the Negative Quantity $-gh$ by the Affirmative 6, must give the Negative $-6gh$, and the like Reason holds for any other Negative by a positive Quantity.

2*dly*. And as in *Logick*, two Negatives make an Affirmative; so in *Algebra*, one Negative Number or Quantity multiplied by another Negative, produceth an Affirmative: For the Sign — being directly contrary to +, must make a Product of a direct contrary Nature when the Multiplier is —, than (as above) when it is +; as subtracting a Negative, is adding really so much as we seem to subtract, as -2 from 4 is 6.

Sect. V. *Division of* Algebra.

I shall make use of the same Letters and their Values as in the last Section, and the Examples shall be such as to prove both themselves and those in the last Section.

Prop. 1. To divide a simple Quantity by a Simple.

Case 1. Divide $bu=300$ by $-b=-12$

$-b)\ bu\ (-u=$ Quote $=25$ for $-12)\ 300\ (-25=-u$

bu

0

$\cdot\cdot$

60

0

Case 2. Divide $dg=40$ by $g=5$

$g)\ dg\ (d$ the Quote; for 5$)\ 40\ (8=d$

dg

0

0

Case 3. Divide $-ch=-200$ by $h=20$

$h)\ -ch\ (-c$ the Quote for 10$)\ -200\ (-20=-c$

$-ch$

0

00

Prop. 2. To divide a Compound by a simple Quantity.

Case 1. Divide $2chu+cd=10000+80$ by $c=20$

$c)\ 2chu+cd\ (2hu+d$

$2chu$

$+cd$

cd

(0) Rests.

Quote $=2hu+d$. 20) 10080 (504

Case

Case 2. Divide $ccd+cb=1600+200$ by $-c=-20$

$-c)\ ccd+cb\ (-cd-b$	Quote $-20)\ 1800\ (-90$
ccd	$=-cd-b$
$+cb$	
$+cb$	
0	

Case 3.
Divide $-2cbu-cd=-10000-80$ by $-c=-20$

$-c)\ -2cbu-cd\ (2bu+d$	Quote $-20)\ -1008\ 0\ (504$
$-2cbu$	$=2cb+d$
$-cd$	
$-cd$	
0	

Prop. 3. To divide compound Quantities by Compound.

Case 1. Divide $ggbb+ggg+bbb+gb=3675$ by $gg+b=35$

$gg+b)\ ggbb+ggg+bbb+gb\ (bb+g$	Quote. $35)\ 3675\ (105=bb+g$
$ggbb+bbb$ Deduct	175
$ggg-bbb+bbb+gb$ rests of the (Dividend.	(0)

Or $=ggg+gb$, if Contracted.
$ggg+gb$ Deduct.
0 Rests.

Case 2. Divide $ggbb+ggg+bbb+gb=3675$
by $-bb-g=-105$

$-bb-g)\ ggbb+ggg+bbb+gb\ (-gg-b$	$-105)\ 3675\ (-35=$ Quote.
$ggbb+ggg$	$=-gg-b.$
$=$ Quote	525
$0+bbb+gb$	0
$+bbb+gb$	
Rests (0)	

Case 3. Divide $-ggbb-ggg-bbb-gb$
by $-gg-b$

$-gg-b)\ -ggbb-ggg-bbb-gb\ (bb+g=$ the	$-35)\ -3675\ (105$
$-ggbb-bbb$ Quote.	175
$-ggg+bbb-bbb-gb=$ the Rest of (the Dividend.	0

Or $-ggg-gb$ if Contracted.
$-ggg-gb$
(0) Rests.

General

General Rule as to the Signs.

It is eaſily ſeen by the nine Caſes above, that if you divide Quantities by thoſe having like Signs with thoſe divided, then the Signs of thoſe put in the Quote are + or Affirmative; but if the Signs of the Dividend and Diviſor are unlike, then the Sign of thoſe Quantities put in the Quote are Negative.

General Rule as to the Quantities.

Proceed in the Method of common Diviſion, putting thoſe Quantities in the Quote which are not in the Diviſor, as in the laſt Caſe; I ask how often $-gg$ in $-ggbb$? the Anſwer therefore is bb, which I put in the Quote, and multiplying the Diviſor by bb, is $=-ggbb-bbb$ (in the laſt Caſe above,) ſo the Remainder is $-ggg+bbb$; (becauſe the Sign muſt be changed of bbb, to deduct it from $-ggg$;) then becauſe here is $+bbb-bbb$, they deſtroy one another, (as is ſhewed under the word *Contraction* in the firſt Section of this Chapter,) ſo there reſts but $-ggg-gh$: And I find I can have $-gg$ of the Diviſor in $-ggg$ of the Dividual $+g$, by which multiplying the Diviſor $-gg-h$, produceth $-ggg\ gh$, which deducted from the laſt compound Quantity contracted, leaves (o) or nothing for a Remainder.

But though this is pretty enough, and ſerves to ſhew the Coherence of Diviſion in *Algebra*, with that in other kinds of Arithmetick; yet it is far from falling in uſe, ſo much as where the Diviſor in Species will not divide any part of the Dividend; or eſpecially where it wont divide it all, but ſome part; and the Reſt as a Fraction of the Diviſor being put under the remaining part of the Dividend.

The firſt Kind of more common Caſes in Diviſion *are theſe.*

To divide gh by r, quoteth $\frac{gh}{r}$; ſo $pr+st-m$ by c, is $\frac{pr+st-m}{c}$, and alſo $cd-c+m$ by $pr+s-tu$, is $\frac{cd-c+m}{pr+s-tu}$, &c. which are done by only putting the Dividend over the Diviſor.

The

The ſecond Kind of more common Caſes in Diviſion *are.*

Where the Diviſor is found in ſome Quantities of the Dividend, but not in all: As $su+stu+g$ being divided by $s+st$, will ſtand as in the Margin.

So alſo to divide $msu+pru+gh-rs$ by $ms+pr$.

$s+st)\ su+stu+gh\ (u+\frac{gh}{s+st}$
$su+stu$ =the Quote.
$+gh$ Reſts.

$ms+pr)\ msu+pru+gh-rs\ (u+\frac{gh-rs}{ms+pr}$
$msu+pru$ =the Quote.
$gh-rs$ Reſts.

The Truth of theſe two Caſes is eaſily demonſtrated, by ſuppoſing the Species to repreſent any Numbers. But becauſe I ſhall a little further have Occaſion to ſhew how to ſolve Equations by Diviſion, I ſhall refer the Numeral Work thereto.

A learned Friend put me in Mind of giving this for my laſt Example in *Diviſion*.

$a+b)\ a^7+b^7\ (a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6$ = the Quote, which
a^7+a^6b $a+b$ (I prove thus.

$-a^6b+b^7$
$-a^6b-a^5b^2$

$+a^5b^2+b^7$
$+a^5b^2+a^4b^3$

$-a^4b^3+b^7$
$-a^4b^3-a^3b^4$

$+a^3b^4+b^7$
$+a^3b^4+a^2b^5$

$-a^2b^5+b^7$
$-a^2b^5-ab^6$

$+ab^6+b^7$
$+ab^6+b^7$

0

$a^7-a^6b+a^5b^2-a^4b^3+a^3b^4-a^2b^5+ab^6$
$+a^6b-a^5b^2+a^4b^3-a^3b^4+a^2b^5-ab^6+b^7$

Sum $=a^7+b^7$ = the Product contracted.
= the Dividend.

And

And *Note*, That the Differences of any two like Powers of two Quantities, will always be divided by the Difference of the Quantities without any Remainder, as $a^4 - b^4 \div a - b = a^3 + a^2b + ab^2 + b^3$.

2. When the Exponents of the Powers are odd Numbers, the Sum of the Powers can be divided by the Sum of the Quantities without Remainder, as $a^3 + b^3 \div a + b = a^2 - ab + bb$. And as in the above Example at large.

3. Also when the Powers are even, the Difference of the Powers can be divided by the Sum of the Quantities; as $a^6 - b^6 \div a + b = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5$.

Sect. VI. *Of raising the Powers of Numbers and Quantities, and of Extracting the Roots.*

1. By the following Table you have the several Ways of expressing the Powers with their Exponents, both of Numbers and Species; what is said of the Powers of 9 being expressed by their Exponents, being to be taken also in the other Roots 8, 7, 6, 5, 4, 3, 2, and 1, for as 9^{10} is the 10th Power of 9, which by the Table is 3486784401: So is 8^{10} = the 10th Power of 8, which you see is 1073741824, which are the two various Ways of expressing the Powers of Numbers.

2. For the Powers of Quantities, whether Simple as those of u, or compounded by Multiplication as those of ub, viz. uu or u^2, and uuu or u^3, &c. for the second and third Powers of u, or of ub, whose Square is $uubb$, or better u^2b^2, the Cube $= u^3b^3$; you have the best way of Notation in this Table.

A TABLE *shewing the several Ways of expressing the Powers, both of Numbers and Species.*

Exponents.	1st Power.	2d Power.	3d Power.	4th Power.	5th Power.	6th Power.	7th Power.	8th Power.	9th Power.	10th Power.
	Root or 1	Square or 1	Cube or 1	Biquadrat or 1	Surfolid or 1	Squared Cube or 1	2d Surfolid or 1	Squared Biquadrat or 1	Cubed Cube or 1	Squared Surfolid or 1
	2	4	8	16	32	64	128	256	512	1024
	3	9	27	81	243	729	2187	6561	19683	59049
	4	16	64	256	1024	4096	16384	65536	262144	1048576
	5	25	125	625	3125	15625	78125	390625	1953125	9765625
	6	35	216	1296	7776	46656	279936	1679616	10077696	60465176
	7	49	343	2401	16807	117649	823543	5764801	40353607	282475249
	8	64	512	4096	32768	262144	2097152	16777216	134217728	1073741824
	9	81	729	6561	59049	531441	4782969	43046721	387420489	3486784401
Or more Brief.	9	9^2	9^3	9^4	9^5	9^6	9^7	9^8	9^9	9^{10}
In Species thus:	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
	x	xx	xxx	$xxxx$	$xxxxx$	$xxxxxx$	$xxxxxxx$	$xxxxxxxx$	$xxxxxxxxx$	$xxxxxxxxxx$
Or rather	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}
If Compound thus:	xb	$xxbb$	$xxxbbb$	&c.	. . .	. . .	. . .	. . .	. . .	. . .
Or better thus:	xb	x^2b^2	x^3b^3	x^4b^4	x^5b^5	x^6b^6	x^7b^7	x^8b^8	x^9b^9	$x^{10}b^{10}$
	bcd	$b^2c^2d^2$	$b^3c^3d^3$	$b^4c^4d^4$	$b^5c^5d^5$	$b^6c^6d^6$	$b^7c^7d^7$	$b^8c^8d^8$	$b^9c^9d^9$	$b^{10}c^{10}d^{10}$

And the Notation of Powers with fractional Exponents is thus:

Roots.	Squares.	Cubes.	Biquadrat.	Surfolid.	Spuared Cube.	Second Surfolid.	Squared Biquad.	Cubed Cube.	Squared Surfolid.
$a^{\frac{1}{2}}$	a	$a^{1\frac{1}{2}}$	a^2	$a^{2\frac{1}{2}}$	a^3	$a^{3\frac{1}{2}}$	a^4	$a^{4\frac{1}{2}}$	a^5
$a^{\frac{1}{3}}$	$a^{\frac{2}{3}}$	a	$a^{1\frac{1}{3}}$	$a^{1\frac{2}{3}}$	a^2	$a^{2\frac{1}{3}}$	$a^{2\frac{2}{3}}$	a^3	$a^{3\frac{1}{3}}$
$a^{\frac{1}{4}}$	$a^{\frac{1}{2}}$	$a^{\frac{3}{4}}$	a	$a^{1\frac{1}{4}}$	$a^{1\frac{1}{2}}$	$a^{1\frac{3}{4}}$	a^2	$a^{2\frac{1}{4}}$	$a^{2\frac{1}{2}}$, &c.
Or with Literal Exponents, thus:									
$a^{\frac{m}{n}}$,	$a^{\frac{2m}{n}}$,	$a^{\frac{3m}{n}}$,	$a^{\frac{4m}{n}}$,	$a^{\frac{5m}{n}}$,	$a^{\frac{6m}{n}}$,	$a^{\frac{7m}{n}}$,	$a^{\frac{8m}{n}}$,	$a^{\frac{9m}{n}}$,	$a^{\frac{10m}{n}}$, &c.

Some

Some Obſervations *upon this Table.*

1. Any two Exponents added, give that of the Product of the reſpective Powers: As where the Root is 3, and the Exponents 4 and 5. I ſay, the Sum of 4 and 5 =9, which is equal to the Exponent of the Rectangle of the reſpective Powers 243×81=19683.

2. Double any Exponent, gives the Exponent of the Square of the reſpective Power, as in the 4th Power of 5, which is 625. I ſay, double the Exponent is 8, which is the Exponent of the Rectangle of 625×625, *viz.* of 390625.

3. I obſerve that 3, 4, *ec* times any Exponent, gives the Exponent of the 3d, 4th, *&c.* Powers: As three times the Exponent 2 is 6, ſo the Cube of the Powers 9, 16, *ec* under the Exponent 2, gives 729, 4096, *ec* the reſpective Powers under the Exponent 6.

4. If an 8th Power be to be divided by a 2d Power, the Exponent 2 taken from the Exponent 8, leaves the Exponent 6; ſo any of the Numbers under the Exponent 8, (as ſuppoſe 256,) divided by any of thoſe reſpectively under the Exponent 2, (as for inſtance 4,) the Quote will be reſpectively the Powers under the Exponent 6, (in this Example 64.)

5. The Cube *ec* Root of any Power, is the Power reſpectively of the Exponent of that Power given, divided by 3*ec*; ſo the Cube Root of 262144 (its Exponent being 9, a third of which is 3,) is 64, found under the Exponent 3, and againſt the Power given. So likewiſe the Surſolid Root of 1024, (or any of the Numbers under the Exponent 10,) is 4, (or the reſpective Numbers under the Exponent 2, the Quote of 10 divided by 5; for 4^5=1024 the Proof. So much for raiſing the Powers of Quantities compounded by *Multiplication*; I proceed to ſhew,

III. *How to raise the Powers of Quantities compounded by* Addition *or* Subſtraction, *i. e. from a Binomial or Reſidual Root, and to extract Roots.*

$a+b$ = the Root or 1ſt Power.
$a+b$

$aa+2ba+bb$ = the Square or 2d Power of $a+b$.
$a+b$

$aaa+2baa+bba$
$+\ baa+2bba+bbb$

$a^3+3baa+3bba+bbb$ = the Cube or 3d Power of $a+b$.
$a+b$

$a^4+3ba^3+3bbaa+ab^3$
$+\ ba^3+3bbaa+3ab^3+b^4$

$a^4+4ba^3+6b^2a^2+4ab^3+b^4$ = the Biquadrat or 4th Power of $a+b$.
$a+b$

$a^5+4ba^4+6b^2a^3+4a^2b^3+ab^4$
$+\ ba^4+4b^2a^3+6a^2b^3+4ab^4+b^5$

$a^5+5ba^4+10b^2a^3+10a^2b^3+5ab^4+b^5$ = the Surſolid or 5th Power of $a+b$.
$a+b$

$a^6+5ba^5+10b^2a^4+10a^3b^3+5a^2b^4+ab^5$
$+\ ba^5+5b^2a^4+10a^3b^3+10a^2b^4+5ab^5+b^6$

$a^6+6ba^5+15b^2a^4+20a^3b^3+15a^2b^4+6ab^5+b^6$ = the Squared Cube of $a+b$.
$a+b$

$a^7+6ba^6+15b^2a^5+20a^4b^3+15a^3b^4+6a^2b^5+ab^6$
$+\ ba^6+6b^2a^5+15a^4b^3+20a^3b^4+15a^2b^5+6ab^6+b^7$

$a^7+7ba^6+21b^2a^5+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7$ = the 2d Surſolid of $(a+b$.

$a^8+8ba^7+28b^2a^6+56a^5b^3+70a^4b^4+56a^3b^5+28a^2b^6+8ab^7+b^8$ = the 8th (Power.

$a^9+9ba^8+36b^2a^7+84a^6b^3+126a^5b^4+126a^4b^5+84a^3b^6+36a^2b^7+9ab^8+b^9$ (= the 9th Power.

$a^{10}+10ba^9+45b^2a^8+120a^7b^3+210a^6b^4+252a^5b^5+210a^4b^6+120a^3b^7+45a^2b^8+10ab^9+b^{10}$ = the 10th Power of $a+b$.

Obſervations

Obſervations from this laſt Table of Powers.

1. That the Sum of the Square of the Parts of any Number or Quantity more the double Rectangle is = the Square of the whole, as *per Euclid.*

2. That the foregoing Example of Powers may ſerve as a Table for any one to know any Power of any other Binomial Roots by, tho' of other Letters, *mutatis mutandis*; the Unciæ and Powers being the ſame as in this.

3. That the 10th Power is eaſily produced from the 9th, or any ſubſequent Power from the preceding, by firſt finding the Unciæ as *per* the Rules foregoing, or as under the 5th Head following, and conſidering the Courſe of the Powers that precede that which you would find: By which Method I have produced the 8th, 9th, and 10th Powers without any Multiplication.

4. And that the Reader may not want a proper Way of expreſſing in Words any Power, I ſhall give an Example how the 10th Power is properly Read; which is thus:

The ſquared Surſolid of u, more 10 times the Rectangle of b in the cubed Cube of u; more 45 times the Square of b in the ſquared Biquadrat of u; more 120 times the 2d Surſolid of u in the Cube of b; more 210 times the ſquared Cube of u in the Biquadrat of b; more 252 times the Surſolid of u in the Surſolid of b; more 210 times the Biquadrat of u in the ſquared Cube of b; more 120 times the Cube of u in the 2d Surſolid of b; more 45 times the Square of u in the ſquared Biquadrat of b; more 10 times u in the cubed Cube of b; more the ſquared Surſolid of b.

5. *A very brief Way of finding the Uncia of any Powers by that of the next preceding Power is this*; (which I have not ſeen given before.)

A Second Way of finding the Unciæ.

Add the Unciæ of the 1ſt and 2d Members (for Example of the 9th Power to find the Unciæ of the 10th) together they make 10, which is that of the 2d Member of the 10th Power, 36 in the 3d, and 9 in the 2d Members of the 9th Power is the 3d Member in the 10th or 45; 36 and 84 in the 3d and 4th Members is 120 belonging to the 4th Member of the 10th Power, *&c.* of the reſt.

And the Powers themſelves, and their proper Exponents are found by the like Method, obſerving, That in which of the 2 Powers ſoever added,

added, is the greateſt Exponent, that Exponent muſt be put in the Sum.

Thus to make the 10th Power by theſe Rules from the 9th. The Unciæ of the 1ſt and 2d Members is 10= the 2d in the 10th; and the Sum of the Powers is bu^9; (becauſe 9 is found the higheſt Exponent of u and b is only the 1ſt Power.) So $10bu^9$ is the 2d Member of the 10th Power. The 3d Member of the 10th is the 2d and 3d Members of the 9th, *&c.* done as in that laſt mentioned.

And this will be made plain by a Numeral Example of finding the Unciæ to the 15th Power of a Binomial, by an eaſy Addition of only two Numbers; which Unciæ, with thoſe of all the intermediate, are found much ſpeedier than that of the 15th Power alone can be by the Method before-mentioned; which requires Subſtraction, Multiplication, and Diviſion, and thoſe pretty tedious ones before thoſe of the 15th Power can be exhibited by this Method.

A New Table of the Unciæ to the 15th Power of a Binomial Root, made by Addition only.

1	1														
1	2	1													
1	3	3	1												
1	4	6	4	1											
1	5	10	10	5	1										
1	6	15	20	15	6	1									
1	7	21	35	35	21	7	1								
1	8	28	56	70	56	28	8	1							
1	9	36	84	126	126	84	36	9	1						
1	10	45	120	210	252	210	120	45	10	1					
1	11	55	165	330	462	462	330	165	55	11	1				
1	12	66	220	495	792	924	792	495	220	66	12	1			
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1		
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1	= the Unciæ the 14 Pow
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1 = the Unciæ the 15 Pow

This Method, to me, is intirely New, and my own Contrivance: But ſee a Third Way a little farther.

6. It is obſervable, that the Unciæ encreaſe till you come to the middle, and then decreaſe; there being the ſame Unciæ to the Right-hand of the higheſt, as towards the Left; ſo that when you

you have found half the Unciæ, you have all: Obſerving farther, That there are two higheſt Unciæ in the middle of the odd Powers, as the 3d, 5th, 7th, &c. Powers.

And that the Exponents of ſeveral Members after you come to the higheſt, do change, as in the 10th Power in the former Table above; the Member toward the Left-hand of the higheſt is $210u^6b^4$; but toward the Right of the higheſt, the Power is $210u^4b^6$, which is the ſame, only the Exponent's changed: So the 2d Member towards the Left-hand is $120u^7b^3$; but 'tis $120u^3b^7$: The 3d $=45u^8b^2$; but 'tis $45u^2b^8$. } toward the Right.
The 4th $=10u^9b$; but 'tis $10ub^9$, &c. }

7. You may ſee that four of the Unciæ, *viz.* the two firſt and two laſt are always known in all Powers, being one; and the Exponent of the Power. And by theſe Rules the Unciæ, the Powers, and Exponents of any Binomial Root may be found with much leſs Trouble than the common Way, or any I have ſeen before.

8. But the Unciæ of any Term, or Member of any of the Powers in the Table above, may be found without any of the previous Unciæ of Powers or their Terms, by eaſy Operations in Multiplication and Diviſion, as by theſe 6 Examples.

A Third Way of finding the Unciæ.

Term or Member.	Power.	Factors.	Products.	Quote or Unciæ required.
4 of the	4	1,2,3 =	6 the Diviſor	= 4
		4,3,2 =	24 the Dividend	
4	5	1,2,3 =	6 the Diviſor	= 10
		5,4,3 =	60 the Dividend	
5	10	1,2,3,4 =	24 the Diviſor	= 210
		10,9,8,7 =	5040 the Dividend	
7	10	1,2,3,4,5,6 =	720 the Diviſor	= 210
		10,9,8,7,6,5 =	151200 the Dividend	
5	15	1,2,3,4 =	24 the Diviſor	=1365
		15,14,13,12 =	32760 the Dividend	
4	16	1,2,3 =	6 the Diviſor	= 560
		16,15,14 =	3360 the Dividend	

Here it may eaſily be ſeen that I tell the Figures from one to within one of the Term given, and multiply them together *for a Diviſor*.

And

And telling as many from the Power given backward; I multiply them one in another for a Dividend; whence the Unciæ of any Term of any Power ariſeth by Diviſion.

For ſuppoſe p = the Exponent of any Power, and t = the Term, and u = the Unciæ ſought, the Rule will be

$\frac{p \times p-1 \times p-2 \times p-3, \&c.}{t-1 \times t-2 \times t-3 \times t-4, \&c.} = u$ (in this Example the Unciæ of the 5th Term or Member of the 10th Power = 210.)

9. Any Number may be ſquared by the Canon exhibited in the Square, or 2d Power of the above Table of the Powers raiſed from a Binomial Root. For Example; What is the Square of 1234^9?

The Analytical Square or Canon directing us how to work, is $uu + 2ub + bb$. Now ſuppoſe 1200, the 2 firſt Figures = a

And 34 the 2 next the Right-hand = b

It follows, That $u^2 = 1200^2 = 1440000$

$2ub$ or $u \times b \times 2$ is = 81600 = $1200 \times 34 \times 2$

bb (or 34×34) is = 1156

Sum = 1522756 = $uu + 2ub + bb$. = 1234^2

10. And by the Cube or 3d Power we have a Rule how to Cube any Number. Example: What is the Cube of 2232? The Canon = $u^3 + 3u^2b + 3ub^2 + b^3$.

I ſay 2000 = u

200 = b

8000000000 = uuu or 2000^3

2400000000 = $3uub$ = $2000^2 \times 200 \times 3$

240000000 = $3ubb$ = $2000 \times 200^2 \times 3$

8000000 = bbb = 200^3.

10648000000 = Sum or 2200^3 or uuu in the 2d Operation; where

2200 = u

and 30 = b

43560000000 = $3uub$, = $2200^2 \times 30 \times 3$

594000000 = $3ubb$ = $2200 \times 900 \times 3$

27000000 = bbb = 30^3.

11089567000 = Sum, or 2230^3. So in the 3d Operation 2230 =

and 2 =

2983740000 = $3uub$ = $2230^2 \times 2 \times 3$

26760 = $3ubb$ = $2230 \times 4 \times 3$

8 = bbb = 2^3.

11119431168 = 2232^3 = $uuu + 3uub + 3ubb + bbb$.

Thi

This proves that the third Power in the foregoing Table is the true Cube of $a+b$, as is here demonstrated by Numbers working with the Species exactly according to the Canon, without having regard to the way of placing Numbers in common *Multiplication*.

11. But that which renders the Powers of a Binomial Root more admirably useful is, that they shew plainly the Reason of all those mysterious Rules given in Extracting the Root of any Power, as the Square, Cube, Biquadrat, *&c.* Roots in the former Part of this Book mentioned: For Example,

In Extracting the Square Root of 1522756.

Canon for the Square Root. $=aa+2ab+bb$.

1522756 (1000$=a$
1000000 $=aa$ deduct.

÷ this Rest by $2a=2000$) 522756 Rest, (200$=b$=the Quote.

400000 $=2ab$.
40000 $=bb$.

440000 $=2ab+bb$, deduct from the last Rest.

÷ by $2a=2400$) 82756 remains, & now 1000 + 200=1200$=a$
(30$=b$, the Quote.

72000 $=2ab$.
900 $=bb$.

72900 $=2ab+bb$, deduct from the last Remainder.

÷ by $2a=2460$) 9856 $=$Rest; & now 1230=the 3 Roots$=a$.
(4$=b$=the Quote or last in the root sought.

9840 $=2ab$.
16 $=bb$.

9856 $=2ab+bb$, deduct from the last Rest.

0 Rests.

And the Reason of Extracting the Cube Root, will appear in the following Example, proceeding according to this Canon $aaa+3aab+3ab^2+b^3$.

Extract the Cube Root of 111194311168 (2000=*a*.
8000000000 =*aaa*, deduct.

12006000) 3119431168 resteth, which divide by $3a+3a$, (200=*b*. (gives 200=*b*.

6000 =3*a*, the Treble Root.
12000000 =3*aa*, = the Treble Square of (the Root *a*.
12006000 = $3a+3aa$, the Divisor.

8000000 =*bbb* or 200^3.
2400000000 =3*abb*=6000×40000.
2400000000 =3*aab*=4000000×200×3.

2648000000 =*bbb*+3*abb*+3*aab*=the subtrahend (to deduct from the last resolvend.

14526600) 4714431168 resteth the second Resolvend.

(30=*b*, =the Quote.

6600 = Treble Root 2200=3*a*.
14520000 =3*aa*=2200²×3.

14526600 =$3a+3aa$=the Divisor for 2d (Resolvend, gives 30=*b*.

27000 =*bbb*=30³.
5940000 =3*abb*=6600×900.
435600000 =3*aab*=4840000×30×3.

441567000 =*bbb*+3*abb*+3*aab*=the subtrah.

14925390) 29864168 = third Resolvend Rests.

(2=*b*, the Quote.

6690 =3*a*, the Treble Root =2230×3.
14918700 =3*aa*=4972900×3.

14925390 =$3a+3aa$, the Divisor for the 3d (Resolvend.

8 =*bbb*=2³.
26760 =3*abb*=6690×4.
29837400 =3*aab*=14918700×2.

29864168 =*bbb*+3*abb*+3*aab*, the Sum or Subtrahend, which taken from the 3d Resolvend.

(0) there Rests (0)

So the Root is found 2000+200
+30+2, And

And the Proceſs according to the Canon of the third Power of $a+b$, is repeated three times, *viz.*

1ſt,	2000 = a,	and 200 = b	or	2000	
2dly,	2200 = a,	and 30 = b	+	200	= 2232, the Root required.
3dly,	2230 = a,	and 2 = b	+	30	
			+	2	

Theſe Examples, according to the 2d and 3d Powers of the Binomial Roots in the Table above, fully ſhew the Reaſon of the Method uſed in the Extraction of the Square and Cube Roots. And by the Obſerving of the Canon given by the Powers, the Root of any other, even the ſquared Surſolid may be extracted: As I did that of the Biquadrat in *Sect.* 6. of *Chap.* 1. Article the 2d; having never ſeen it done but only proceeding purely as the 4th Power directs.

IV. The Roots of Algebraick Quantities are either thoſe of Rationals or Surds. 1. The Roots of Rational Quantities, whether theſe Quantities be compounded by Multiplication of Single, Binomial, or Reſidual Roots are thus expreſs'd.

1. How Single Roots of Powers are expreſs'd in Species and Numbers.

	Powers. in Species.	in Numb.	Their Roots. Species.	Numb.
Square	$uu =$	4	$u =$	2
Cube	$bbb =$	27	$b =$	3
Biquadrat	$c^4 =$	256	$c =$	4
Surſolid	$d^5 =$	3125	$d =$	5
Squared Cube	$h^6 =$	46656	$b =$	6
2d Surſolid	$k^7 =$	823543	$k =$	7

2. How Binomials and Reſidual Roots are extracted.

Binomial Powers. Species.	N°.	Their Roots. Species.	N°.	Reſidual Powers. Species.	N°.	Their Roots. Species.	N°.
$uu+2ub+bb =$	25.	$u+b =$	5	$dd-2du+uu =$	9.	$d-u =$	3
$uu+2u+1 =$	9.	$u+1 =$	3	$bb-2b+1 =$	4.	$b-1 =$	2
$16hh+8hk+kk =$	961.	$4h+k =$	31	$16hh-8hk+kk =$	289.	$4h-k =$	17
$bbuu+2buu+uu =$	64.	$bu+u =$	8	$bbuu-2buu+uu =$	16.	$bu-u =$	4

3*dly*, Surd Roots, or Ropts of Surd Quantities are expreſs'd thus by their own Radical Signs.

Simple Surd Quantities.

	Powers. Roots.	Powers. Roots.
Square Root of	$x = \sqrt{x}$ or in N°.	$171 = \sqrt{171}$
Cube Root	$l = \sqrt[3]{b}$	$315 = \sqrt[3]{315}$
Biquadrat.	$c = \sqrt[4]{c}$	$517 = \sqrt[4]{517}$
Surſolid	$d = \sqrt[5]{d}$	$735 = \sqrt[5]{735}$
Squared Cube	$g = \sqrt[6]{g}$	$379 = \sqrt[6]{379}$
2d Surſolid	$b = \sqrt[7]{b}$	$1231 = \sqrt[7]{1231}$
Squared Biquad.	$k = \sqrt[8]{k}$	$339 = \sqrt[8]{339}$
Cubed Cube	$l = \sqrt[9]{l}$	$1129 = \sqrt[9]{1129}$
Squared Surſol.	$m = \sqrt[10]{m}$ . . .	$1057 = \sqrt[10]{1057}$

Compound Surd Quantities.

Powers.	Roots.
$x+b$	$\sqrt{x+b}$
$x+b+c$	$\sqrt[3]{x+b+c}$
$x+b+c+d$	$\sqrt[4]{x+b+c+d}$
$x+b+c+d-g$	$\sqrt[5]{x+b+c+d-g}$
$x+b+c-d+g+b$	$\sqrt[6]{x+b+c-d+g+b}$
$x+b+c-d+g-b-k$	$\sqrt[7]{x+b+c-d+g-b-k}$
$x+b-c+d+g+b+k+l$ . . .	$\sqrt[8]{x+b-c+d+g+b+k+l}$
$x+b+c+d-g-b+k+l+m$ . .	$\sqrt[9]{x+b+c+d-g+k+l+m}$
$17r+gb-bc+m$	$\sqrt[10]{17r+gb-bc+m}$

It muſt be obſerved, That the Sign connecting the Reſidual Root is always the ſame with that of the double Rectangle, *i. e.* Negative as under the 2d above.

Having fully ſhewn the Nature of Raiſing the Powers of Quantities, and Extracting the Roots, and illuſtrated the ſame by Numbers,

Numbers, it naturally leads me to *The Arithmetick of Surd Quantities*; which I ſhall alſo demonſtrate by Numbers.

SECT. VII. *The Algorithm of Surd Quantities.*

In regard there is often Occaſion for Working the Roots of Species or Numbers, as by Addition, Subſtraction, Multiplication, *&c.* of them; and that it would be tedious and leſs accurate to find near the real Roots of Numbers, and impoſſible to be done of all Algebraick Quantities; therefore the Buſineſs of this Section is to give Rules for ſuch Operations in Powers without regarding the Roots farther than by the Radical Signs prefixed: So that the compleat Powers of Quantities, or Numbers may hereby be added, ſubſtracted, *&c.* as well, (and much ſooner) than if the real Roots were known, and the Work done by them. See Article 7. Sect. 1. of this Chapter.

I. *Reduction of Surd Quantities.*

Reduction is here taught firſt, becauſe Quantities are required to be reduced to a common Radical Sign before thoſe that have different Signs can be multiplyed, divided, *&c.*

Example 1. Reduce $\sqrt{u}$ and $\sqrt[4]{c}$ to Surds having a common Radical Sign; or Surds Heterogeneal to Homogeneal. See the Operation.

$\underline{4_2 \quad 4}$ The common Radical Sign.
$\underline{2, \quad 1}$ = The loweſt Terms of the given Radical Signs or Exponents.
$\underline{\sqrt[2]{u}, \quad \sqrt[4]{c}}$ = The Surds given to be reduced.
$\underline{\sqrt[4]{uu}, \sqrt[4]{c}}$ = The Anſwer or the Surds given in a common Radical Sign.

By the Operation 'tis plain, that firſt I take half the Radical Sign 2 (given) and put it over the other given, or 4; and half the given Radical Sign 4, and place it over the other given or 2, which are the loweſt Terms of the given Radical Sign.

2*dly*, I multiply the Radical Signs given by thoſe ſtanding over reſpectively, and they produce 4 = the common Radical Sign required.

3*dly*, I

3*dly*, I raiſe the Surd Roots given to the Powers of the loweſt Terms of the given Radical Signs which ſtand over the reſpective Roots; as *u* raiſed to the 2d Power is *uu*, *c* in the 1ſt Power is $=c$, and placing the common Radical Sign before this *uu* and the *c*, I have the Anſwer $\sqrt[4]{uu}$ and $\sqrt[4]{c}$.

Example 2d ſhall be in Numbers, as to reduce $\sqrt[2]{9}$ and $\sqrt[4]{16}$ to a common Radical Sign.

See the Work by the ſame Rule as above; and in Multiplication and Addition 'tis ſhewn how theſe prove each other, or how the Totals agree, and conſequently prove the Truth of this Method of Reduction.

4, 4=Common Radical Sign.
2, 1=Leaſt Terms of given Radical Signs, placed interchangeably.
$\sqrt[2]{9}, \sqrt[4]{16}$=Surds given.
$\sqrt[4]{81}, \sqrt[4]{16}$=Surds required, or thoſe given reduced.

II. *Multiplication of Simple Surds.*

In this Article I ſhall ſhew,

1. How to multiply a Surd by a Rational Number or Quantity.
2. To multiply a Surd in others of the ſame kind, or the ſame Radical Sign.
3. To multiply Surds together having different Radical Signs.

1. *To multiply a Surd in a Rational Number or Quantity.*

This is expreſſed two ways, for Example, to multiply $\sqrt[4]{c}$ by *b*, the Anſwer is very often expreſſed thus, $b\sqrt[4]{c}$ or *b* in the Biquadrat Root of *c*. But actually to multiply them is done by this

Rule.] Raiſe *b* up to the Power of the Surd given, as here *b* in the fourth Power is *bbbb*, and putting the ſame Radical Sign before, is $\sqrt[4]{bbbb}$, which being multiplied by $\sqrt[4]{c}$, gives $\sqrt[4]{bbbbc}$.

Or in Numbers, ſuppoſe $c=16$, and $b=8$, then $b^4=4096$, and $\sqrt[4]{16}$ multiplied in $\sqrt[4]{4096}=\sqrt[4]{65536}$ the Product, and that Root being Extracted is $=16$, the Product in a Rational Number.

To

To prove both which Ways of working, by raiſing the Power of the Rational Number, to be Genuine,

I ſay, c being $=16$, its $\sqrt[4]{}$, $=2$, which multiplied by $b=8$, gives 16 as before.

2. *To multiply a Surd Number by a Surd, which are Homogeneal, or when the Radical Signs are the ſame.*

Rule.] Multiply the one by the other, and the Product is the Anſwer, prefixing one of the Radical Signs given.

Example. Multiply $\sqrt[3]{dr}$ by $\sqrt[3]{u}$, and the Product is $=\sqrt[3]{dru}$, and putting $u=8$; $r=9$; and $d=3$. It is thus done in Numbers, $\sqrt[3]{27}$ by $\sqrt[3]{8}=\sqrt[3]{216}=6$ the Product.

Proved thus: By the real Roots without Surds. I ſay $\sqrt[3]{27}=3$; and $\sqrt[3]{8}=2$, and 2 times 3 is 6, as before in the Numerical Way laſt above, which is the ſame as the Literal.

3. *To multiply Heterogeneal or Surds, having different Radical Signs.*

Rule.] Reduce them to a common Radical Sign, and then work as in the laſt.

Example. Multiply $\sqrt[4]{c}$ by $\sqrt[2]{u}$. In a common Radical Sign, the Surds are $\sqrt[4]{uu}$ and $\sqrt[4]{c}$; whoſe Product is $\sqrt[4]{cuu}$.

Or in Numbers: $c=16$; and $u=9$; or $\sqrt[4]{16}$ to be multiplied by $\sqrt[2]{9}$, which in a common Radical Sign are $\sqrt[4]{16}$ and $\sqrt[4]{81}$, whoſe Product is $\sqrt[4]{1296}=6$.

Rule proved thus: By the real Roots, for $\sqrt[4]{16}=2$; and $\sqrt[4]{81}=3$; and $3\times2=6$.

III. Diviſion *of Surd Quantities.*

There is nothing of Difficulty in this Rule; for if the Quantities of the Dividend can be divided by thoſe of the Diviſor, and the Radical Signs are the ſame, work as is taught in *Diviſion of Algebra*, or of whole Numbers; as $\sqrt{ubc}\div$ by $\sqrt{bc}$ the Quote $=\sqrt{u}$, &c.

Or

Or in Numbers; $\sqrt{144} \div \sqrt{9} = \sqrt{16} = 4$, or $= 12 \div 3$. For Proof,

2. If the Radical Signs are not the ſame, they may be reduced to a common Radical Sign, and then divide the Power of the Dividend by the Diviſor; or put it over that of the Diviſor, as if the $\sqrt[4]{cuu}$ be divided by $\sqrt[2]{u}$. In a common Radical Sign they are $\sqrt[4]{cuu}$ for the Dividend, and $\sqrt[4]{uu}$ for the Diviſor; ſo the Anſwer is $\sqrt[4]{\frac{cuu}{uu}} = \sqrt[4]{c}$. And this proves the laſt Example in the laſt Rule, and is farther proved by dividing the Numeral Rectangle there by one Factor, for the Quotient will be the other Factor.

IV. Addition *of Surd Quantities.*

This Rule is placed after *Multiplication*, becauſe that is often uſed in this Rule of *Addition*. I ſhall ſhew

1. How to add when there are the ſame Quantities and Radical Signs.

2. When there are different Quantities or Species, but the ſame Radical Signs, which Quantities are ſuppoſed Commenſurable.

3. How to add Quantities, having the ſame Species, but different Radical Signs, or when both the Radical Signs are different, and the Quantities incommenſurable.

1. To add $\sqrt[4]{u}$, $\sqrt[4]{u}$, and $\sqrt[4]{u}$.

Rule.] Multiply any 1 of the Surds by the whole Number of them, as here $\sqrt[4]{u} \times$ by $3 = \sqrt[4]{81u}$, (obſerving the firſt Rule of *Multiplication* laſt above,) which is the Anſwer.

Or in Numbers: Add $\sqrt[4]{16}$, $\sqrt[4]{16}$, and $\sqrt[4]{16}$. Here 3 the Number of Roots raiſed to the fourth Power is 81; and $\sqrt[4]{16} \times \sqrt[4]{81} = \sqrt[4]{1296} = 6$ proved thus; $\sqrt[4]{16} = 2$, and 3 times 2 is 6, or $2+2+2$, which proves the Rule.

2. When two unequal Surds are Commenſurable, and have the ſame Radical Signs, and when divided by their greateſt common Meaſure, are reduced to ſuch Quantities as are the compleat Powers of ſome Quantities or Numbers; they may be added thus, ſuppoſe $\sqrt[3]{40}$ to be added to $\sqrt[3]{135}$.

2*d Rule.*]

2d Rule.] Divide the Surds by their common Measure $\sqrt[3]{5}$, and they are reduced to $\sqrt[3]{8}$ and $\sqrt[3]{27}$, the Cube Roots of which are 2 and 3; the Sum of which is 5, which Sum of the Roots multiplied is $5\sqrt[3]{5}=\sqrt[3]{625}$ the Answer.

And for Proof, Extract the Cube Roots of the Surds given in the common way of Extraction, and you'll find the Sum $=\sqrt[3]{625}$ $=8.55$.

How to add Quantities having the same Surds, but different Radical Signs; or when the Radical Signs are either different or the same, and the Surds are incommensurable.

Rule.] There is no Occasion for farther Operation, than to put the Radical Sign + between the given Surd Roots: As

To add $\sqrt{cc}$ to the $\sqrt[3]{cc}$ and $\sqrt[4]{cc}$; the Sum is $\sqrt{cc}+\sqrt[3]{cc}+\sqrt[4]{cc}$.]

Or the Sum of $\sqrt{5}$, the $\sqrt[3]{6}$ and $\sqrt[4]{13}$, is $=\sqrt{5}+\sqrt[3]{6}+\sqrt[4]{13}$.

Or $\sqrt{3}+\sqrt{7}+\sqrt{12}$, *&c.* is the Sum of such Surds as $\sqrt{3}$, $\sqrt{7}$ and $\sqrt{12}$, *&c.*

There is a Rule which some pretend helps us to the Sum of these kind of Surds, when the Radical Sign is $\sqrt[2]{}$. The Rule is this; *To the Square Root* of the Sum of the Squares of the Surds given, add the double Rectangle of these Surds, and the Sum is the Answer.

Example. To $\sqrt{12}$, add $\sqrt{8}$; according to the Rule, the Sum of the Squares of the Surds is $=20$, its Root $\sqrt{20}$; and the double Rectangle of the Surds is $2\sqrt{96}=8\times12\times4=\sqrt{384}$, by the first Example of *Multiplication* of Surds. So by this Rule, the Sum of $\sqrt{12}+\sqrt{8}$ is reduced to $\sqrt{20}+\sqrt{384}$. I must confess I do not see the Use of the Rule, nor of the Alteration of the Surds, since to say the Answer is $\sqrt{12}+\sqrt{8}$ is more true and intelligible, than to call the Sum $\sqrt{20}+\sqrt{384}$.

V. Subtraction *of Simple Surds.*

1. When the Surd Roots are commensurable or reduceable to Rational Quantities, as in the second Example in *Addition*; work

as is there directed, but when you come to the Roots, inſtead of taking their Sum, take their Difference, and multiply as in the ſame Example. To know the Difference between $\sqrt[3]{40}$ and $\sqrt[3]{135}$: Theſe $\div\sqrt[3]{5}$ reduces them to $\sqrt[3]{8}$ and $\sqrt[3]{27}$, whoſe Roots are 2 and 3, and the Difference of thoſe Roots is 1, which multiplied by the common Meaſure $\sqrt[3]{5}$, gives $\sqrt[3]{5}$ for the Remainder or Difference.

But when the Surds are incommenſurable, as in the laſt Example in *Addition*, you have nothing to do but place the Sign (—) between the Surds given; ſo the Difference of $\sqrt{b}$ above $\sqrt{g}$, is $\sqrt{b}-\sqrt{g}$.

VI. Addition *and* Subſtraction *of Compound Surds.*

1*ſt*, For commenſurable Surds, as thoſe under the ſecond Rule of *Addition* above, take this Example.

To $\sqrt{45}+\sqrt{294}$
Add $\sqrt{125}-\sqrt{96}$ Sum $=\sqrt{320}+\sqrt{54}$.

Here the Sum of $\sqrt{45}$ and $\sqrt{125}=\sqrt{320}$; and of $\sqrt{294}$ and $-\sqrt{96}=\sqrt{54}$ by the ſaid ſecond Rule.

2*dly*, For mixt Numbers, as Surds and Rational Numbers; ſee this Example.

To $331+\sqrt[4]{768}-\sqrt[4]{80}$	Or from $331+\sqrt[4]{768}+\sqrt[4]{80}$
Add $153+\sqrt[4]{243}+\sqrt[4]{1280}$	Take $153+\sqrt[4]{243}-\sqrt[4]{1280}$
Sum $=484+\sqrt[4]{7203}+\sqrt[4]{89}$	Reſts $=178+\sqrt[4]{3}+\sqrt[4]{1296}$

I need give no further Examples, becauſe the *Addition* and *Subſtraction* of theſe Surds are done by the ſelf-ſame Rules as thoſe called *Simple Surds*, as by the Rule under the ſecond Example, and in *Subſtraction* of thoſe Surds.

The ſecond Example laſt foregoing is done thus: To add $\sqrt[4]{768}$ to $\sqrt[4]{243}$; I divide them by the common Meaſure $\sqrt[4]{3}$, which reduceth them to $\sqrt[4]{256}$ and $\sqrt[4]{81}$, whoſe Roots are 4 and 3, whoſe Sum

Sum is 7p so $7\sqrt[4]{3}$ is the Answer, or $7^4\&\sqrt[4]{3} = \sqrt[4]{7203}$, and so of the Reſt.

The Demonſtration of the Rule for this laſt, as under Article IV. Rule 2.

This I ſhall do by this plain Inſtance of Numbers adapted on purpoſe, as in the Margin; where $\sqrt{64} + \sqrt{64}$, are given to have their Sum found.

$\sqrt{4})\ \sqrt{64} + \sqrt{64} =$ the ſuppoſed Surds given.
Quotes $\sqrt{16} = 4$ Roots.
Quote $= + \sqrt{16} = 4$

$8 =$ Sum of the Roots.
and $8\sqrt{4} = \sqrt{256} = 16$ the Anſwer.
$= \sqrt{64} + \sqrt{64} = 8 + 8.$

1. Theſe divided by $\sqrt{4}$, are reduced to the Root $\sqrt{16} + \sqrt{16}$, whoſe real Roots are $4 + 4 = 8$. And $\sqrt{4} \times 8$, as by the firſt Rule of *Multiplication* above, gives the $\sqrt{256}$, whoſe Square Root being 16, is = the Sum of the Roots of the two Surds given.

VI. Multiplication *of Compound Surds.*

Three Examples or Varieties done in Species two Ways.

The three Examples in Species proved by Numbers.

Example 1.

To multiply $b \sqrt{ac}$ in $ac + \sqrt{b}$ } See the Operation, alſo in Numbers { $4 \sqrt{64}$ / $2 + \sqrt{16}$ } Multiply as the Species.

$abc\sqrt{ac} + b\sqrt{abc} =$ the Prod. or in Numbers $= 8\sqrt{64} + 4\sqrt{64}\sqrt{6} = 192$

Or reduced into all Surds. $= \sqrt{aaabbccc} + \sqrt{abbbc}$

For $4 \times 8 (= \sqrt{64}) = 32$
& that $32 \times 2 + 4 (= \sqrt{16}) = 6 = 192$

Example 2.

Example 2d proved thus:

To multiply $b \sqrt{ac}$ in $ac\sqrt{b}$

$4\sqrt{64}$ } Multiply as the
Multiply by $2\sqrt{16}$ } Species are.

$abc\sqrt{abc} =$ the Product; or in Numbers $= 8\sqrt{64}\sqrt{16} = 256$

Or reduced to Surds. $= \sqrt{aaabbbccc}$

For $4, \times, 8 (= \sqrt{64}) = 32 = 256$
And $32 \times 2 \times 4 (= 2\sqrt{16}) = 8$

Example 3.

To multiply $b+\sqrt{ac}$ in $ac+\sqrt{b}$

$abc+ac\sqrt{ac}+b\sqrt{b}+\sqrt{abc}$

Or more in Surds $= abc+\sqrt{aaacccc}+\sqrt{bbb}+\sqrt{abc}$

Or in Numbers thus: $4+\sqrt{64}$ × in $2+\sqrt{16}$ } By the 3d Example.

$=8+2\sqrt{64}+4\sqrt{16}+\sqrt{64}\sqrt{16}=72$

And so 'tis in Reality for $4+\sqrt{64}=12$

And $2+\sqrt{16}=6$

And $12 \times 6=72$

1. *Note*, That in multiplying b, multiplied in the Square Root of ac, by ac more the Square Root of b; I say $b\sqrt{ac}$ by $ac=abc\sqrt{ac}$, and stands so in the Example 1. More b (in the Multiplicand) $\sqrt{abc}$, &c.

2. *Note*, In like manner by the Numbers, Example 1. I say 2 times $4\sqrt{64}$ is $8\sqrt{64}$. 2*dly*, $4\sqrt{64}$ by $\sqrt{16}$, is $=4\sqrt{64}\sqrt{16}$; so the Product is $=8\sqrt{64}+4\sqrt{64}\sqrt{16}$. Now the Square Root of 64 is 8, and of 16, 4, so that this Product is $8\times8=4\times8\times4=192$: And that this is right multiplied, will appear by the Numbers given, or $4\times$ in 8, (which 8 is the Square Root of 64,) gives 32 for the Value of the Multiplicand.

And 2, more 4, (which 4 is the Square Root of 16,) makes 6, which is the Value of the Multiplier; so 6 times 32 is $=192$ as before, which is a full Proof of the Rule by the first Example, and the rest are performed and proved much after the same manner, which it would be needless to insist farther upon.

Example 4. To multiply $\sqrt[3]{u-cd}$ by $\sqrt[3]{u+cd}$: See the Operation in the Margin.

$$\begin{array}{l} \sqrt[3]{u-cd} \\ \sqrt[3]{u+cd} \\ \hline \sqrt[3]{uu}-\sqrt[3]{ccdddu} \\ \quad +\sqrt[3]{ccdddu}-\sqrt[3]{ccdd} \\ \hline \text{Prod.} = \sqrt[3]{uu}+\sqrt[3]{ccdddu}-\sqrt[3]{ccdddu}-\sqrt[3]{ccdd} \\ \hline \text{Product contracted} = \sqrt[3]{uu-ccdd} \end{array}$$

In Numbers demonstrated thus; $u=8, c=1, d=2$.

Then $\sqrt[3]{8-2}$

by $\sqrt[3]{8+2}$

$$\begin{array}{l} \sqrt[3]{64}-\sqrt[3]{64} \\ \quad +\sqrt[3]{64}-4 \\ \hline \sqrt[3]{64}-4 = \text{the Sum or Product contracted.} \end{array}$$

Now the Cube Root of $64=4$, and $4-4=0$

So the $\sqrt[3]{8}$ = the Multiplicand $=2$, and $2-2=0 \times 2$ in the Multiplier $=0$ (the Proof.

5. Hence

5. Hence it appears, that if you multiply two Surd Numbers together, whoſe Radical Signs are $\sqrt{\ }^2$, the Square Root of the Product is the Anſwer in a Rational Number. But if the Root cannot be Extracted without a Remainder, divide the Product by the greateſt Square Number, that will do it without Remainder; ſo the Root of that Diviſor in the Quote is the Anſwer, as in the firſt Caſe, $\sqrt{16}$ in $\sqrt{9} = \sqrt{144} = 12$ the Anſwer. And in the ſecond Caſe, $\sqrt{72}$ in $\sqrt{8} = \sqrt{576}$, which $\div 144$ quoteth 4, and the $\sqrt{144} = 12$; ſo that $12\sqrt{4}$ is the Anſwer.

VII. Diviſion *of Compound Surds.*

This will appear very eaſy to thoſe who underſtand common Diviſion in Species, as before taught, and Multiplication of Surds.

Example, Divide $\sqrt{bu} + \sqrt{du}$ by $\sqrt{u}$; ſee the marginal Work.

Quote.

$\sqrt{u}$) $\sqrt{bu} + \sqrt{du}$ ($\sqrt{b} + \sqrt{d}$

$\sqrt{bu}$

$\sqrt{du}$

$\sqrt{du}$

0 Reſts.

Example 2. To divide $abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc}$ by $b + \sqrt{ac}$, or in Numbers, $8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}$, $\sqrt{16}$ by $2 + \sqrt{16}$.

Done Literally thus:

$b + \sqrt{ac}$) $abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc}$ ($ac + \sqrt{b}$ = the Quote.

$abc + ac\sqrt{ac}$

$b\sqrt{b} + \sqrt{abc}$

$b\sqrt{b} + \sqrt{abc}$

Reſts = 0

Done Numerally thus:

$2 + \sqrt{16}$) $8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}$ $\sqrt{16}$ ($4\sqrt{64}$ = Quote.

$8 + 4\sqrt{16}$

$2\sqrt{64} - 4\sqrt{16} + 4\sqrt{16}$

Or Contracted, and the Reſt brought down. } $= 2\sqrt{64} + \sqrt{64} \cdot \sqrt{16}$

$2\sqrt{64} + \sqrt{64} \sqrt{16}$

0 Remains.

There

There is nothing of Difficulty in these Operations, observing to change the Signs of what you subtract: As in the Numeral Example, I am to take $4\sqrt{16}$ from $2\sqrt{64}$, therefore I say that $2\sqrt{64}-4\sqrt{16}$ is the Remainder, to which if I bring down $4\sqrt{16}$, the $+$ and $-$ destroy the $4\sqrt{16}$: So that bringing down to the said Remainder $2\sqrt{64}$, the rest of the Dividend, which is $\sqrt{64}\sqrt{16}$, I find that $2+\sqrt{64}$ is contained in $2\sqrt{64}$; $\sqrt{64}$ times, which I put in the Quote, and multiply and deduct, so the Remainder is (0).

And the Truth is thus proved, *1st*, In that it agrees with the Literal Example. *2dly*, As it is the Product of Example the Third foregoing, divided by the Multiplier. And *3dly*, And most plainly thus:

The Value of the several Members of the Numeral Dividend are,

8	= 8
$2\sqrt{64}$, or 2×8	= 16
$4\sqrt{16}$, or 4×4	= 16
$\sqrt{64}\sqrt{16}$, or 8×4	= 32
	72 = Sum

Value of the Divisor $2+\sqrt{16}=2+4=6$) 72 = Sum (12 = Quote
(= 4 + 8 = 4 + $\sqrt{64}$ for Proof.

Sect. VIII. *Algebraical Fractions.*

Although the Business of Fractions in Species are often used in the Solution of Analytical Questions: Yet since the Reader of this Part is supposed to have perfectly learnt *Reduction of Vulgar Fractions*, Chap. 2. Sect. 1. and there being so very little Difference between the Method of working Fractions in Numbers and Species, therefore I shall not need to enlarge much on this Section.

1. *Note*, The fifth Case is done thus: Multiply the Quantity given by the proposed Denominator, and place the Product over the Denominator for the Answer.

2. And the sixth Case is only placing the Integeral Quantity given as a Numerator, and a Unit for a Denominator.

3. And that the Quantities omitted, which are found in both the Terms, brings a Fraction into its lowest Terms.

The

The Cases of Reduction *of* Algebraical Fractions.

Number of Cases given.	To reduce.	Examples, two to each Case.	The Answers.	Done by Rules Ch. 2 Sect 1. & Cases.
1	Mixt Quantities to Fractions.	$ub+\frac{b}{c}$	$\frac{bcu+b}{c}$	1
		$u+\frac{b+c}{d}$	$\frac{du+b+c}{d}$	
2	An improper Fraction to a whole or mixt Quantity.	$\frac{bcu+b}{c}$	$bu+\frac{b}{c}$	2
		$\frac{du+b+c}{d}$	$u+\frac{b+c}{d}$	
3	A Fraction to its lowest Terms.	$\frac{ubbd}{ubc}$	$\frac{bd}{c}$	4
		$\frac{14udc+21dpu}{7dru}$	$\frac{2c+3p}{r}$	
4	Fractions of different to common Denominators.	$\frac{u}{c}, \frac{m}{r}, \frac{s}{t}$	$\frac{urt}{crt}, \frac{cms}{crt}, \frac{crs}{crt}$	6
		$\frac{u+c}{m}, \frac{b+d}{gg}$	$\frac{ggu+cgg}{ggm}, \frac{bm+dm}{ggm}$	
5	A whole Quantity to a Fraction of the same Value, whose Denominator is given.	$b+c$ to that of Denominator r	$\frac{br+cr}{r}$	
		$u+m$ to that whose Denom. $=r+s$	$\frac{ru+mr+su+ms}{r+s}$	
6	An Integeral Quantity to the Form of a Fraction.	bu	$\frac{bu}{1}$	
		$cu-pr+bb$	$\frac{cu-pr+bb}{1}$	
7	A Compound to a simple Fraction.	$\frac{u}{b}$ of $\frac{c}{d}$ of $\frac{r}{ns}$	$\frac{cru}{bdns}$	
		$\frac{3ub+cd}{m}$ of $\frac{r-5st}{b+n}$	$\frac{3bru+cbr-15bstu-5cdst}{bm+mn}$	3

II. Addition, Subſtraction, Multiplication, *and* Diviſion *of* Algebraical Fractions; *two* Examples *to each Rule.*

	Propoſitions or Examples.	Solutions or Anſwers.	Rules.
Addition	1. To $\frac{am}{n}$ add $\frac{2bb}{m}$	Sum $= \frac{amm+2bbn}{mn}$	Reduce the Fractions to a common Denominator, and add the Numerators as in Vulgar Fractions. Note, $-r$ and $+gr$ deſtroy each other.
	2. To $\frac{n-r}{g}$ add $\frac{r}{g+n}$	$\frac{gn-gr+nn-rn+gr}{gg+gn}$	
Subſtraction	1. From $\frac{amm+2bbn}{mn}$ Take $\frac{2bb}{m}$	Reſts $= \frac{am}{n}$	Reduce to a common Denominator and work as in Vulgar reducing the Remainders to their loweſt Terms
	2. From $\frac{gn-gr+nn-rn+gr}{gg+gn}$ Take $\frac{r}{g+n}$	Reſts $\frac{n-r}{g}$	
Multiplication	1. $\frac{2bn}{c}$ by $\frac{b}{n}$	Product $\frac{2bbn}{cn}$	Multiply the Numerators give $2bn$, and the Denominator $= cn$ as in Vulgar Fraction
	2. $n+\frac{b}{r}$ by $\frac{3mp}{s}$	Prod. $= \frac{3mprn}{rs} + \frac{3bmp}{rs}$	reducing $n+\frac{b}{r}$ into a Fraction, as by the firſt Reduction
Diviſion	1. Divide $\frac{2bbn}{cn}$ by $\frac{b}{n}$	Quote $= \frac{2bn}{c}$	Work as in Vulgar, and reducing the Quotes to loweſt Terms, ſhall be as you ſee.
	2. $\frac{3mprn+3bmp}{rs}$ by $\frac{3mp}{s}$	Quote $= n+\frac{b}{r}$	

Theſe *Examples* above are very eaſy to ſuch as have ſufficiently acquainted themſelves with *Vulgar Fractions*, and the two Examples in *Subſtraction* prove thoſe in *Addition*, as thoſe in *Diviſion* do

do thoſe in *Multiplication*; there is no need to illuſtrate theſe by Numbers, becauſe that is done by comparing theſe with the like Examples in *Vulgar Fractions*. So that I proceed to

SECT. IX. *Rules to reſolve Simple Equations.*

The Buſineſs of this is, (when unknown Quantities are intermixt with known,) to bring the unknown Quantity to poſſeſs ſolely one Side of the Equation (commonly that toward the Left-hand) now theſe complex Quantities being ſo either by the unknown Quantity, being added to, ſubſtracted from, multiplied in, or divided by known, or by moſt or all of theſe together; therefore Equations are reſolved, or the Value of the unknown Quantity is diſcovered by *Addition*, *Subſtraction*, *Multiplication*, *Diviſion* and *Evolution*, &c. and that in a contrary Way to that by which the known and unknown Quantities are connected and placed together, *i. e.* thoſe of *Subſtraction* ſolved by *Addition*; *Addition* by *Subſtraction*; *Multiplication* by *Diviſion*; and thoſe divided and ſtanding Fraction-ways are ſolved by *Multiplication*, as by theſe Examples.

Simple Equations Reſolved.

1. *By Addition* (*Note* $b=20$, $c=12$, $d=8$) And that x is put for the unknown Quantity required.

Literally	Numerally
$x-d=b-c$	$x-8=20-12$
Add d — d Add	Add 8 — 8 Add
Sum $x=b-c+d$ the Anſwer.	Sum $x=20-12+8=16$

2. *By Subſtraction.*

$x+d=b+c$	$x+8=20+12$
Deduct d, — d Deduct	Deduct 8, — 8 Deduct
Reſts $x=b+c-d$ the Anſwer.	$x=20+12-8=24$

3. *By Multiplication.*

$\frac{x}{4}=\frac{dc}{b}$	$\frac{x}{4}=\frac{96}{20}$
$x=\frac{4dc}{b}$ = the Anſwer.	Anſwer $x=\frac{96\times4}{20}=19\frac{1}{5}$

4. *By Diviſion.*

$du+cu=5b$

$u=\frac{5b}{d+c}$ the Anſwer.

$8u+12u=20\times5$

$u=\frac{100}{8+12}=5$

---

5. *By Addition, and Subſtraction, or Tranſpoſition*

$u+c-d=b$

$u=b-c+d$ the Anſwer.

$u+12-8=20$

$u=20-12+8,=16$

---

6. *By Addition, Subſtraction, and Multiplication.*

$\frac{u-d}{2}+c=b-d$

$u-d+2c=2b-2d$

$u=2b-d-2c$ the Anſwer.

$\frac{u-8}{2}+12=20-8$

$u-8+24=40-16$

$u=40-16+8-24$

Or $u=8$ . . . .

---

7. *By Addition, Subſtraction, Multiplication, and Diviſion.*

$\frac{cu+du}{b}+d-b=5c$

$cu+du+bd-bb=3bc$

$cu+du=3bc-bd+bb$

$u=\frac{3bc-bd+bb}{c+d}$ the Anſwer.

$\frac{12u+8u}{20}+8-20=3\times12$ or 36

$12u+8u+20\times8-20\times20=36\times20$

$12u+8u=720-160+400$

$u=\frac{720-160+400}{12+8}$ (or $u=48$)

Eight General Rules for reſolving Simple Equations, applied to the foregoing Examples.

1. *When one or more known Negative Quantities are connected with an unknown Affirmative, they are ſeparated, and the unknown made to poſſeſs ſolely one Side of the Equation (which is the ſame as to ſay that ſuch Equations are Reſolved) by* Addition *only.* So in the Example of reſolving by *Addition*, I add the Quantity connected with (u), as (d) to both Parts of the Equation, and the Sum is very plainly $u=b-c+d$.

The Reaſon of which is plain, and ſhews the Foundation of Tranſpoſition: (which Word ſee in the Alphabetical Account near the beginning of this Section) For adding the ſame Quantity or Number to both Parts of the Equation does preſerve the Equality; but this Advantage

vantage is gained thereby, that $+d$ added to $-d$, deſtroys the Quantity $-d$, ſo that it vaniſhes and leaves u in Poſſeſſion of one Side of the Equation, and that $(+d)$ being added to the other part of the Equation, makes $b-c+d$, to which (u) is therefore found equal. And the Reaſon appears ſtill more plain in the Numerical Operation; for if $u-8$ be $=8$, (or $20-12$,) it muſt follow, that $u=8+8$ or 16, and this is no more than what is taught in *Subſtraction of intire natural Numbers.*

2. *When one or more known affirmative Quantities are connected with an unknown, that Equation is reſolved by ſubſtracting ſuch known Quantities from each part of the Equation.* Thus in the Example of the Reſolving by *Subſtraction*, (d) taken from $+d$ (which is connected with u) leaves nothing but (u) on that Side of the Equation; and d deducted from the other part of the Equation, there reſts $b+c-d$, to which therefore (u) is equal, and the Equation is Reſolved.

The Reaſon of this is evident from what was ſaid of adding in the firſt Rule above; *for if equal Numbers or Quantities be either added to or taken from equal, the Sums or Remainders will be equal.* And the Reaſon plainly appears alſo in the Numerical Work; for if $u+8$ be $=32$, it follows that $u=32$, abating only the 8.

3. When in an Equation, there are ſeveral known Quantities on the ſame Side with the unknown, ſome Affirmative, and ſome Negative; it follows from the above Rules, that you may tranſpoſe ſuch known Quantities to the other Side of the Equation, changing their Signs, as in the fifth Example: And this you may do both ways to the Right, or thence toward the Left-hand of the $=$, and that with either known or unknown Quantities, keeping cloſe to the Rules above, your own Reaſon, and the Nature of the Queſtion: I mean, you will not think it reaſonable to make the unknown Quantity a Negative, by tranſplacing it, when the Reaſon and Nature of the Queſtion does dictate, that what is on the other Side of the Equation, is ſo far from being equal to $-u$, that it ought to be at leaſt equal to u, or to ſome abſolute Number or Quantity which $-u$ is not.

4. When in an Equation, the unknown Quantity is divided by ſome Number or known Quantity, poſſeſſing one Side of the Equation; then every Member of ſuch Equation muſt be multiplied by the Denominator of the Fraction, whereof the unknown Quantity is the Numerator, as in the third Example; I multi-

ply $\frac{u}{4}$ by 4, produceth u; and $\frac{dc}{b}$ by 4, produceth $\frac{4dc}{b}$; and ſo the Equation is reſolved, the unknown Quantity being cleared and found $= \frac{4dc}{b}$, as the Numeral Work farther demonſtrates; for equal Quantities multiply'd by equal, produce equal, as well as adding and ſubſtracting.

The Reaſon is very evident: For if a fourth Part of u be $= \frac{dc}{b}$, it muſt follow, that the whole (u) muſt be equal to four times the $\frac{dc}{b}$; and the like holds for any other Diviſor of the unknown Quantity, or Denominator as aboveſaid.

5. Hence it appears, that in multiplying by the Denominator of ſuch a Fraction, 1ſt, *That every Member of both Parts of an Equation muſt be multiplied.* And 2dly, *That the Numerator of that Fraction is multiplied by putting it down as a whole Number*, as $\frac{1}{2}x$ by 2, is $= 1$; $\frac{3}{4}$ by 4, is $= 3$; $\frac{5}{6}$ by 6, is $= 5$, &c.

6. *When Quantities known and unknown ſtand multiplied together in an Equation poſſeſſing one Side thereof, ſuch Equations are ſolved by* Diviſion; *in which Caſe the Quantities ſo multiplied in the unknown, are to be your Diviſor*; as in the fourth Example or Diviſion above. To ſeparate u (in the two Members $du+cu$) from the known Quantity d and c; I divide both Parts of the Equation by $d+c$, and the Quotient is $u = \frac{5b}{d+c}$; *for equal Quantities divided by one and the ſame Quantity, muſt exhibit equal Quotients*; and at the ſame time you free one Side of the Equation of complex Quantities, and ſhew the known, which the unknown Quantity is equal to. Thus in the Numeral Example, the Matter is plain, for where $8u+12u = 100$; $20u$ is $= 100$; therefore u only muſt be equal to $\frac{100}{20}$, which is the ſame as in the Proceſs, according to the Rule above; for $8u+12u \div 8+12 = u$; and 20×5, or $100 \div 8+12 = 5$.

But Note, That when you have two Members, one mixt with a known Quantity, the other intirely unknown; in this Caſe, the known $+$ or -1 is your Diviſor, as if $8u-u = 49$, then $u = \frac{49}{8-1}$; and if $6u-u = 25$, then $u = \frac{25}{6-1} = 3$.

7. When (as in the ſeventh Example) 'tis found that unknown Quantities are ſo mixt with known, as to have Products, Quotients,

ents, Sums, and Remainders; you have nothing to do but to work as by theſe foregoing Rules, as in $\frac{cu+du}{b}+d-b=3c$: For if the whole is multiplied by b, it gives the next Step $cu+du+bd-bb=3bc$. And if $bd-bb$ be tranſpoſed, (as being all known Quantities,) the third Step ſtands thus; $cu+du=3bc-bd+bb$. And if the third Step be divided by $c+d$, (becauſe thoſe are the known Quantities that are multiplied in the unknown,) the fourth Step or Quote, which is the Anſwer is $u=\frac{3bc-bd+bb}{c+d}$; each of which Steps appears ſtill more plain in the Literal Work, which ſhews in this laſt Equation $u=48$.

8. There is one other Thing to be obſerved, but that is rather in compoſing or forming the Equation, than in the Reſolution, *i. e.* If $u.\ c::b.\ d.$ what is the Value of u?

Here the ſecond Step is found, as being the Rectangle of the two Extreams, which is equal to that of the means. And dividing by d, gives the Canon. See the Words *Analogy* in the beginning of this Chapter, and *Equation*. And if you would reduce an Equation into an Analogy, it is as u to c or b; ſo is the other to d in the two Examples above, or

$u.\ c::b.\ d$
$ud=bc$
$u=\frac{cb}{d}$ the Anſwer.

$\left\{\begin{array}{l} u.\ b::c.\ d \\ u.\ c::b.\ d, \text{ \&c.} \end{array}\right.$

Sect. X. *Reſolution of Quadratick Equations.*

What ſimple and adfected Quadraticks are, is ſhewed in the foregoing Definition of Terms near the beginning of this Chapter; where under the Words *Simple Quadraticks*, I have ſaid enough relating to that kind. As for adfected Quadratick Equations, (which Word ſee,) there are by ſome Algebriſts reckoned three Sorts, but I make but two really diſtinct; to one of which, all other adfected Equations are reducible. I ſhall give Examples,

1*ſt*, Of the two diſtinct Kinds of Square adfected Equations.
2*dly*, Of thoſe reducible thereto, and how performed.

II. *Shew how theſe Equations are Reſolved by compleating the Square.*

III. *How Reſolved by Subſtitution; how done four different Ways, two of which (being the beſt) are new.*

1*ſt*, The

1*ſt*, The two diſtinct Forms of Quadratick Equations are theſe only.

	In Species.	In Numbers.
1*ſt*,	$uu+du=bc$	$uu+6u=135$
2*dly*,	$uu-du=\frac{bc}{5}$	$uu-6u=27$

2dly, Equations reducible to one of the two Forms above, are theſe and ſuch like.

Equations given.		Reduced to theſe Quadraticks.
1ſt,	$du-uu=m$	$uu-du=-m$, as the 2d Form.
2dly,	$u+b=\frac{c-d+b}{2cu}$	$uu+bu=\frac{c-d+b}{2c}$, as the 1ſt Form.
3dly,	$u=\frac{c+db}{3bu}+b$	$uu-bu=\frac{c+db}{3b}$, as the 2d Form.
4thly,	$u=\frac{cu+tu}{3bzu}-bb$	$uu+bbu=\frac{c+t}{3b}$, the 1ſt Form *cc.*

Notes upon theſe Reductions.

1*ſt*, That the firſt is performed by changing the Signs of all the Members, (as is directed in Rule the third foregoing to *Simple Equations*,) which makes $-m=-du+uu$, which is the ſame as to ſay $uu-du=-m$, as in the ſecond Form; though ſome make this a diſtinct Sort, but with no more Reaſon than the three ſubſequent; for the Square is compleated, the Root extracted, and the Canon produced exactly as in one of the two Kinds above-mentioned.

2*dly*, The ſecond Equation above is reduced, as taught in the 4th and 6th Rules to the Examples of *Simple Equations*, multiplying by $2cu$, and dividing by $2c$ each part of the Equation.

3*dly*, The third Equation above is reduced by the ſecond, fourth, and ſixth Rules to the Examples of *Simple Equations*, tranſpoſing b, multiplying by $3bu$, and dividing by $3b$.

4*thly*, The fourth is reduced by multiplying by $3buu$, expunging z in each Member, and dividing the reſt by $3b$.

II. *The*

II. *The Steps of resolving Quadratick Equations by compleating the Square.*

Equations proposed.	Squares compleated. 1st.	Roots extracted. 2dly.	Canons exhibited. 3dly.
$uu+du=bc$	$uu+du+\frac{dd}{4}=bc+\frac{dd}{4}$	$u+\frac{d}{2}=\sqrt{bc+\frac{dd}{4}}$	$u=\sqrt{bc+\frac{dd}{4}}-\frac{d}{2}$
$uu-du=\frac{bc}{5}$	$uu-du+\frac{dd}{4}=\frac{bc}{5}+\frac{dd}{4}$	$u-\frac{d}{2}=\sqrt{\frac{bc}{5}+\frac{dd}{4}}$	$u=\sqrt{\frac{bc}{5}+\frac{dd}{4}}+\frac{2}{d}$
$uu+2cu=3b$	$uu+2cu+cc=3b+cc$	$u+c=\sqrt{3b+cc}$	$u=\sqrt{3b+cc}-c$
$uu+u=2b$	$uu+u+\frac{1}{4}=2b+\frac{1}{4}$	$u+\frac{1}{2}=\sqrt{2b+\frac{1}{4}}$	$u=\sqrt{2b+\frac{1}{4}}-\frac{1}{2}$
$uu+6u=135$	$uu+6u+9=135+9$	$u+3=\sqrt{135+9}$	$u=\sqrt{135+9}-3$
$uu-6u=\frac{135}{5}$	$uu-6u+9=\frac{135}{5}+9$	$u-3=\sqrt{\frac{135}{5}+9}$	$u=\sqrt{\frac{135}{5}+9}+3$
$uu+u=90$	$uu+u+\frac{1}{4}=90+\frac{1}{4}$	$u+\frac{1}{2}=\sqrt{90\frac{1}{4}}$	$u=\sqrt{90\frac{1}{4}}-\frac{1}{2}$

Rules to be observed in this Method (which is the best) of the Resolution of Quadratick Equations, applied to the Examples above.

1st, *Whatsoever known Quantities are multiplied in the Root, or unknown, they are called the Co-efficients*; as d, $2c$, and 1 in the second Terms of the four first Examples: In the fourth of which there being no Co-efficient, therefore 1 must be taken for it, because a Unit is supposed to stand before every Quantity, as u is $= 1u$, $c=1c$, &c.

2dly, *To complcat any Square having only the two first Members given*, is done by adding the Square of half the Coefficient: Thus in the first and second Examples, half the Coefficients $\frac{d}{2}$, (the

(the Square of which is $\frac{dd}{4}$,) added to the two other Members ſtanding towards the Left-hand, the compleat Squares are $uu+du+\frac{dd}{4}$ and $uu-du+\frac{dd}{4}$. And in the third and fourth Examples $2c$ and 1 are the Coefficients, the half of which are c and $\frac{1}{2}$; the Squares of which Halves are cc and $\frac{1}{4}$, the former of which compleats the Square of the two Members towards the Left-hand of the third Example, and the latter (or $\frac{1}{4}$) added, doth compleat the Square of the fourth Example: See at the beginning of this Chapter.

3*dly*, And becauſe this Addition of the Square of half the Coefficients is made to one part of the Equations, where the unknown Quantities are; therefore it muſt be added alſo to the other part of the Equations, *That ſo the Equality may be continued*, (*as is ſaid in the firſt and ſecond Rules to the Examples of* Simple Equations,) and thus the ſecond Column is made in all the ſeven Examples above.

4*thly*, The third Column from the Left-hand, are the Roots of thoſe in the ſecond.

1ſt, *For in all compleat Squares, the Square Roots are exactly thoſe of the firſt and third Member.*

2*dly*, *The Sign connecting the Quantities of the Root, is always the ſame as that before the ſecond Term of the compleat Square, or of the Equation given:* Thus the Roots of

Squares.	Roots.
$uu+du+\frac{dd}{4}$ in the ſecond Column,	is $u+\frac{d}{2}$ in the third Column.
$uu-du+\frac{dd}{4}$ is	$= u-\frac{d}{2}$
$uu+2cu+cc$	$= u+c$
$uu+u+\frac{1}{4}$	$= u+\frac{1}{2}$, &c.

And for a Proof, any of theſe Roots being multiplied in it ſelf, produces the reſpective Square above.

3*dly*, The

3dly, The Root of the 1ſt Part being extracted, in order to diſcover the Value of the ſingle unknown Quantity; therefore the Roots of the Quantitys known, ſtanding on the other part of the Equation, (as is ſaid under the 3d of adding the Square of half the Co-efficient) muſt have their Roots extracted; which is done by prefixing the Radical Sign, becauſe they are generally Surds.

4thly, To reduce the 3d Column to the 4th, or to the Canons or Anſwers, you have nothing to do but to tranſpoſe the Root of the 3d Member of the compleat Square, as $\frac{d}{2}$, c, and $\frac{1}{2}$ in the 4 firſt Examples changing the Sign as taught.

A farther Demonſtration or Proof of the Method for the Solution of Quadraticks, by the Numeral Operations above.

In the Numeral Work theſe Numbers are equal to the Letters in the Specious, *viz.* $d=6$, $b=45$, $c=3$, and u is found equal to 9.

Here (proceeding by the Rules for Solution above) in the Equation $uu+6u=135$, u in the 4th Column is found $=\sqrt{135+9}-3$.

Now $135+9$ is $=144$, the Square Root of which is 12, whence take 3, and the reſt is $=9$, ſo $u=9$.

And u being found $=9$ by the Canon, grounded upon the Method of Proceſs aboveſaid; if ſuppoſing $u=9$, the 2 Parts of the Equations propoſed prove equal, it fully proves the Truth of the Method for Solution.

So $uu+6u=81+54$ in the 5th Equation given, is $=135=$ the other part of that Equation given.

$uu-6u=81-54$ in the 1ſt part of the 6th Equation is $=27=$ the 2d part of that Equation.

And $uu+u=81+9$ in the 1ſt part of the 7th Equation is $=90$ in the 2d part of that Equation, *&c.*

Thus I have given the moſt plain and demonſtrative Rules and Examples for the underſtanding and proving the Method of ſolving Square Equations by the way of compleating the Square. I proceed to ſhew,

III. *The Method of Solving Quadratic Equations by Subſtitution.*

N° of Steps.	*Steps towards the Anſwer.*	*How each Step is produced.*
1	$uu + cu = 18d$	= *Equation given.*
2	for u is put $a - \frac{c}{2}$ in this 1ſt Caſe.	
3	$aa - ac + \frac{cc}{4}$	= □ $a - \frac{c}{2} = u$ or $a - \frac{c}{2}$ ⊙
4	$ac - \frac{cc}{2}$	$= a - \frac{c}{2} \times c - cu$
5	$aa - xc + \frac{cc}{4} + ac - \frac{cc}{2} = 18d$	= 3d Step + 4th.
6	$aa - \frac{cc}{4} = 18d$	= 5th Contracted.
7	$a = \sqrt{\frac{4}{18d + \frac{cc}{4}}}$	= 6th uu, and $\frac{cc}{4}$ +
8	$x + \frac{c}{2} = \sqrt{18d + \frac{cc}{4}}$	$u + \frac{c}{2}$ put for a
9	$u = \sqrt{18d + \frac{cc}{4}} - \frac{c}{2}$	$\frac{c}{2}$ in the 8 +

A 2d Example ſee towards the right hand.

N° of Steps.	*Proceſs toward an Anſwer.*	*Rules for each Step.*
1	$uu - cu = 9d =$	*the Equation given.*
2	for u is put $a + \frac{c}{2}$ in this 2d Caſe	$u = a + \frac{c}{2}$
3	$aa + ac + \frac{cc}{4}$	= □ $a + \frac{c}{2} = u$ or $a + \frac{c}{2}$ ⊙
4	$- ac - \frac{cc}{2}$	$= a + \frac{c}{2} \times -c$
5	$aa + ac + \frac{cc}{4} - ac - \frac{cc}{2} = 9d$	= 3d + 4th Steps
6	$aa - \frac{cc}{4} = 9d$	= 5th Contracted
7	$a = \sqrt{\frac{4}{9d + \frac{cc}{4}}}$	= 6th uu and $\frac{cc}{4}$ +
8	$u - \frac{c}{2} \sqrt{9d + \frac{cc}{4}}$	$u - \frac{c}{2}$ put for in the 7th Step
9	$u = \sqrt{9d + \frac{cc}{4}} + \frac{c}{2}$	$= - \frac{c}{2} +$ in the 8th Step

I have here inſerted the Operations at large, to make them more plain and eaſy; which the 3d and 6th Columns from the right hand do ſo fully contribute to, that I have little to add, only,

1. To put the Reader in mind, That the Signs connecting the ſubſtituted Quantities are always contrary to that which connecteth the 2 Members of the Square given, as is plain by the 2 Examples above.

2. That any Letter as well as a (if it be not in the Equation given) + or leſs half the Co-efficient, as above, may be ſubſtituted.

3. In the 8th Step I omit a, and ſubſtitute in place thereof $u + \frac{c}{2}$ in the 1ſt Example, becauſe if $a - \frac{c}{2} = u$, then by tranſpoſing $- \frac{c}{2}$, a is $= u + \frac{c}{2}$, and in the 2d Example $a = u - \frac{c}{2}$.

A

A New and Better Way.

But tho this is the only Way I know extant of ſolving theſe Equations by Subſtitution; *yet I am ſure 'tis a better, to make the Vowel and half the Co-efficient in the Quantitys given to be your Subſtitute:* and then it will fall right at the laſt, remembring that half the Co-efficient is to be added or deducted, and 'twill ſave ſome Steps of working. For example, putting $u - \frac{d}{2}$ for u, in the Equation $uu + du = bc$.

Here it only needs to be noted, that when the Sign connecting the 2 firſt Terms of the Equation given is +, I make the half Co-efficient added to the Surd Root of the Anſwer —; and the contrary.

$$uu - du + \frac{dd}{4} = \square\ u - \frac{d}{2}$$

$$du - \frac{dd}{2} = u - \frac{d}{2} \times d.$$

$$uu - \frac{dd}{4} = bc; \left\{\begin{array}{l}\text{the Sum of the 2}\\ \text{laſt contracted.}\end{array}\right.$$

$$u = \sqrt{bc + \frac{dd}{4}} - \frac{d}{2} = \text{the Canon}$$

But in truth this way of ſolving Equations by Subſtitution, is at the beſt not ſo brief and eaſy a Method as that by compleating the Square; for if that be well conſidered, there ariſes from it,

IV. *Theſe Examples and Rules for Solving moſt Quadratic Equations by Inſpection only.*

Examples. Note, $c=3$, and u found $=9$.	*Demonſtrations of the Truth of the Solutions or Canons.*
1. Equations propoſed. $uu + 10u = 57c$ Canon .... $u = \sqrt{57c + 25} - 5$	$\sqrt{57c + 25} - 5 = 9$: ſo $u = 9$ as *per* Canon 1. and $57c = 171$: ſo is $uu + 10u = 171$ as *per* Equation 1.
2. Equations propoſed. $uu - 6u = 9c$ Canon ..... $u = \sqrt{9c + 9} + 3$	$\sqrt{9c + 9} + 3 = 9$: ſo $u = 9$ as *per* Canon 2. $9c = 27$: ſo is $uu - 6u = 27$ as *per* Equation 2.

The Rules for Solving a Quadratic Equation by Inſpection.

1. What I mean here, *by Inſpection*, is the putting down only the Canon or Anſwer without any previous Operation: To do which, take

take the Root of the Square of the unknown Quantity, which here is *u*.

2. Make the Sign of Equation, and to the right hand of that, place the Root of the known Quantity or Quantitys given, with the Sign √ before, as in Equation the 1ſt, $u = \sqrt{57c}$.

3. Square half the Co-efficient, and add to the laſt mentioned Quantity with the Sign + always before it, and it will be $u = \sqrt{57c + 25}$.

4. To the laſt add half the Co-efficient, when the Sign is —, and ſubſtract it when the Sign before the Quantity where the Co-efficient is, is +; as in this Example I put —5, becauſe 10*u* is +10*u*: ſo is the Canon $u = \sqrt{57c + 25} - 5$.

And thus you may give the Canon for any Quadratic Equation when reduced into either of the two Forms above.

Here is the proper place for giving the Geometrical Conſtruction of Simple and Quadratic Equations; which I omit for theſe four Reaſons: 1. Becauſe it is done by ſo many other Authors. 2. For that this Treatiſe is deſigned to be as eaſy as poſſible, in order to inſtruct and encourage the Tyro, rather than to make a ſhew of Learning. 3. It cannot be expected, where ſo many things are contained in one ſmall Volume. And, 4. It is ſufficiently proved and demonſtrated above, by comparing the Parts of the Canon and thoſe of the Equation given, as grounded on the Solution, eſpecially in the Numeral Operations.

Sect. XI. *Containing for Exerciſe, ſome Queſtions reſolv'd by Simple and Quadratic Equations, and by various Poſitions.*

I. By *Simple Equations.* I had the following propoſed to me, which the Queriſt my Friend had a real Occaſion to know, as falling in his Buſineſs.

Problem 1.] 365 Pounds is received for the Uſe of a publick Building, and the Receiver was to have 4 *d per* Pound for what he paid in: what was the Poundage, and what paid in?

Steps.

Steps.	*Done Literally thus.*	*The Columns of Rules to both Ways*	*Numerally thus:*	Steps.
1	u = the Poundage, and b is = 365		u = the Poundage.	1
2	$b - u$ = the Sum paid in.		365 — u = the Sum paid in.	2
3	$60u = b - u$		$60u = 365 - u$	3
4	$61u = b$	— u in 3d + the 4th ÷ 61	$61u = 365$	4
5	$u = \frac{b}{61}$ the Canon.		$u = \frac{365}{61}$ = the Canon.	5
	So $\frac{b}{61}$ = the Poundage.		$365 \div 61 = l.\ 5 : 19 : 8$	
	And $b - \frac{b}{61}$ = the Sum paid in.		and $365 - \frac{365}{61} = 359 : 0 : 4$	

Problem 2.] *The whole Length of a May-Pole is* 16 *Yards, the major part of which is broke by the Wind, tho not quite off, but so that the Top of the Pole extends from the Fracture to the Ground at* 8 *Yards distant from the Appearance of the Pole above the Surface of the Earth in which it was placed: how much is broken off, and how much is standing?*

I put $16 = k$ and $8 = n$ —

Steps.		*Rules*, viz.		Steps.
1	u = the Perpendicular (or (Part standing.		u = the part standing.	1
2	$k - u$ = Hypothenuse.		$16 - u$ = the Hypothenuse	2
3	$nn + uu = kk - 2ku + uu$	n ⊙ + 1st ◎ + 2d ◎	$8^2 + u^2 = 16^2 - 2 \times 16u + u^2$	3
4	$2ku = kk - nn$	$2ku$ +, nn + 4th ÷ $2k$	$2 \times 16u = 16^2 - 8^2$	4
5	$u = \frac{kk - nn}{2k}$ = the Canon		$u = \frac{16^2 - 8^2}{2 \times 16}$ = the Canon	5

Demonstration of both.

$k^2 = 256 - n^2 = 64 = 192 \div 2k = 32 = u = b$. | $16^2 = 256 - 8^2 = 64 = 192 \div 32 = 6$ = the part (standing.

Vid. *Euclid*'s Elem. *Lib.* 1. *Prop.* 47.

Problem 3.] *Numbers in Geometrical Progression being given to find the Rule for the Aggregate of any Series. This is shewn in* Progression, *Chap.* 2. *page* 82.

Problem 4.] *Supposing the Pendulum of a Clock which vibrateth Seconds (or* 60 *times in a Minute) be* 39.2 *Inches in length; how long must a Pendulum*

dulum be to, vibrate as often in a Minute as it hath Inches in length? This also was a Question propoſed for me to anſwer, which I did thus:

I put $k=60$ Seconds: and $n=39.2$ Inches.

Steps.	$n=$ the N° of Vibrations requir'd	*Rules for the Work.*	$u=$ N° of Vibrations.
1			
2	$k\,k.\quad n.\quad u\,u.\quad \frac{k\,k\,n}{u\,u}$ the Analogy.	Or $uu.\ kk :: n.\ \frac{kkn}{uu}$ in a direct Proportion.	$60^2.\ 39.2 :: uu.\ 39.2\times 60^2 \div uu$
3	$u=\frac{k\,k\,n}{u\,u}$	$uu\ (ww)=\frac{kkn}{uu}$ or $\sqrt{uu}=$ &c.	$u=\frac{60^2\times 39.2}{u\,u}$
4	$u\,u\,u=k\,k\,n$	3d Reduced.	$uuu=60^2\times 39.2$
5	$u=\sqrt[3]{k\,k\,n}$	$=\sqrt[3]{}$ of the 4th.	$u=\sqrt[3]{60\times 39.2}$ *viz.* of (141120

$kk=3600, \times n\ (=39.2)=141120$; and $\sqrt[3]{141120}=52$ near.

$\sqrt[3]{141120}=52=$ the Vibrations in a Minute, And Inches for the Length of ſuch a Pendulum.

Problem 5.] *What Age is that Perſon when* $\frac{2}{3}$ *of the Time from his Birth is equal to* $\frac{7}{8}$ *of what he wants of* 60?

I put $60=k$, $\frac{2}{3}=n$, and $\frac{7}{8}=t$.

Steps.	$u=$ the Perſon's ſuppoſed Age.	*Rules for the Steps.*	$u=$ his Age.
1			
2	$k-u$ the time he wants of 60.	or $60-u$	$60-u$
3	$\frac{u}{n}=\frac{k-u}{t}$	$u\div n$, & $k-u\div t$	$u\div\frac{2}{3}=60-u\div\frac{7}{8}$
4	$u=\frac{kn-nu}{t}$	the 3d × by n	$u=60\times\frac{2}{3}-u\times\frac{2}{3}\div\frac{7}{8}$
5	$tu=kn-nu$	the 4th × by t	$\frac{7u}{8}=60\times\frac{2}{3}-u\times\frac{2}{3}$
6	$tu+nu=kn$	nu in 5, +	$\frac{7u}{8}+\frac{2u}{3}=60\times\frac{2}{3}$
7	$u=\frac{kn}{t+n}=$ the Canon.	6th $\div$ $t+n$	$u=60\times\frac{2}{3}\div\frac{7}{8}+\frac{2}{3}$

So that $\frac{kn}{t+n}=$ the Age; and $k-\frac{kn}{t+n}=$ it wants of 60.

And $\frac{kn}{t+n}\div n=k-\frac{kn}{t+n}\div t$ for Proof.

$u=25\frac{3}{3}\frac{5}{7}$ *Years*, the Age, and $60-25\frac{3}{3}\frac{5}{7}=34\frac{2}{3}\frac{2}{7}$ *Years*, the want of 60.

And $25\frac{3}{3}\frac{5}{7}\div\frac{2}{3}=34\frac{2}{3}\frac{2}{7}$ *Years* $\div\frac{7}{8}$ for a Proof.

Problem 6.] *Two Persons,* A *and* B, *have each a certain Number of Pounds, so that if* B *give* A 3 *of his, then they will have each a like Number: But if* A *give* B *three of his Pounds, then* B *will have* 5 *times as many as* A: *What Number of Pounds had each at first?*

I put $n=3$ and $k=5$.

1	$u=B$ hath at first.	u suppoſed	$u=B$ hath at firſt.
2	$u-2n=A$ hath at firſt.		$u-3\times 2$
3	$u-n=B$ will have.	1ſt $-\frac{2n}{2}$	$u-3$
4	$u-n=A$ will have.	*ibid.* as 3d.	$u-3$
5	$u-3n=A$ will after have.	$=u-2n-n$	$u-9$
6	$u+n=B$ will then have.		$u+3$
7	$ku-3kn=u+n$	5th $\times k+$ 6th	$5u-3\times 5\times 3=u+3$
8	$ku-u=n+3kn$	$u+$ & $3ku+$	$5u-u=3+3\times 5\times 3$
9	$u=\frac{n+3kn}{k-1}$ the Anſwer.	8th $\div k-1$	$u=\frac{3+3\times 5\times 3}{4}$ Anſwer.

$\frac{n+3kn}{k-1}=4n$, which B had.	$\frac{3+3\times 5\times 3}{4}=12$, which B at 1ſt had.
And A had $2n$: for $2n-n=n$: and $n+4n=5n$.	And A had 6: for $6-3=3$; and $3+12=5\times 3=15$.

Problem 7.] *A Hare,* 50 *of her Steps or Paces before a Greyhound, takes* 4 *Steps to the Dog's three; but then the Dog steps as much at twice as the Hare does at thrice: How many Steps must the Dog make before he catch the Hare?* This old Algebraic Queſtion I anſwer thus:

I put $k=50$, $n=3$, $s=2$, and $b=4$.

1	$u=$ the ſuppos'd Anſwer (or Dog's Steps.	u	
2	$n.\ b::u.\ \frac{bu}{n}$ all the Hare's (actual Steps		$3.\ 4::u.\ \frac{4u}{3}$
3	$n.\ s::\frac{bu}{n}+k.\ \frac{bsu}{nn}+\frac{ks}{n}$ the (Dog's Steps.		$3.\ 2::\frac{4u}{3}+50.\ \frac{8u}{9}+\frac{100}{3}$
4	$u=\frac{bsu}{nn}+\frac{ks}{n}$		$u=\frac{8u}{9}+\frac{100}{3}$
5	$nnu-bsu=kns$	the 4th $\times nn$, &c.	$9u-8u=\frac{900}{3}$. *Ergo*
6	$u=\frac{kns}{nn-bs}$ the Canon for (Anſwer.	the 5th $\div nn-bs$	$u=\frac{900}{3}=300$ Steps, the (Anſwer.

Problem 8.] A Ciſtern that hath four Vents of different Dimenſions containeth 72 Barrels of Liquid: If one Vent or Cock be opened, the Liquid will be ſpent in 6 Hours; if a ſecond be opened alone, it will run out in 8 Hours; if the third Cock be turned alone, the Liquid will be exhauſted in 9 Hours; and the fourth Cock or Vent is ſo ſmall, that if it alone be opened, it will be twelve Hours before the Ciſtern is empty: Now the Queſtion is, *in what Time the Ciſtern will be void of Liquid*, if all the four Vents be opened at the ſame Moment of Time. This Queſtion (which I have not, that I know of, ſeen before anſwer'd by Algebra) I ſolve thus; tho there is ſomewhat a ſhorter way: but I deſign it plain to the Learner, and if 72 or k be omitted and ſuppoſed 1, it will come to the ſame Canon, as you may make tryal at your leiſure, for k is thrown out even in this Example.

For 72 I put k, $6=n$, $8=s$, $9=t$, $12=r$.

	u = the unknown Time or ſuppoſed	(Anſwer.	u
1	$u.\ k :: u.\ \frac{ku}{n}$		$6.\ 72 :: u.\ \frac{72u}{6}$
2	$s.\ k :: u.\ \frac{ku}{s}$		$8.\ 72 :: u.\ \frac{72u}{8}$
3	$t.\ k :: u.\ \frac{ku}{t}$		$9.\ 72 :: u.\ \frac{72u}{9}$
4	$r.\ k :: u.\ \frac{ku}{r}$		$12.\ 72 :: u.\ \frac{72u}{12}$
5	So that $\frac{ku}{n}+\frac{ku}{s}+\frac{ku}{t}+\frac{ku}{r}=k$		$\frac{72u}{6}+\frac{72u}{8}+\frac{72u}{9}+\frac{72u}{12}=72$
6	$\frac{u}{n}+\frac{u}{s}+\frac{u}{t}+\frac{u}{r}=1$	{5th Contracted or : by k}	$\frac{u}{6}+\frac{u}{8}+\frac{u}{9}+\frac{u}{12}=1$
7	$u+\frac{nu}{s}+\frac{nu}{t}+\frac{nu}{r}=n$	the 6th × n	$u+\frac{6u}{8}+\frac{6u}{9}+\frac{6u}{12}=6$
8	$su+nu+\frac{nsu}{t}+\frac{nsu}{r}=ns$	the 7th × s	$8u+6u+\frac{6u\times8}{9}+\frac{6u\times8}{12}=6\times8$
9	$stu+ntu+nsu+\frac{nstu}{r}=nst$	the 8th × t	$8u\times9+6u\times9+6u\times8+\frac{6u\times8\times9}{12}=6\times9$
10	$rstu+nrtu+nrsu+nstu=nrst$	the 9th × r	$8u\times9\times12+6u\times9\times12+6u\times8\times12+6u\times8\times9=6\times8\times9\times12$
11	$u=\frac{nrst}{rst+nrt+nrs+nst}$ = Anſw.	the 10th ÷ all that are × u	$u=\frac{6\times8\times9\times12}{8\times9\times12+6\times9\times12+6\times8\times12+6\times8\times9}$

And $\frac{nrst}{rst+nrt+nrs+nst}=\frac{5184}{2520}$ Hours.

Numerat. = 5184; Denom. = 2520 = 2:3:2:3:2:5 which is the accurate Anſwer.

Problem 9.] How shall I find out a Canon for discovering the Discount of Money? See the Operation.

d = Days to come. c = 100 *l.*
p = Principal. y = a Year or 365 Days.
r = Rate of Interest. u = the Discount sought.

$$y.\ r :: d.\ \frac{dr}{y}$$

$$\text{And } c + \frac{dr}{y}.\ \frac{dr}{y} :: p.\ u$$

$$cu + \frac{dru}{y} = \frac{dpr}{y}$$

or $cuy + dru = drp$, and consequently $u = \frac{dpr}{cy + dr}$ the Canon for Discount.

Problem 10.] To find a Canon for discovering the present Worth of Money due at the end of any Days. Symbals as above.

$$y.\ r :: d.\ \frac{dr}{y}$$

$$c + \frac{dr}{y}.\ c :: p.\ u$$

$$cu + \frac{dru}{y} = cp$$

$cuy + dru = cpy$; and consequently $u = \frac{cpy}{cy + dr} =$ the Canon for present Worth.

These Canons apply'd under *Sect.* 8. *Chap.* 3. in finding the Discount of Money much sooner than by the common Way of the Rules of Proportion.

II. *To resolve Questions by Quadratic Equations, which they form.*

Problem 1.] There are three Numbers in Geometrical Proportion continued; the Mean $=42$, the Difference between the Extremes $=112$: What are the two Extremes?

I put $k=112$; $n=42$.

1	$u=$ the leſſer Extreme.		$u=$ the leſſer Extreme. 1
2	$u+k$ the greater.		$u+112=$ the greater. 2
3	$uu+ku=nn$	2d × 1ſt $=u^2$	$uu+112u=42^2$ 3
4	$u=\sqrt{nn+\frac{kk}{4}}-\frac{k}{2}$ *Anſw.*	3d w; and $\frac{k}{2}+$	$u=\sqrt{42^2+56^2}-56$ 4

$\sqrt{nn+\frac{kk}{4}}-\frac{k}{2}=14=$ the leſſer. | $\sqrt{42^2+56^2}-56=14$ the leſſer.

And $14+k=$ the greater Extreme. | And $14+112=126$ the greater.

And for Proof $14\times126=42^2=1764$.

Problem 2.] To divide a Line into Extreme and Mean Proportionals: or (which is one Uſe thereof) to find the Side of the Dodecahedron by having that of the Octahedron given, whereof this is the greater Proportional Part.

A Line is thus divided, when the Square of the greater Part is $=$ the Rectangle under the whole Line and the leſſer Part.

I put $k=$ to the Side of the Octahedron $(=ab)$ 120. 

1	$u=$ the greater part (or the ſide of the Dodecahedr.		$u=ac$ of the Line ab. 1
2	$k-u=$ the leſſer part (as cb)		$120-u=cb$ of the Line ab. 2
3	$uu=kk-ku$	the 1ſt ⊙; and $k\times$ in 2d	$uu=120^2-120u$ 3
4	$uu+ku=kk$	$ku+$	$uu+120u=120^2$ 4
5	$u=\sqrt{kk+\frac{kk}{4}}-\frac{k}{2}=ac$ (Anſw.	the 3d w and $\frac{k}{2}+$	$u=\sqrt{120^2+\frac{120^2}{4}}-\frac{120}{2}$ 5

So that $u=ac$ of the Line $ab=$ the Side requir'd. | And $\sqrt{120^2+\frac{120^2}{4}}-\frac{120}{2}=74.16$ (near.

Pro-

Problem 3.] A Merchant sold a Ship for *l.* 2400, and gained after the Rate of what the Ship cost by laying out 10000 Pounds: What did the Vessel cost him?

I put $s = 2400$, and $t = 10000$.

1	u = the supposed Cost.		u	1
2	$s - u$ = the Gain.		$2400 - u$	2
3	$u. s - u :: t. \frac{st - tu}{u}$		$u. 2400 - u :: 10000. \frac{2400 - u \times 10000}{u}$	3
4	$u = \frac{st - tu}{u}$	} or the 1st = 4th } Proportional.	$u = \frac{2400 - u \times 10000}{u}$	4
5	$uu + tu = st$	4th $\times u$ and $tu +$	$uu + 10000\,u = 24000000$	5
6	$u = \sqrt{st + \frac{tt}{4}} - \frac{t}{2} = 2000$	5th Step *ww*, &c.	$u = \sqrt{24000000 + 25000000} - 5000 (= 2000$	6

And for Proof 2000. 400 :: 10000. 2000

Problem 4.] Admit *h i n* (*Fig.* 9. *Plate* C.) to be the Curve of an Hyperbola, and that there are given

$t\ (= i\,o)$ the *Latus transversum* $= 7.4$
$l\ (= i\,h)$ the *Latus rectum* $= 1.76$
$o\ (= z\,h)$ the greatest Ordinat $= 4.68$

What is the Length of the Abscissa, $(i\,z)$ for which I put (u) because unknown?

1	$oo = lu + \frac{luu}{t}$ = the Property of the Curve.		$4.68^2 = 1.76 \times u + \frac{1.76 \times uu}{7.4}$	1
2	$ltu + luu = oot$	1st × by t	$1.76 \times 7.4 \times u + 1.76 \times uu = 4.68^2 \times 7.4$	2
3	$uu + tu = \frac{oot}{l}$	2d ÷ by l	$uu + 7.4 \times u = \frac{4.68^2 \times 7.4}{1.76}$	3
4	$u = \sqrt{\frac{oot}{l} + \frac{tt}{4}} - \frac{t}{2} = 6.7$	the 3d *ww*, & $\frac{t}{2} +$	$u = \sqrt{\frac{4.68^2 \times 7.4}{1.76} + \frac{7.4^2}{4}} - \frac{7.4}{2} = 6.7$	4

Problem 5.] What Number is that, whose Square multiply'd by 10, being added to its Biquadrate, the Sum makes 30251?

I put $k=30251$, and $10=n$.

1	$u=$ the Answer unknown.		u
2	$u^4+nuu=k$		$uuuu+10uu=30251$
3	$uuuu+nuu+\frac{nn}{4}=k+\frac{nn}{4}$	2d $+\frac{n^2}{2}$ or $+\frac{nn}{4}$	$u^4+10uu+\frac{10^2}{4}=30251+\frac{10^2}{4}$
4	$uu+\frac{n}{2}=\sqrt{k+\frac{nn}{4}}$	3d w	$uu+\frac{10}{2}=\sqrt{30251+\frac{10^2}{4}}$
5	$u=\sqrt{\sqrt{k+\frac{nn}{4}}-\frac{n}{2}}$	{4th Step w and $\frac{n}{2}+$}	$u=\sqrt{\sqrt{30251+\frac{10^2}{4}}-\frac{10}{2}}=13$

And for Proof $13^2=169\times10=1690+13^4\ (=28561)=30251$.

The last Canon for the Answer $=u$, is thus to be read: That if half n be deducted from the Square Root of $k+\frac{nn}{4}$, the Square Root of the Remainer is $=u=13$.

Problem 6.] A Farmer left his Son and a Nephew *l.* 400 to be so divided between them, that their Parts being alternately divided by each other, the Sum of the Quotients will be 30; but the greater part he devised to his Son: What did he leave to each?

Some make this Question, which I propose as above, to be solved by Various Positions, others by the Doctrine of Surds: But I am sure it is most naturally reducible to the 2d of my 2 Cases of Quadraticks before mention'd. And because I have not seen it done before, I shall insert the Solution at large as it is most easily understood, and that by the Numeral Way; but shall wave the Literal Method on purpose that the Reader may try his own Proficiency therein.

1	u = the Son's Share.	
2	$400-u$ = the Nephew's Share.	
3	$\frac{u}{400-u} + \frac{400-u}{u} = 30$	
4	$u + \frac{160000-800u+uu}{u} = 12000-30u$	
5	$un + 160000-800u+uu = 12000u+30uu$	
6	$160000 = 12800u-32uu$	$\left\{\begin{array}{l}\text{the 5th contracted,}\\ \text{and } 2uu-800u+\end{array}\right.$
7	$5000 = 400u-uu$; and *ergo*	the 6th $\div$ by 32
8	$uu-400u = -5000$ an Equation of the (2d ſort.	$\left\{\begin{array}{l}5000 \text{ (in the 7th} +\\ \text{and } 400u-uu +\end{array}\right.$
9	$uu-400u+40000 = 35000$	$\left\{\begin{array}{l}\frac{400}{2}\text{@ and added}\\ \text{to each part of}\\ \text{the Equation.}\end{array}\right.$
10	$u-200 = \sqrt{35000}$	the 9th w
11	$u = \sqrt{35000}+200$.	-200 in the 10th +

And $\sqrt{35000}+200$ = Son's Share = *l.*387.083; and $400-387$, &c.
$= 12.917$

And for Proof $387.083 \div 12.917 = 29.966$; and $12.917 \div 387.083$ $= .034$; and $.034+29.966 = 30$: which anſwers in every part the Requiſits.

III. *The Reſolution of Queſtions which require Various Poſitions.*

Having in *Sect.* 9, & 10. given Rules for ſolving Simple and Quadratic Equations; there was no need in *Sect.* 11. that I ſhould repeat any thing farther than what I have done in the middle Column of Rules: But in this Head of Various Poſitions having ſaid nothing thereon before, I ſhall make it as plain as I can.

1. Then

1. Then by various Positions we mean Questions which have more Numbers than 1 sought for, and are discovered by assuming a Vowel for each Number sought.

2. And we proceed in the Work first with one unknown Number, not regarding the rest but as known: And when we have found the first unknown Quantity equal to some others, we omit that Quantity, and take instead thereof what it is equal to. Examples, and Rules thereupon, will make the matter plain.

Problem 1.] There is a Composition of 20 Integers, whose total Value is 82; but there are 2 different prized Ingredients in the 20, 1 of 4, the other of $4\frac{1}{4}$: How much of each Simple is there in the said Composition, and what the Value of each Ingredient in the whole of it, to make up the 82?

I put $q=20$, $4=k$, $4\frac{1}{4}=n$, and $82=t$.

Steps.		*Rules for the Operations.*		Steps.
1	$u=$ the Quantity at 4		u	1
2	$a=$ the Quantity at $4\frac{1}{4}$		a	2
3	$ku+an=t$	the 1st and 2d $\times k$ & n	$4u+4\frac{1}{4}a=82$	3
4	$ku=t-an$	$a\,n$ +	$4u=82-4\frac{1}{4}a$	4
5	$u=\frac{t-an}{k}$ *	4th $\div$ by k	$u=\frac{82-4\frac{1}{4}a}{4}$	5
6	$\frac{t-an}{k}+a=q$	{2d part of 5th, + $a=q$}	$\frac{82-4\frac{1}{4}a}{4}+a=20$	6
7	$t-an+ak=kq$	the 6th $\times$ by k	$82-4\frac{1}{4}a+4a=80$	7
8	$t-kq=an-ak$	$-an+ak+$ in 7th	$82-80=4\frac{1}{4}a-4a$	8
9	$a=\frac{t-kq}{n-k}=$ the Canon.	the $8\div n-k$	$\frac{82-80}{4\frac{1}{4}-4}=a$	9

Hence appears the Canon to find the Quantity at $4\frac{1}{4}$, *viz.* $\frac{82-80}{4\frac{1}{4}-4}$

$=2\div\frac{1}{4}=8$. and $20-8=12=a$.

For Proof of which, 12 at $4=48$; and 8 at $4\frac{1}{4}=34$: Now $48+34=82=$ the total Value.

Problem 2.] A Person dying, left by Will to 2 Sons each a Sum of Money; so that if to 5 times what he left the elder you add 3 times what

what he gave to the younger, the Sum will be 1675 : And if from 7 times the Share of the elder Brother you deduct 8 times the Share of the younger, the Remainer will be 210: What Sum did the Father leave to each ?

I put $s=1675$, $d=210$, $k=5$, $n=3$, $m=7$, and $t=8$.

1	u=the Legacy of the (Elder.		u	1
2	a=that of the Younger.		a	2
3	$ku+an=s$.	1ſt×k; 2d×n=s	$5u+3a=1675$	3
4	$u=\frac{s-an}{k}$ *	3d÷k; & an+	$u=\frac{1675-3a}{5}$ *	4
5	$\frac{ms-amn}{k}-at=d$	{2d part, 4th × by $m-at=d$}	$\frac{1675\times7-3a\times7}{5}-8a=210$	5
6	$ms-amn-akt=dk$	5th × by k	$1675\times7-3a\times7-8a\times5=210\times5$	6
7	$ms-dk=amn+akt$	{$amn-akt+$ and $dk+$}	$1675\times7-210\times5=3a\times7+8a\times5$	7
8	$\frac{ms-dk}{mn+kt}=a=175$. *Ergo*	7th÷$mn+kt$	$\frac{1675\times7-210\times5}{3\times7+8\times5}=a=175$, and	8
9	$\frac{s-175n}{k}=u=230$.	4th compared with 8th.	$\frac{1675-175\times3}{5}=u=230$	9

And for Proof $230\times5+175\times3=1675$. And $230\times7-175\times8=210$, as *per* the Queſtion.

Problem 3.] A Surveyor having meaſured three Fields of Encloſure, hath loſt the Contents : But remembers, that if 847 Acres be added to the 1ſt, the Sum will be equal to that of the other two Fields ; if 847 Acres be added to the 2d Encloſure, the Sum will be double

double to the Content of the 1ſt and 3d; and if 847 Acres be added to the 3d Field, it will make treble the 1ſt and 2d: How many Acres is in each Field?

I put a=1ſt, e=2d, u=3, and k=847.

1	$a+k=e+u$; ſo that $a=e+u-k$	k + in 1ſt.	$a+847=e+u$ or $a=e+u-847$	
2	$e+k=2a+2u$ $e=2a+2u-k$	k + in 2d.	$e+847=2a+2u$	$e=2a+2u\ \left[-847\right.$
3	$u+k=3a+3e$ $u=3a+3e-k$	k + in the 3d.	$u+847=3a+3e$	$u=3a+3e\ \left[-847\right.$
4	$e=2e+2u-2k+2u-k$	{the 1ſt×in 2 +2u−k in 2d.}	$e=2e+2u-\overline{847\times2},+2u-847$	
5	$e=3k-4u$	{the 4th contracted.}	$e=\overline{847\times3},-4u$	
6	$u=9k-12u+3u-3k+9k-\ \left[12u-k\right.$	{the 5th×3 +3u −k +, the 5th ×3−k.}	$u=\overline{847\times9},-12u+3u-\overline{847\times3}\}$ $+847\times9-12u-847$	
7	$u=14k-21u$	6th Step contracted	$u=847\times14-21u$	
8	$u=\frac{7k}{11}$ *	{21u + in the 7th and ÷ 22, &c.}	$u=\frac{847\times7}{11}=539$	*Acres.* =3d Field.
9	$e=3k-\frac{28k}{11}$ or $\frac{5k}{11}$ *	{=3k−4u as in the 5th, and u in the 8th is $\frac{7k}{11}\times4$=9th}	$e=\frac{847\times5}{11}=385$	=2d Field.
10	$a=3k-\frac{21k}{11}-k$	{a = as *per* 1ſt compar. with Steps 8, & 5.}	$a=\overline{847\times3}-\frac{847\times21}{11}-847$	
11	$a=2k-\frac{21k}{11}=\frac{k}{11}$ *	{the 10th Step contracted.}	$a=\frac{847}{11}=77$	=the 1ſt Field.

Now $\frac{k}{11}$ =a the 1ſt Field	*Acres* =77
$\frac{5k}{11}$ =e the 2d Field	=385
$\frac{7k}{11}$ =u the 3d Field	=539

And for Proof, *Acres*
77+847=924=385+539 =the 2d and 3d.
385+847=1232=77+539×2 =2d and 3×2.
539+847=1386=77+385×3 =1ſt and 2d×3.

Note

Notes on the three Operations to the laſt three Problems above.

The middle Column has ſuch plain Directions, as may ſuffice a docile Reader; but that I may render every thing plain, I ſhall obſerve that

In the firſt Operation, every Step is as eaſy as in common ſimple Equations, till you come to the ſixth Step, which is formed thus:

Since u (an unknown Quantity in the 5th Step) is found $= \frac{t-an}{k}$, and ſince $u+a$ (the 2 Quantities of the different Ingredients in the 20) is equal to $q=20$; therefore that I may get rid of u, I take what it is equal to, and ſay $\frac{t-an}{k}+a=q$, and then proceeding by the common Rules laid down, I find $a=\frac{t-kq}{n-k}$, which are all known Quantitys: And this Canon doth ſhew $a=8$.

Therefore u muſt be $20-8=12$. Or you may find it by a more artificial way, by having recourſe to the 5th Step, where $u=\frac{t-an}{k}$ *ergo* $u=\frac{t-8n}{k}$, which you'll find $u=12$.

In the 2d Operation, I proceed as in common ſimple Equations to the 4th Step, and there, becauſe $u=\frac{s-an}{k}$ I omit u, and take what it is equal to, $\frac{s-an}{k}$, which according to the Tenor of the Queſtion being multiply'd by m, and made leſs by at, the Remainder is $=d$; and ſo proceeding by the uſual Rules, as before taught, I find $a=\frac{ms-dk}{mn-kt}=175$: And if you have recourſe to the 4th Step, and inſtead of an there, put $175n$ (a being found $=175$) you will have $u=\frac{s-175n}{k}$, or $u=230$. And

In the 3d Operation, the three firſt Steps are compoſed according to the very Words of the Problem, and the 2d part of thoſe Steps are the ſame, only having k tranſpoſed towards the right, there is ſhew'd what the 3 ſeveral unknown Quantitys are ſimply equal to.

Now for the 4th Step I argue thus: Conſidering that in the 2d Step $e=2a+2u-k$, and that a is $=e+u-k$, therefore $e=2e+2u-2k$ (which is twice the firſt Step, $=2a$ in the 2d) $+2u-k$, which are the reſt of the 2d Step.

Which 4th Step is in the 5th reduced to $3k-4u$.

Then I proceed to diſcover the Value of u by compoſing the ſixth Step, thus: u being in the third Step towards the right hand

$= 3a + 3e - k$; and the 1ſt Quantity in what a is $=$, being e; and e being in the 5th Step $= 3k - 4u$, therefore I ſay $3a$'s is firſt 3 times $3k - 4u$, *i.e.* $9k - 12u$: and the Remainer of what a is $=$, being $u - k$, *ergo* that $\times$ by the 3, I ſay $3a = 9k - 12u + 3u - 3k$. Now the reſt of what u is $=$ being $3e$, and e in the ſaid 5th Step being $3k - 4u$; therefore (to make the Proceſs plain) I repeat $3e$ or $9k - 12u$. And there being farther $- k$ to make up what u is $=$; therefore I ſay in the 6th Step $u = 9k - 12u + 3u - 3k + 9k - 12u - k$, which contracted and reduced, is eaſily found (as *per* the middle Column in the Example) $u = \frac{7k}{11}$. Therefore e in the 5th Column; I put $= 3k$, $- 4$ times what u is found equal to in the 8th Step.

Laſtly, a being $= e + u - k$; I ſay it is $= e$ in the 9th Step, $+ u$ (in the 8th Step) $- k$, or $a = 2k - \frac{21k}{11}$, which is $= \frac{k}{11}$.

Sect. XII. *Concerning Cubical Equations.*

A Cubical Equation may be known by having commonly (for the firſt Term) the Cube of the unknown Quantity; the 2d Term, being the Square of that Quantity multiply'd in a Coefficient or ſome known Quantity; the third is the unknown Quantity ſought in a known, as thus and the like; $uuu + cuu + du = k$, u being the Root or Quantity ſought.

A Cubical Equation is formed from 1, 2 or 3 Binomials, *&c.* involved or multiply'd in each other; repreſenting either a compleat Cube or Parallelopiped, whoſe Baſe is either a Geometrical Square, or a Parallelogram: in which laſt, if $u + b$ be the greater Side, $u - b =$ the leſſer Side of the Baſe, and $u + d =$ the Length; the Quantitys multiply'd produce this Cubical Equation, *viz.* $uuu + duu - bbu - bbd = k$ the Content. To find the Dimenſions, which depend on the Value of u, it muſt be done by the Rules following.

2. In a Quadratic Equation, by the Methods of Reſolution foregoing, you gain 1 Root; and by dividing the Equation given by that Root you find the other, if there be 2 different Roots.

Thus in the Equation $uu - 16u = -63$, being reſolved gives $u = 9$: And if the $uu - 16u + 63$ (the 63 being tranſpoſed) be divided by $u - 9$ (the 9 being tranſpoſed) the Quote will be $u - 7$ or $u = 7$. So are $u - 9$ and $u - 7$ found the 2 Roots or Dimenſions of that Equation, *i.e.* 9 and 7.

I. To

3. I. *To resolve Cubic Equations by Tentation and Depression.*

Having by tryal, as in the following Example, found 1 Root, you must divide the Cubic Equation given by that Root, and that will depress the Equation into a Quadratic; which Quadratic you may find the Roots of, as is shewn last above.

Example 1.] What are the 3 Roots or real Dimensions of this Cubical Equation $u\,u\,u - 57\,u\,u + 1040\,u = 6000$?

To answer this; I must first find by tryal a Root, which being multiply'd, *&c.* according to the given Equation, will be 6000.

To which end I suppose 10 *to be* 1 *Root* $= u$.

Then uuu (or 10^3) is $= 1000$
The 3d Term $1040\,u = 10400$ } Sum $= 11400$
2d Term $- 57\,uu$ (or 57×10^2) deduct. $= 5700$ } Rests 5700

The Remainer should be 6000, but 'tis only 5700; therefore the 10 supposed is too little.

2dly, I suppose therefore a Root to be $12 = u$.

Then uuu (or 12^3) is $= 1728$
$+ 1040\,u \ldots\ldots = 12480$ } Sum $= 14208$
$- 57\,uu$ (or 57×144) $\ldots\ldots\ldots = 8208$ } Rests 6000

So that I find $12 = u$ or $u - 12$ to be 1 Root.

By which Root having divided the Cubic Equation given, I depress it to the Quadratic Equation $u\,u - 45\,u = -500$. Which Equation being resolved, gives $u = 25$, or $u - 25$ for another Root. And dividing that Quadratic by this $u - 25$, the Quote or 3d Root is found $u - 20$, or $u = 20$. See the Operation.

$u-12)\,uuu-57uu+1040u-6000\,(=uu-45u=500=$ *the Quote or Quadratic.*

$uuu-12uu$

$-45uu+1040u$
$-45uu+540u$

$500u-6000$
$500u-6000$

or $u = \sqrt{.6.25} + 22.5 = 25$ *Quadratic resolved, (or a 2d Root.*

$u-25)\,uu-45u+500\,(u-20$ *(= a 3d Root.*

$uu-25u$

$-20u+500$
$-20u+500$

0

So that the 3 Roots being found 12, 20 and 25; you may prove the Truth by putting u equal to each, transposing the Roots, and multiplying the Quantitys one in another: for the 2d Rectangle will be the Cubical Equation given, transposing the Member next the right hand of the last Product.

1 *Note*, That in a Cubic Equation if the ſingle Root or Square of the unknown Quantity be multiply'd in more than one known Quantity, with Signs between the Rectangles, as ſeveral Members of the Equation; where the ſaid unknown Root is found ſquared in 2 *&c.* ſuch Members, they make but 1 Term: and ſo do all thoſe where the unknown Root is found ſingle make another; as $u^3 - u^2b - u^2d + uc + ur + d^3 = 0$. In this $-u^2b - u^2d$ is but 1 Term of the Equation, and $uc + ur$ is one other Term.

The Proof of the foregoing Work.

$u - 25$
$u - 20$

$uu - 25u$
$\quad - 20u + 500$

$uu - 45u + 500$
$u - 12$

$uuu - 45uu + 500u$
$\quad - 12uu + 540u - 6000$

$uuu - 57uu + 1040u - 6000$
or $uuu - 57uu + 1040u = 6000 =$ (*the Equation given, for Proof.*

2 *Note*, That in the Cubic ſuppoſe $u^3 = uc + ur + d^3 = 0$; the 2d Term here is ſaid to be wanting, and $uc + ur$ is the 3d Term.

Example 2.] Admit this Equation were to be reſolved; $uuu - 30.5uu + 309u = 1039.5$: I ſuppoſe 1 Root = 12. and working as by the Equation, I find

$u^3 = 12^3 = 1728$
$-30.5u^2 = -4392$
$+309u = 3708$

} The Sum as *per* Addition of Algebra is 1044, which is more than the 1039.5.

2*dly*, I therefore ſuppoſe a leſſer Number, *viz.* 11 = a Root, then

$u^3 = 11^3 = 1331$
$-30.5uu = -3690.5$
$+309u = 3399$

} The Sum of which is = 1039.5 = the Sum in the Equation given: ſo that 11 is 1 Root.

3*dly*, Dividing the Equation given (with the 1039.5 tranſpoſed) by $u - 11$, I depreſs it to the Quadratic $uu - 19.5u = -94.5$

4*thly*, This Quadratic being reſolved is $u = \sqrt{95.0625} - 94.5 + 9.75 = 10.5$ a 2d Root.

5*thly*, Dividing this Quadratic Equation by the 2d Root $= u$ or $u - 10.5$, the Quote or 3d Root is found 9. And for Proof, the 3 Roots 9, 10.5, and 11, (putting each $= u$, and tranſpoſing it) multiply'd one in another, will give the Cubic Equation propoſed to have its Root extracted, *viz.* $u^3 - 30.5u^2 + 309u = 1039.5$

From theſe 2 Examples it may be obſerved, That whether you happen to pitch upon the leaſt Root (upon your Tryals) as in the firſt Example, or upon the greateſt of the 3, as in the 2d Tryal of the 2d Example; the Method of Proceſs is the ſame: and ſo it would, if you happen firſt upon the middle Root. Theſe

These 2 Examples are of Regular Cubics having all their Terms.

4. I ſhall give one Example of an Irregular Solid Equation: as ſuppoſe $uuuuu + .10uu = 20.61293$.

To reſolve this, I can eaſily ſee that 2 is more than the Root; for 10 times the Square of 2 (alone, without the Surſolid) is 40. Therefore I take 1.5 for the Root, and (working with that according to the Equation) I find it too great; I therefore try 1.4, which is alſo a ſmall matter too much: ſo that working according to the Equation with 1.3, I find it juſt $= 20.61293$. So that I diſcover the Root to be $1.3 = u$. for $1.3^5 + 10 \times 1.3^2 = 20.61293$.

5. *Biquadratic Equations* are reſolved after the ſame manner as the Regular Cubics in the 2 firſt Examples: Firſt, by ſuppoſing a Root till you pitch upon the right, and depreſſing the Biquadratic thereby into a Cubic; and then finding another Root by Tentation whereby to divide the Cubic to depreſs it to a Quadratic, *&c.* as in the ſaid 2 firſt Examples above, for all Regular Biquadratics (or ſuch as have all their Terms.) Thus, *for example*, to reſolve or find the Roots of the Biquadratic $uuuu - 29uuu + 308uu - 1420u = 2400$.

(1.) By Tentation I find 8 to be 1 Root, and dividing the Equation given by $u - 8$, I depreſs it to the Cubical Equation $uuu - 21uu + 140u = 300$.

(2.) I make tryal again, and find 10 to be another Root; ſo dividing the Cubic Equation by $u - 10$, I depreſs it to the Quadratic $uu - 11u = -30$, which being reſolved, gives a 3d Root or $u = 6$.

(3.) Dividing the ſaid Quadratic Equation by $u - 6$, I find the Quote to be $= u - 5$ or $u = 5$, the 4th Root: ſo are 8, 10, 6 and 5 the 4 Roots required.

There are 3 other Ways of reſolving or finding the Roots of all Cubic and Biquadratic Equations, done more Mathematically: as,

II. By Subſtitution, Deduction, and Diviſion.

III. By way of Conſtruction, with the help of the Parabola and Circle.

IV. By Approximation or Converging Series.

II. *To reſolve Cubic Equations by Subſtitution.*

It has been ſhew'd how to reſolve Quadratics by Subſtitution, and much after the ſame manner is a Canon found whereby to reſolve Cubic, *&c.* Equations.

Example 1.] Admit the Value of u is required in this following Equation, *viz.* $uuu + duu - gu = k$, or $uuu + 120uu + 300u = 3714544$: Here 'tis plain that d is put for the given 120; g for 300; and k for 3714544.

The

The first Step toward the Answer is to substitute $b+c$ in the place of u in every Term of the Equation, in the Power that (u) is put.

Then will this be the Canon for Resolution.

$bbb+3bbc+3bcc+ccc$ (instead of uuu)
$bbd+2bcd+ccd$ .... (instead of duu)
$-bg-cg$ (in the place of $-gu$)

The whole Process and Rules are in the Margin, whereby u is found = 124.

1. Having your Canon before you, and also that $d=120$, $g=300$, proceed as the Canon directs you, first pointing over every third after the first Digit of the given Number; the 3 Points shewing the Root sought, will consist of 3 places, and therefore the 1 (=the first b) is 100.

2. You must compose a Number to deduct from that given, by taking those in the Canon which have not c found in them; as $b^3+bbd-bg=2170000$, which deducted reduceth the Number given to 1544544, which is your first Dividend, as by Step 7.

3. Take for a Divisor (in order to discover the Value of c) the Sum of the Numbers answering to the Sumbals mix'd with or × by c, such are those in the 8th, 9th, 10th, 11th, and 12th Steps; which Sum is

Step.	*An Example of the Resolution of a Cubic.*
	. . .
1	3714544 (100=b
2	1000000=bbb } Add.
3	1200000=bbd }
4	2200000=$bbb+bbd$=1+2d Step.
5	30000=$-bg$ deduct from the 4th.
6	2170000 rests, which deduct from the 1st Step.
7	1544544 ÷ =the 1st—the 6 Step ÷ by 12
8	30000=$3bb$ } Add.
9	..300=$3b$
10	24000=$2bd$
11	..120=d
12	—300=$-g$
13	÷ 54120=the Sum, or Divisor.
	(20=c
14	600000=$3bbc$ } Add.
15	120000=$3bcc$
16	..8000=ccc
17	480000=$2bcd$
18	48000=ccd
19	—6000=$-cg$
20	1250000=Sum deduct from the 7th Step.
21	294544 ÷, rests of b, or 1st Step.
	Now 120=b
22	43200=$3bb$ } Add.
23	..360=$3b$
24	28800=$2bd$
25	..120=d
26	—300=$-g$
27	÷ 72180=Sum, by which to ÷ the 21 Step.
	(4=c
28	172800=$3bbc$ } Add.
29	..5760=$3bcc$
30	....64=ccc
31	115200=$2bcd$
32	..1920=ccd
33	—1200=$-cg$
34	294544=Sum, deduct from the 21 Step.

54120 your Diviſor, which may be had in the 6 Figures next the left hand of the Dividend 2 times, (which 2 muſt be 20, becauſe it is the middle of the 3 that will be in the Root, as is obſerved above) this 20 is $= c$.

4. You muſt not make any farther Uſe of the Quantitys in the 1ſt, 2d, and 4th Steps, but having found the Value of c, you proceed to find your 2d Subtrahend in the 20th Step with the ſeveral Quantitys in the Canon; which you ſee in the 14th, 15, 16, 17, 18, and 19th Steps.

5. And having found your 2d Diviſor by the very ſame Quantitys that you had the 1ſt, (and obſerving that here $b = 120$, and not to 100) you may eaſily finiſh the Operation by the Example, which is only a Repetition of the Method above ſhew'd, and you find the 3 ſeveral Roots put together is $124 = u$ required: which I ſhall prove both from the Equation given, and by the Canon, to be the true Anſwer.

1. *By the Equation* $u^3 = 124^3 = 1906624$
$+120uu = 1845120$
$-300u = -37200$

Sum $= 3714544 = k$ in the Equation given.

2. *The Proof by the Canon* $bbb + 3bbc + 3bcc + ccc + bbd + 2bcd + ccd - bg - cg$.

Note, $b = 120$; $c = 4$
$d = 120$; $g = 300$

bbb	$= 1728000$		
$+3bbc$	$= 172800$		
$+3bcc$	$= 5760$		
$+ccc$	$= 64$	Sum	$= 3751744$
$+bbd$	$= 1728000$	$-bg$	$= 36000$
$+2bcd$	$= 115200$	$-cg$	$= 1200$
$+ccd$	$= 1920$		

-37200

3714544 reſts $= k$

Biquadratics are reſolved after the ſame Method, raiſing the Power of $b + c$ (which is ſubſtituted) accordingly in making the Canon; and proceeding by that, as has been in the laſt Example obſerved.

Example 2.] Of an irregular Cubic Equation, as ſuppoſe $uuu + 32334u = 823975 = k$, or $u^3 + fu = k$.

1. Make your Canon as before, ſubſtituting $b + c$ in the 2 Members of the Equation (as u is in power) in the place of u.

2. Where the Equation has not all its Terms, and the Co-efficient is large, you muſt put ſuch a Number firſt in the Root,

Root, as being multiply'd by the Coefficient, and added to its own Cube, the Sum will be as near as poſſible to, and leſs than that given on the right ſide of the Equation; as in the Example finding 50 $=b$ much too great (as in the 4th Step) I try $30=b$, which I ſtill find too much, as in the 7th Step. Then putting $20=b$, I find its Cube as in the 8th Step, and the Coefficient 32334 by 20, the Sum is deducible as in the 10th Step from the 1ſt, and then you proceed with the Remainer as in the laſt Example, and the Value of $u=25$.

Step.	
	The Canon is $bbb+3bbc+3bcc+ccc+bf+cf=k$.
1	823975 (80 $=b$ (30 $=b$ (20 $=b$
2	$125000=bbb$
3	$1616700=bf$
4	$1741700=bbb+bf$ > 1ſt Step, and cannot be deducted.
5	$27000=30^3=b^3$
6	$970020=bf$
7	$997020=b^3+bf$ > 1ſt Step.
8	$8000=b^3=20^3$
9	$646680=bf$
10	654680 deduct from the 1ſt Step.
11	169295 ÷ reſts of the 1ſt Step.
1	$1200=3bb$ }
13	$60=3b$ } Add
14	$32334=f$ }
15	÷ 33594 = the 3 laſt Steps a Diviſor. (5 $=c$
16	$6000=3bbc$ }
17	$1500=3bcc$ } Add.
18	$125=ccc$ }
19	$161670=cf$ }
20	169295 Sum, deduct from 11th Step.
	0 reſts.

3. It ſometimes happens where the Coefficient is very large, and the Root of the higheſt Power (or Value of u) is ſmall, that you muſt make that Figure 1ſt put in the Root one place (or more) leſs than the Points over the Number given (as in the 1ſt Step above) denote. See the following Example.

Example 3.] Admit $u^3+82334\,u=2073975$, what is the Value of u? 82334 being $=f$, and $b+c$ ſubſtituted in the room of u.

Here

Steps; w...

Here 'tis plain the Number given admits of 3 Cubical Points, but the Root u (or $b+c$) is but 25.

1. For if I ſhould put 1 as the Root of 2 next the left hand; that 1 (there being 3 Points) muſt be 100. Now if I multiply the Co-efficient by 100, *i.e.* if I ſuppoſe two Cyphers put towards the right hand of it, it will be 8233400, wch (without the Cube of the 100, which muſt as *per* the Examples above be added) is much too great to be taken from the 2073975 in the firſt Step.

Step.	*The Canon as before* $= b^3 + 3b^2c + 3bcc + c^3 + bf + cf$
1	2073975 (20 $= b$
2	8000 $= b^3$
3	1646680 $= bf$
4	1654680 Sum, from the 1ſt Step.
5	419295 reſts of Step 1ſt.
6	1200 $= 3bb$
7	60 $= 3b$
8	82334 $= f$
9	83594 $=$ Sum or Diviſor. (5 $= c$
10	6000 $= 3bbc$
11	1500 $= 3bcc$
12	125 $= ccc$
13	411670 $= cf$
14	419295 $=$ Sum, from the 5th Step.
	0 reſts.

2. Seeing a leſſer Digit than 1 cannot be put firſt, or in hundreds place in the Root; I muſt therefore conclude, that the Root will have but two places.

3. I therefore ſuppoſe 10 in the Root, by putting 1 Cypher towards the right hand of the Co-efficient (to repreſent bf) which would be 823340, which (by doubling the 8 next the left hand I find 16) may eaſily be taken from 20, (next the ſame left hand of the Number given) I therefore put the double of 10 (or 20) in the Root, and ſo proceed; all the difficulty in this caſe being to put the firſt true Figure in the Root: to which end I have not ſeen any Rules that contribute to do ſo eaſily and certainly as theſe above.

Example 4.] *Of Reſolving Cubics by Subſtitution*, &c.

Admit $uuu + 350u^2 = 1225125$. Or that $uuu + duu = k$; and that $b + c$ is put $= u$.

This Example is of an irregular Cubic compoſed of 2 Members different from the former; where tho there be 3 Cubical Points (or Ternaries) yet the Root has but 2 places, *viz.* 55.

For if we ſuppoſe $b =$ but 1 in the 3d place (or 100) you will find $bbb + bbd$ to be much more than can be taken from the Number pointed; and $10 = b$ is much too little: therefore I try $50 = b$, and find that to be the Root, that produces (by its Cube added to bbd) the next Number to and leſs than 1225125 given, and therefore I proceed as by the Canon, as directed in Example the 1ſt, laſt foregoing.

Step.	
	The Canon is $bbb + 3bbc + 3bcc + ccc = u^3$ $bbd + 2bcd + ccd = dtu$.
1	1225125 (50 $= b$
2	125000 $= bbb$
3	875000 $= bbd$
4	1000000 = Sum.
5	225125 = 1ſt—4th Step, a Dividend.
6	7500 $= 3bb$
7	150 $= 3b$
8	35000 $= 2bd$
9	350 $= d$
10	43000 = Sum, a Diviſor. (5 $= c$
11	37500 $= 3bbc$
12	3750 $= 3bcc$
13	125 $= ccc$
14	175000 $= 2bcd$
15	8750 $= ccd$
16	225125 = 11 + 12 + 13 + 14 + 15 thSteps.
	reſts 0 5th—16th Step.

III. *To reſolve Cubic Equations by way of Conſtruction.*
Vid. *Plate* A. *Fig.* 27.

This being done by the Parabola and Circle, it is neceſſary to ſhew the making of the Parabola, &c.

A Parabola is one of the 5 Sections of a Cone, being made by cutting a Cone by a Plane parallel to the Side which is oppoſite to that wherein you cut; *and it is Geometrically formed thus:*

1. Draw the Line (t *v* F ⊙ d *y*) the Axis of the Section. Then croſs the ſame at right Angles with a Line at pleaſure, as (a *p*) and according as you would have the ſame (or if the a *p*, which is called the

the *Latus rectum*, or Parameter, be given) ſuppoſe 1 inch, ſet half an inch from *F* to *a*, and to *p*.

Then take half of one of thoſe Halfs (or $\frac{1}{4}$ of the Parameter, or of 1 inch) and ſet it from *F* to *v*, and from *v* to *t*, ſo is *F* = the Focus, and *v* the Vertex of the Parabola.

Then draw as many Lines as you pleaſe parallel to *a p* (as *o o l* *&c.*) with a Ruler on purpoſe for drawing parallel Lines; and the Diſtance *t* to *o* in the Axis extends from *F* the Focus to *l* in the ſame Parallel forming the Curve (*v p l l l*, *&c.*) which you may draw through the Points (*l*, *l*, *&c.*) And having found the Curve on one ſide of the Axis, as *v l*, ſet it off on the ſame Parallel, on the other ſide of the Axis, as *v o*, *v o*, *&c.* and draw the Curve (*o o v*) ſo is the Section finiſhed, and (*a F p*) is the *Latus rectum*, *o o l* an Ordinat Applicate (or whole Ordinat) *o o* the Semi-Ordinat, (which is commonly called the Ordinat) any (*v o*) in the right Line is called an Abſciſſa.

Now ſuppoſe a Cubic Equation where $\left.\begin{matrix} uuu + buu - pu - q = 0 \\ \text{Or } uuu + 3uu - 2u - 2 = 0 \end{matrix}\right\}$ ſo $\begin{cases} b = 3 \\ p = 2 \\ q = 2 \end{cases}$

1*ſt*, To find the Value of *u*. From the ſame Scale by which you ſet off the *Latus rectum* of your Parabola, take 3 (the Co-efficient multiply'd in the Square of the Root ſought) and applying it at right Angles to the Axis (*v y*) towards the right hand becauſe + 3, it will cut *b* in the Curve.

2*dly*, Draw a Line at pleaſure cutting a ſuppoſed Line (*v b*) in the middle at right Angles, as *q q*.

3*dly*, Where that Line interſects the Axis, as at ⊙ ſet half *p* downward, becauſe — *p*, *viz.* 1, from the ſame Scale extends from ⊙ to *n*.

4*thly*, On (*n*) erect a perpendicular Line (*n m*) which will cut the infinite Line *q q* in *r*.

5*thly*, From *r* ſet half *q* = 1 towards the right hand becauſe — *q*, and it extends (by the ſame Scale) from *r* to *s*, which is the Center of the Circle ſought.

6*thly*, Extend the Compaſſes from *s* to *b*, and deſcribe the Arch (λ *e b*) which cutting the Curve at (*e*) the perpendicular Line (*e e*) meaſured on the ſame Scale, gives *u* = 1, which is + 1 or affirmative, becauſe the Section (*e*) is in the right hand part of the Curve. See *Philoſoph. Tranſactions*, N° 188 & 190; Dr. *Haley*'s Method.

Sect. XIII. *Approximation or Converging Series.*

WHAT is meant hereby, is ſhewn under the words *Converging Series* in the foregoing Alphabetical Account at the beginning of this Chapter, and will be farther explained by the Rules and Examples following: as $u - ub + ub^2 - ub^3 + ub^4 - ub^5$, &c. is a Series; which I multiply by $u^2 - b^2$ as followeth.

$$\begin{array}{l} u - ub + ub^2 - ub^3 + ub^4 - ub^5 \\ u^2 - b^2 \\ \hline u - b + u b^2 - u b^3 + u b^4 - u b^5 + u b^6, \text{ \&c.} \\ \qquad - u b^2 + u b^3 - u b^4 + u b^5 - u b^6, \text{ \&c.} \\ \hline u - b = \text{the Product.} \end{array}$$

Here it is plain, that if the Series was continued never ſo far, the Negatives would deſtroy the Affirmatives: ſo that it had been the ſame in the Product, if you had run it only to $u - b$ there. But my Intent in this Section is not to ſhew the way of repreſenting or expreſſing by a Series, a Product, Root, &c. for tho that be a pretty Speculation, yet 'tis not ſo eaſy to diſcover the Roots, &c. therefrom. But I chuſe rather here to ſhew the great Uſe of Converging, in finding with much eaſe and expedition the Real Roots themſelves.

II. *To find the Square Root,* as ſuppoſe of 13.

Here I will put s for the Square or Surd Number given; r = the 1ſt two Figures in the Root; a = the 2d Approach to the true Root; and e = the 3d Approach; the ſeveral Approaches being the coming nearer and nearer to the Truth of the Root by ſo many Diviſions, by 1 of which ſeveral places in the Root are gained, as is ſhewed under the fifth Head following.

If in the Example above it be conſidered what Number ſquared will be next to and leſs than 13, or what 2 Digits will be next to

13.00 and put r for thoſe 2 places it will then be $s - rr \div 2r = a$ the 2d Approach.

$$\overline{s - \overline{r+a}^2} \div \overline{\overline{r+a} \times 2} = e \text{ the 3d Approach, } \mathcal{C}c.$$

Or in Numbers $13 - 3.6^2 \div 3.6 \times 2 = .00555 = a$ the 2d Approach, $3.6 = r$.

$13 - 3.60555^2 \div 3.60555 \times 2 = .000001275\,46 = e$, and $r + a + e$. Or $3.6 + .00555 + .000001275\,46$ which is $= 3.605551275464 - =$ the Square Root required to 13 places, (but ſee more under General Head the 5th following.) Or e will be $= \frac{s - rr - 2ra - aa}{2r + 2a}$ which facilitateth the Operation as to ſquaring 3.60555, $\mathcal{C}c$.

Or inſtead of 3.6 at the 1ſt Approach, you may, as in the common Way of Extraction, put 3, whoſe Square is next to and leſs than that of any other Digit deducible from 13.

Prop. 2. *To find the Square Root of a larger Number*, as of 371.00071 $= s$, $\mathcal{C}c$. as before.

$371.00071 = s$. $19 = r$ the 1ſt Approach.

$-361 = rr$

$10.00071 \div 2r\ (= 38) = .26 = a$ the 2d Approach, and $r + a = 19.26$

Again $371.00071 - 19.26^2 = .05311 \div 19.26 \times 2 = 0013787 = e$ the 3d Approach.

And $r + a + e = 19 + .26 + .0013787 + = 19.2613787 =$ the Root ſought.

So by the ſame Rules of Converging Series the Square Root of 4712345 is found 2170.7936+

III. *Approximation, or the Extraction of the Cube Root by the Method of Converging Series.*

I ſhall alſo proceed in this with as much Plainneſs as I can, obviating what only is to the purpoſe.

1. Suppoſe then I would know the Cube-Root of $13 = s$, for the firſt in the Root which is known here I put r. Then the Proceſs, as in common Extraction, (which is the beſt way in this Caſe, as being moſt accurate and certain for the 2d in the Root) will be

$$s - r^3 \div 3rr = R = .3 \text{ the 2d in the Root.}$$

Or ſuppoſing 2.3 put in the Root together (for I can eaſily ſee that 2.3 is the Root of the next Cube Number to and leſs than 13.000) it will be as before to find the 2d Approach, $2.3 = r =$ the 1ſt Approach.

Then

Then $s - r^3 \div 3r = .05 =$ the 2d Approach, and now $2.35 = R$.
And $s - R^3 \div 3RR = .001333+$; = the 3d Approach. So is
$2.351333+$ = the Root.

Or in Numbers more plainly thus;

$13 - 2.3^3$ ($= 12.167$) $= .833$, $\div 2.3^2 \times 3$ ($= 15.87$) $= .05$ the 2d Approach.

And $13 - 2.35^3$ ($= 12.977875$) $= .022125 \div 2.35^2 \times 3$ ($= 16.5675$)
$= .001333+$ = the 3d Approach.

And the 1st Approach + the 2d + the 3d $= 2.351333+$, which is the Cube Root of 13, done with near 200 Figures fewer than in the common Way of Extraction.—— But see *Head* VIII. following.

2. And if we put $r =$ the known part of the Root, and a for each new Approach, you have

$rr + 2ra + aa =$ the Square of $r + a$
$r^3 + 3r^2a + 3ra^2 + a^3 =$ the Cube of $r + a$

So likewise $r^4 + 4r^3a + 6r^2a^2 + 4a^3r + a^4 =$ the Biquadrate of $r + a$
And $r^5 + 5r^4a + 10r^3a^2 + 10r^2a^3 + 5ra^4 + a^5 =$ the Surfolid of $r + a$

And as in the Square and Cube Roots it appears to be (putting s for the Number given to have its Root extracted)

$s - rr \div 2r = a$ the several Approaches in the Square Root.
$s - r^3 \div 3rr = a =$ the Approaches in the Cube Root.

So for the Biquadrate and Surfolid Roots it will be

$s - r^4 \div 4r^3 = a =$ the Approaches to the Biquadrate Root.
And $s - r^5 \div 5r^4 = a =$ the Approaches to the Surfolid Root.

Hence it appears, That the known part of the 2d Quantity next the left hand in the above-mentioned Powers of $r + a$, is your Divisor. I come next,

IV. *To illustrate by Numeral Examples the Extraction of the Biquadrate and Surfolid Roots by way of Converging Series.*

1. *To find the Biquadrate Root* of 314. The Process is (the 1st r being $= 4$)

1*st*, $314 - r^4$ ($= 256$) $= 58 \div 4r^3$ ($= 256$) $= .2 = a$. So is 4.2 a new r.

2*dly*, $314 - r^4$ ($= 311.1696$) $= 2.8304 \div 4r^3$ ($= 296$) $= .009$; so is $4.209 =$ a new r.

And by repeating the same Method with that last r, you have $4.2099+$ for the Biquadrate Root of 314.

2. *To*

2. *To find the Surfolid Root of* 314 (the firſt $r=3$) the Proceſs will be

1ſt, $314-3^5\ (=243)=71\div 5r^4\ (=405)=.1=a$. So is 3.1 a new r.

2dly, $314-3.1^5\ (=286.29)=27.7185\div 461.75\ (=5r^4)=.05=a$, and $3.15=$ a new r.

3dly, $314-3.15^5\ (=310.1364)=3.8636\div 5r^4\ (=492.28)=.007=a$. So is $3.157=$ a new r.

And proceeding in the ſame Method with that new r, the Surfolid Root of 314 is found $3.1578+$

But there is yet

V. *Another Way of Converging (for Example) to the Square Root.*

For as in the 1ſt Example in the Square Root in this Sect. the Canon was (putting $s=$ the Square Number or Surd given, and r for each Approach to the true Root) $\frac{s-rr}{2r}=r$ the 2d, *&c.* (or here to a) ſo in this Method I am going to ſhew, the Canon is $\frac{s+rr}{2r}=r$ the 2d, *&c.*

Now ſuppoſe $s=13$, it follows that the firſt r (or Figure in the Root) is 3, ſo that $13+3^2\div 6=3.66=r$ the 2d.

$13+3.66^2\div 7.32=3.605955=r$ 3d.

And $13+3605955^2\div 7.21191=3.60555129=r$ 4th.

Note, *The Diviſors are always the double of the r next before.*

From the laſt Example you may note,

1. That though the preceding Number be falſe, yet by every Repetition of Proceſs according to the Canon it convergeth or comes nearer to the true Root or Value of r.

2. That the Places of the Digits in the ſeveral Approaches are true, according to the Progreſſion whoſe Ratio is 2, and the 1ſt Term $=1$. So

	Places.	
In the 1ſt Approach 1 *viz.*	3	} Are true, the reſt not ſo.
2	3.6	
3	3.605	
4	3.60555 12+	
$=r=$ the Square Root of 13 *propè.*		

And if the Operation were repeated, as $3.60555129^2+13\div 3.60555129\times 2$, the Quote will give 16 places true in the Root; for

for every Approach affords double to that preceding. But the Reader may chuſe which of the 2 ways he likes as the beſt, this, or the 1ſt way, which I take to be ſomewhat eaſier and ſhorter in the Operations of Diviſion.

Example 2. *Of this Way of Converging; to extract the Square Root of* 29507710.4132345.

29507710.4132345 (5 = *r* the 1ſt.
25 ....

2 *r* = 10) 545 ..... (54 = *r* the 2d.
45

2950
+2916 = 54^2

2 *r* = 108) 5866 ..... (5432 = *r* the 3d.
466
347
237

29507710 = of the Data.
+29506624 = 5432^2

2*r* = 10864) = 59014334 ..... (5432.1000 = *r* the 4th.
46943
34873
22814
10864

29507710.41323450
+29507710.41000000

2*r* = 108642) 59015420.82323450 (5432.100000297721 = *r* the 5th Anſwer.
469442
348740
228148
108642
0323450
1061660
838820
783260
227660
103760

VI. *But*

VI. *But to perform the laſt Operation by the Method firſt taught making* $a =$ *each Approach, and* $t =$ *the Total of the Approaches; it will ſtand thus:*

$\frac{s-tt}{2t} = a$ 29507710.4132345 = s (5 = t and $t+a=t$ the 2d = 54
10) 45 ..... 04 = a

2950
2916 = tt

and t the 2d + a the 2d = t the 3d = 5432

$2t = 108$) 347 ..... (0032. = a the 2d.
237 t 3d + a the 3d = t 4th
and t the 4th = 5432.1000

29507710
29506624 = tt

$2t = 10864$) 10864 ..... (0000.1000 = a 3d
0132

295077 10.41323450
295077 10.41000000 = tt

$2t = 10864.2000$) ..... = .00323450000 (0000.0000029772 1 = a
1061660 (the 4th.
838820 and a the 4th + t the 4th.
783260
227660 = 5432.1000002977 21 —
103760 = the Square Root ſought.

This Method is ſhorter than the laſt (as I ſaid before) becauſe the Figures in the Approaches are not repeated by dividing: or the Sum of the 1ſt (t) and the (a's) is = the ſaid Root.

And as in the laſt Example the Operations in Diviſion are ſhorter than in that next before; ſo the Canon for the Approaches (next after the 2 firſt, when the Squaring of the Approaches grows operoſe) does ſhorten the Work of Multiplication; or that Prop. of *Euclid* (I think 1, 2.) which ſays, That of two Lines (or Numbers) if one be divided into any Number of Parts, the Sum of the Rectangles of one by the ſeveral Parts of the other, is = the Rectangle of one Whole by the other.

But it being my Design here, only to shew how the large Multiplications for producing the 4th, &c. Approaches may be contracted; I shall explain the said Canon, which is grounded on another Prop. of *Euclid*, *i. e.* Any Line (or Number) divided into two Parts; the Sum of the Squares of the Parts, more the double Rectangle of those Parts is = the Square of the Whole.

So in the last Example putting s the 3d $= b$.
a the 3d $= c$
the Square Number given $= s$
s the 4th $= b + c$

There arises agreeable to the Canon under the 2d Head of Converging toward the Square Root, this Equation.

$$\frac{s - bb - 2bc - cc}{2b + 2c} = a \text{ the 4th.}$$

Now whereas it would be a little tedious to square the whole of s the 4th $= 5432.1000$ (especially if the Cyphers were significant Figures) I therefore always divide it into 2 parts just at the meeting of the Approaches, as s the 3d, and a the 3d, *viz.* into 5432, and .1000, then as the Canon directs:

$cc = .1$
$2bc = \begin{cases} 5432 \\ 5432 \end{cases}$
and $bb = 29506624 = tt$ in the last Approach before,
(ready done.)
295077104.1 = the Sum $= 5432.1^2 = tt$ in the last Example.

In this Operation (this way performed) you save the trouble of working and inserting 12 Figures in 29; and the Divisor $2b + 2c$ is = the 5432.1000×2 (as in the last Example) so that

$$\frac{s - tt}{2t} = \frac{s - bb - 2bc - cc}{2b + 2c} = a \text{ the 4th as before.}$$

But the latter Canon being used, after the 3d, &c. Approaches, it does much facilitate the Operations. And thus I think I have made this matter shorter, more certain in the Approaches, and plainer than has been done before.

VII. *Ano-*

VII. *Another Example of Approximation to the Cube Root* ſhall be to extract the Cube Root of 33148648517294436663656267 = *c*

$\frac{c - s^3}{3s^2} = a$ = the Theorem or Canon.

Root ſought = *r*
Each Approach = *a*
And the Sum of the Approaches = *s*

1 *Note*, You always bring down ſo much of *c*, as will admit of s^3 to be deducted.

2. If any thing had remained upon deducting the laſt s^3, you muſt have put Cyphers, and divided again by $3s^2$. See the Operation, and the Probation thereof.

33148648517294436663656267 (3 = *a* the 1ſt.

$3a^2 = 27$) 61 (..... 02 = *a* the 2d.
7
+ *a* the 1ſt = 32 = *s*.

33148 = part of *c*.
32768 = s^3

$3s^2 = 3072$) 3806 (..... 00123 = *a* the 3d.
7344
12008
2792
+ 32 = 32123 = *s* the 2d.

33148648517294 = part of *c*.
33147310244867 = s^3 (the 2d.)

$3s^2 = 3095661387$) 1338272427 4..... 000004323 = *a* the 4th.
10000787263
7136031026
9447082526
+ *s* the 2d = 32123
= 32123 4323
= *s* the 3d = *r*.

33148648517294436663656267 = *c*
33148648517294436663656267 = s^3 (the 3d.)

0 reſts, *ergo* 32123 4323 = Root required.

This might be proved by the common Way of Extracting, tho with very many more Figures: I shall chuse therefore rather to prove it by Involution of the Root, multiplying by 2 Digits at once, as taught *Compend. 6. Chap. 3.*

321234323=s=the Root.
× 321234323

7388389429
13813075889
7388389429
6745920783
963702969

103191490273268329=s^2
321234323=s

2373404276285171567
4437234081750538147
2373404276285171567
2167021295738634909
3095744708198049 87

331486485172944366636 56267=c=the Proof.

3 *Note*, That tho you put a Figure or 2 too many or too few, or too little or too great, next the right hand of your Quotients, it will be rectify'd in the following Operations, if you proceed as *per* the Theorem. But the safest way is not to proceed farther at one Division than in the Example.

VIII. To illustrate what is said in [*Note* 3.] I shall shew how the first Example of Approximation to the Cube-Root foregoing in this 13th Section, is done by different Approaches and Figures to what it is there: And how to manage when Negative Numbers fall in, as when the Cube of *a* or *s* exceed the Cube given from which they are to be subtracted.

Thus

Thus to extract the Cube Root of 13 $=c$; Approaches $= a$
Sum of the Approaches $= s$
The Theorem $\frac{c-a^3}{3a^2} = a$ the 2d, &c. Root $= r$

13.0 ..... (2.4 $=a$ the 1st
$3a^2=12$) 50

13.000 $=c$
13.824 $=a^3$

$3a^2=17.28$) —.824000 refts (—.0476 $=a$ the 2d + a the 1st.
13280 $=2.3524=s$.
11840

13.
13.0176762 1824 $=s^3$

$3s^2=16.60135718$) —.0176762 1824 0000 (—.0010647 $=a$ the 3d.
10748610600 + 2.3524 $=s=$ 2.3513353
7877962920 $=s$ the 2d.
12374200480

13.
13.0000101554648092 37977 $=s^3$ (the 2d.)

$3s^2=16586333079078 27$)—.0000101554648092379770000 (—.0000006122 78 $=a$ (the 4th.
2036649617910150 + 2.3513353
3780163100023230 $=$ 2.3513346877 22 (= s.
4628964842075760
13116982262601060 $=r$ the Root of the
15045491072462710 (c given *propè*.

1. It may be obſerved from the laſt Operation, That there are different Digits in every Approach, from thoſe in the Approaches under the firſt Example of the Cube Root, in this *Sect.* foregoing. But 2*dly*, That tho the Digits toward the right hand in theſe Approaches be wrong, yet thoſe a's ariſing from the ſubſequent Operations proceed in each nearer and nearer to the Truth.

2. Altho

2. Altho the Cube of s does all along exceed c the Cube Number given, and cannot therefore in the common way be ſubſtracted; yet by the Rules foregoing in Logarithms and Algebra, and under the word *Negative* (Arithmetic) in the Alphabetical Explanation at the beginning of Algebra, it may; (for example) 13.824 may be taken from 13: but then there will always remain ſo much leſs than nothing to be mark'd with the Sign (—) as the Subtrahend (or Cube of s) exceeds (c) *i. e.* in this Example of deducting a^3 (the firſt) from c (or 13.824 from 13) there muſt remain —.824 to be divided by the laſt Approach ſquared and multiply'd by 3. And therefore that Quote muſt be negative, *i. e.* —.0476, which added to $2.4 = a$ the firſt (as *per* the Doctrine of Addition of Algebra) the Sum is $= 2.3524$. And ſo forward of the Management of the other Negative Numbers or Quantities; as is taught in the Arithmetic of Negatives under the word *Negative*, at the beginning of Algebra, *Chap* 10. and alſo *Sect.* 2, 3, 4, 5. of that Chapter; and alſo *Sect.* 3. of *Chap.* 7.

Theſe are the moſt Brief and Accurate Ways that I know of Converging Series, tho I am not ignorant of others which ſome Authors have been fond of exhibiting; but they are neither ſo ſhort nor accurate as the common Way of Extraction. I ſhall give the Inſtance which they have exemplify'd in finding the Square Root of 133225.

Their Way by Converging Series.

$$133225 = c$$

$$r = 3)\ 66612.5 = \frac{c}{2}$$

$$-45 = \frac{rr}{2}$$

$$\text{Divisor } r = 3)\ 216\ (6 = s$$

$$18 = rs$$

$$+18 = \frac{ss}{2}$$

$$= 198 = \text{Sum.}$$

$$r = 36)\ 1812.5\ (5 = s$$

$$180 = rs$$

$$12.5 = \frac{ss}{2}$$

$$1812.5 \text{ Sum.}$$

$$0$$

$$\text{Root} = 365$$

The Common Way of Extraction.

133225 (365 = Root.

6) 432

72) 3625

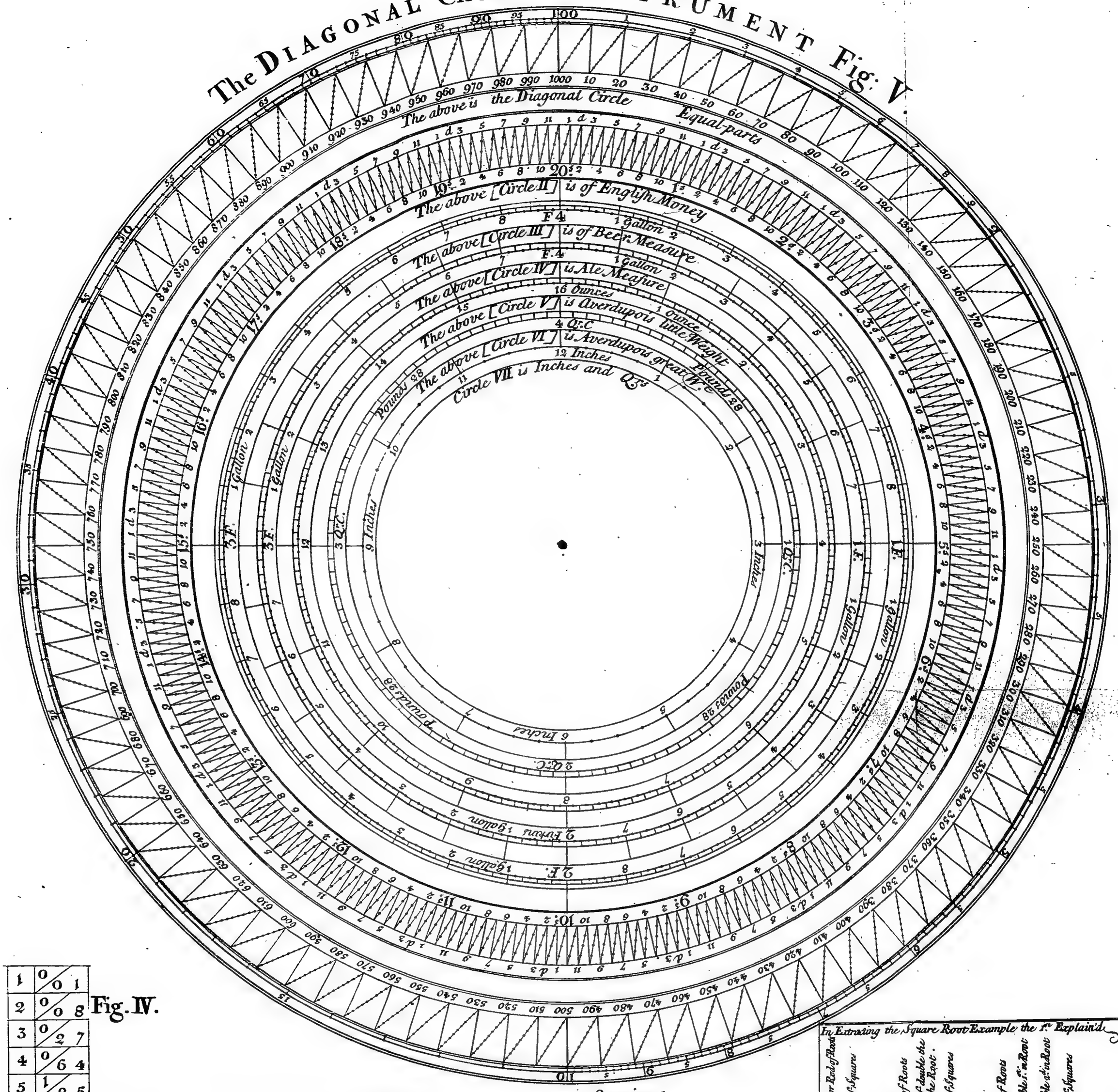
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Done by putting down 20 Figures: Whereas their Way by Converging takes 48, beſides the Signs and Symbals, and is therefore not worthy of Conſideration.

Their

Plate B.

The DIAGONAL Circular INSTRUMENT Fig: V.

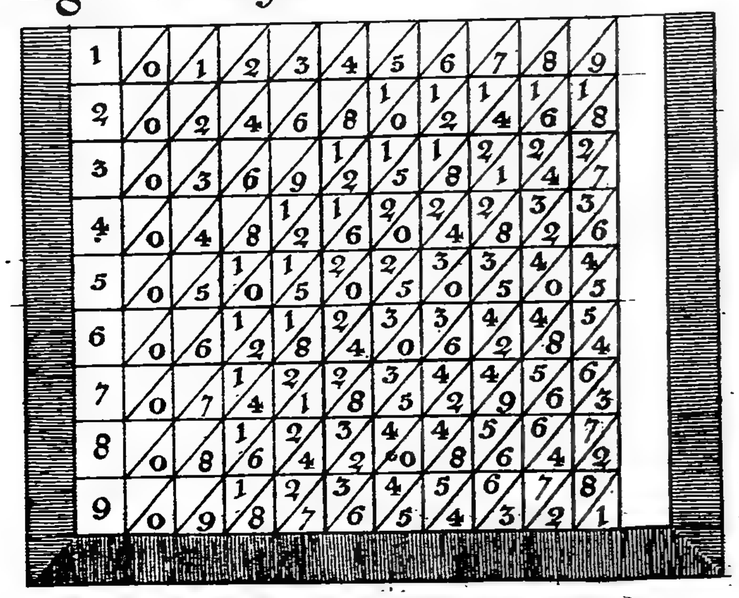


1	0	0 1
2	0	0 8
3	0	2 7
4	0	6 4
5	1	2 5
6	2	1 6
7	3	4 3
8	5	1 2
9	7	2 9

A Rod of Roots
A Rod of their Cubes

Fig. IV.

Fig: I. *A Set of the Bones or Rods.*



Question 1 Tabulated.

In Multiplication

1		3	4	7	2	9
2		6	8	1 4	4	1 8
3		9	1 2	2 1	6	2 7
4	1 2		1 6	2 8	8	3 6
5	1 5		2 0	3 5	1 0	4 5
6	1 8		2 4	4 2	1 2	5 4
7	2 1		2 8	4 9	1 4	6 3
8	2 4		3 2	5 6	1 6	7 2
9	2 7		3 6	6 3	1 8	8 1

Fig: II.

In Extracting the Square Root Example the 1st Explain'd

Index or Rod of Roots	Rod of Squares	Rod of Roots	Rod of double the 1st in the Root.	Rod of Squares	Rod of Roots	Of double 1st in Root	Of double 2d in Root	Rod of Squares
1	1	1	4	1	1	4	6	1
2	4	2	8	4	2	8	1 2	4
3	9	3	1 2	9	3	1 2	1 8	9
4	1 6	4	1 6	1 6	4	1 6	2 4	1 6
5	2 5	5	2 0	2 5	5	2 0	3 0	2 5
6	3 6	6	2 4	3 6	6	2 4	3 6	3 6
7	4 9	7	2 8	4 9	7	2 8	4 2	4 9
8	6 4	8	3 2	6 4	8	3 2	4 8	6 4
9	8 1	9	3 6	8 1	9	3 6	5 4	8 1

200 — 30 — 4 = 234 the Root

Fig: III

Their other Inſtance of Converging, in the Example of 2 takes 25 Digits more in the Operation than the common Way of Extraction, and is not true in the 3d Decimal Place: Wonderful new Invention!

An APPENDIX

Fig. 1.



Fig. 2.

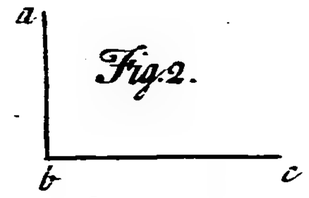


Fig. 3.



Fig. 5.

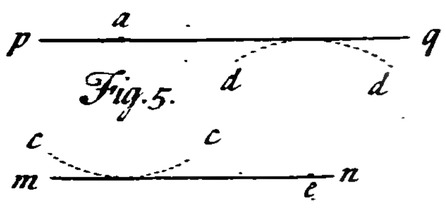


Fig. 7. This upper is the Line of Numbers or Logarithms

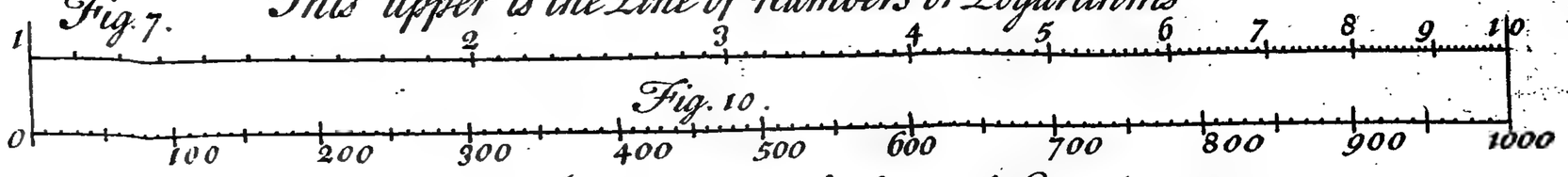


Fig. 10.

This Lower is ye Line of Equal parts

Fig. 12.

6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4	

Fig. 13.

	on ye Index Rod	there Stands on ye other 5 rods these figurs 3 4 7 2 9	Which Added by ye Rule above are these Numbers
against	5	1/5 2/0 3/5 1/0 4/5	173645
against	9	2/7 3/6 6/3 1/8 8/1	312561
against	7	2/1 2/8 4/9 1/4 6/3	243103
against	3	/9 1/2 2/1 /6 2/7	104187
against	2	/6 /8 1/4 /4 1/8	69458
Summ,or	product required = 826376555		

Fig. 14.

1/2	0	1/6	0	3/6	3/6	

Fig. 15.

3/6

Fig. 16.

2/7	2/7

the Index Rod — the Rod of Cubes

Fig. 17.

2/7

Fig. 19.

1	1	8	1

Fig. 20.

1	1	9	6	1

Fig. 21.

1	1	9	7	4	1

Fig. 22.

1	0/0 1
2	0/0 8
3	0/2 7
4	0/6 4
5	1/2 5
6	2/1 6
7	3/4 3
8	5/1 2
9	7/2 9

Place this plate A, — at the end of the Book, next to ye last leaf thereof.

Fig. 18.

Dividend 2373469108 4 Quotient 93·7241.

Divisor 2 5 3 2 4

against these Digits in the Tabulated Divisor are 9	1/8	4/5	2/7	1/8	3/6
3	/6	1/5	9/6	1/2	
7	1/4	3/5	2/1	1/4	2/8
2	/4	1/0	6/4	/8	
4	/8	2/0	1/2	1/8	1/6
1	/2	/5	/3	/2	/4

Products of the Divison by each Fig in ye Quote as in the Rods.

Or 227916 deduct 94309 ---------- 3
Or 75972 deduct 183371 ---------- 7
Or 177268 deduct 61030 ---------- 2
Or 50648 deduct 103828 ---------- 4
Or 101296 deduct 25324 ---------- 1
Or 25324 deduct (0) Rests

Fig. 23

1	7	5	0	1
2	1/4	1/0	0/0	8
3	2/1	1/5	0/2	7
4	2/8	2/0	0/6	4
5	3/5	2/5	1/2	5
6	4/2	[illegible]		
7	4/9	[illegible]		
8	5/6	4/0	5/1	2
9	6/3	4/5	7/2	9

Fig. 4.

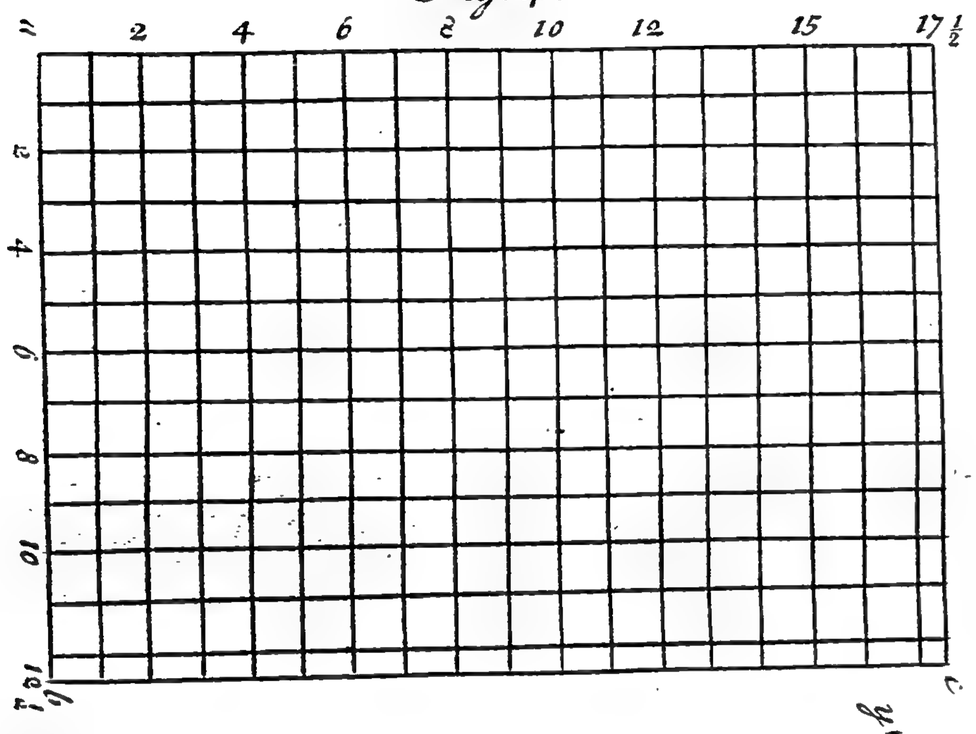


Fig. 24.

	ye Divisor Tabulated				ye Rod of Cubes	
1	9	4	0	8	0/0	1
2	1/8	8	0	1/6	0/0	8
3	2/7	1/2	0	2/4	0/2	7
4	3/6	1/6	0	3/2	0/6	4
5	4/5	2/0	0	4/0	1/2	5
6	5/4	2/4	0	4/8	2/1	6
7	6/3	2/8	0	5/6	3/4	3
8	7/2	3/2	0	6/4	5/1	2
9	8/1	3/6	0	8/1	7/2	9

Fig. 25.

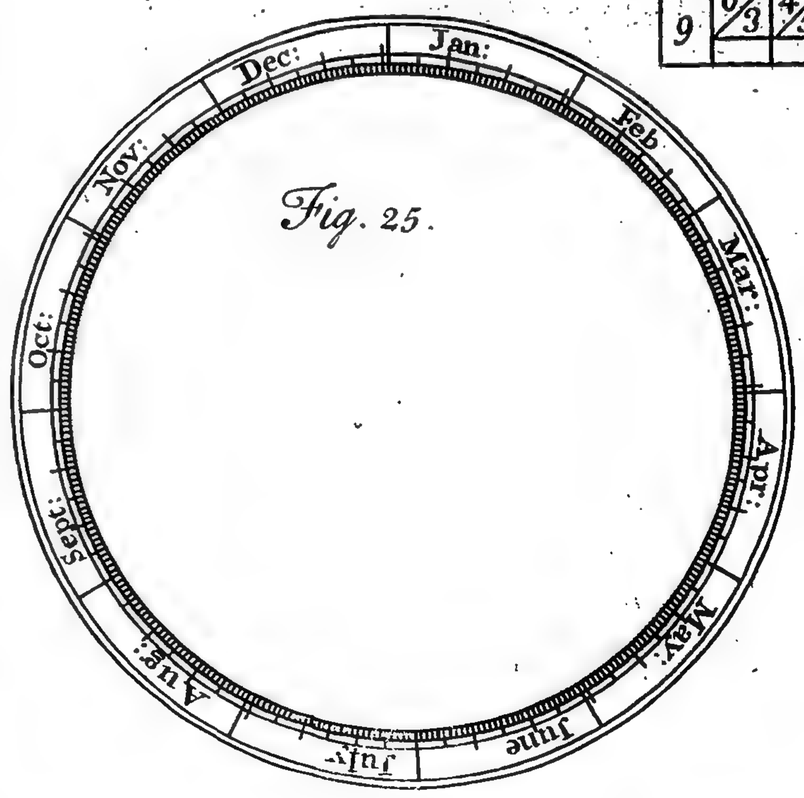


Fig. 9.

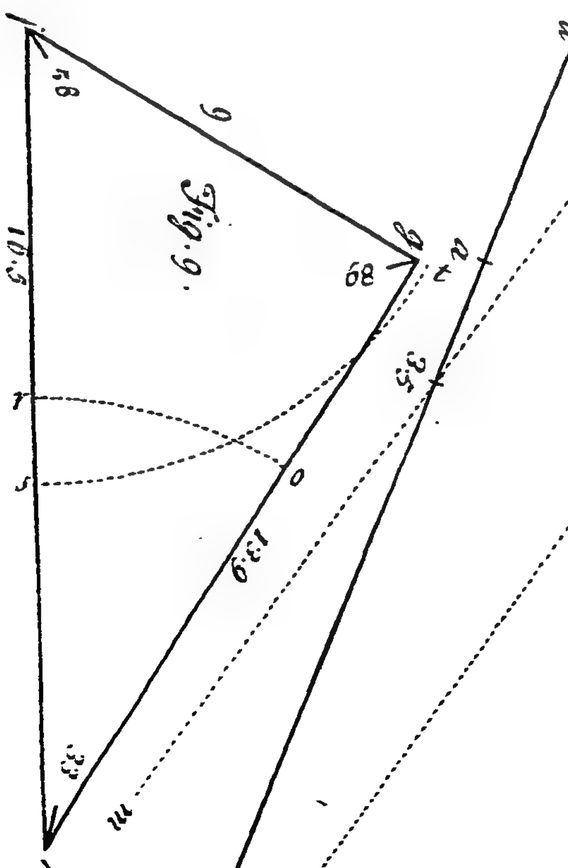
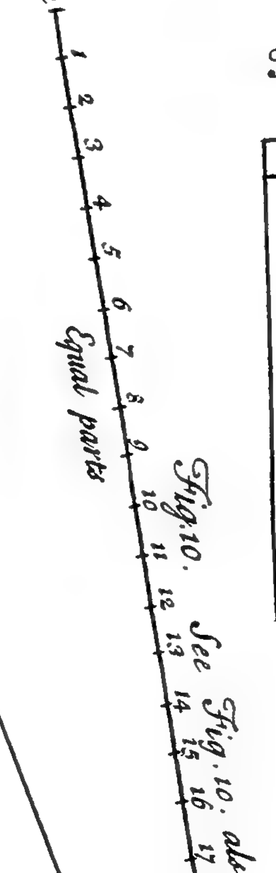


Fig. 6.



Equal parts

See Fig. 10. above

Fig. 8.

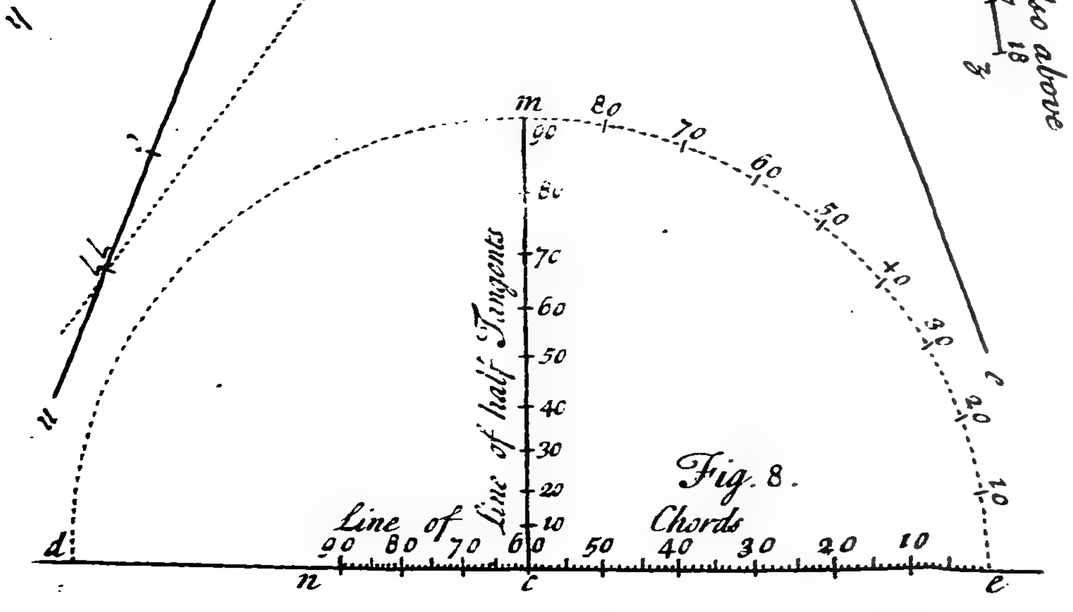


Fig. 27.

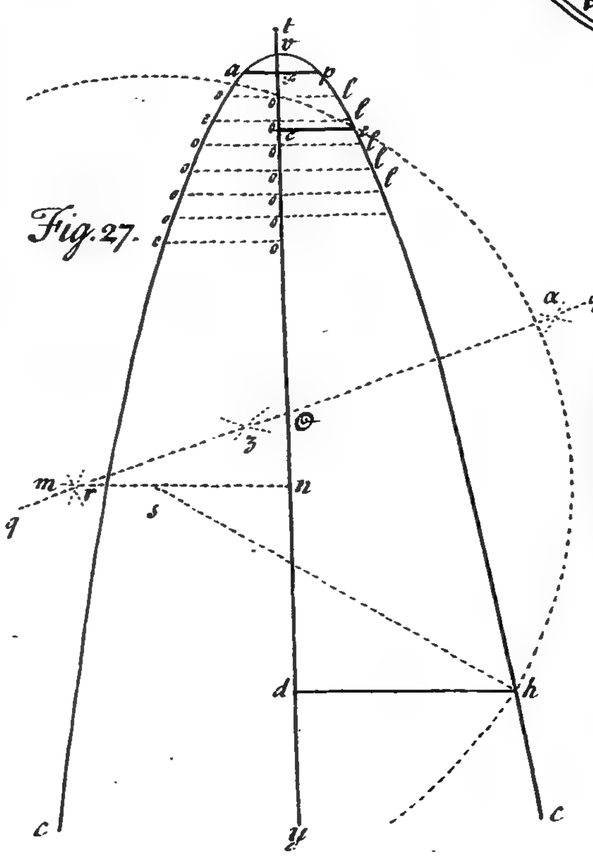


Fig. 26.

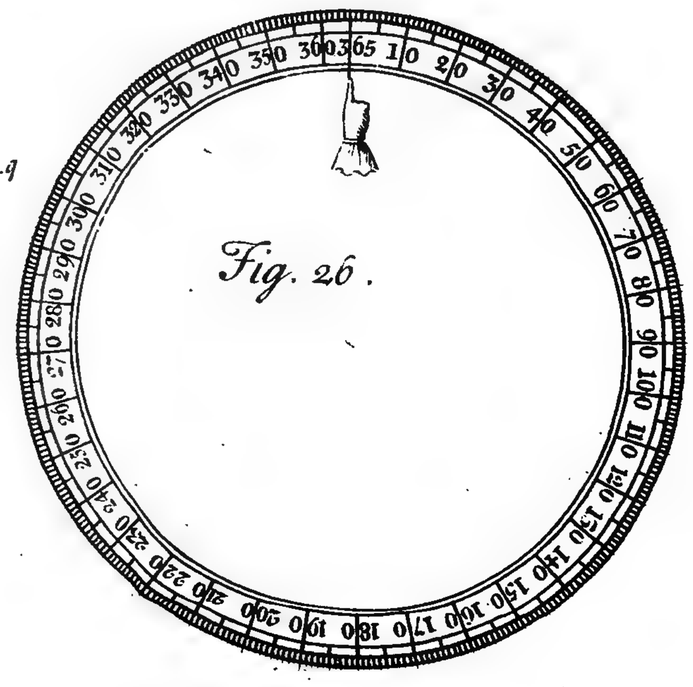


Plate C

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig 8.

Fig. 9.

Fig. 10

Fig 11

Fig. 12

Fig. 13

Fig. 14.

Fig 15

Fig. 16.

Fig. 17.

Fig. 18.

Fig. 19.

Fig. 21.

Fig. 20.

Fig. 22

Fig. 23.

Fig. 24

Plate C. place at ye end of the Book next B. vizt A.B.C.

The 5 platonic bodys.

An APPENDIX *to an Intire System of Arithmetic:*

By EDWARD HATTON *Gent.*

Of the Menſuration of Superficies and Solids.

MY Intent is to be as Brief, and yet as Copious as poſſible; not doubting but the Variety of Figures here meaſured, more than any other Treatiſe of Meaſuring contains, that I have ſeen or believe to be publiſhed, will meet with the Approbation of the Studious in this Geometrical Science.

The Dimenſions I will ſuppoſe taken in Feet and Decimal Parts, as beſt anſwering that Accuracy and Brevity which will be expected from me: But at the Cloſe of this Appendix I ſhall give my Advice what Inſtruments are moſt proper for the ſeveral kinds of Workmen; and alſo by what Denomination the ſeveral kinds of Work is valued.

CHAP. I.

To meaſure any Superficies.

A *Superficies* is properly *defined* to conſiſt only of Length and Breadth or Girt, without taking any Notice of Thickneſs.

SECT. I. *The Geometrical Square and Parallelogram, Rombus and Romboide.*

Propoſition 1. *To meaſure the Geometrical Square.*

Definition. This is a Figure conſiſting of 4 equal Sides, and as many right Angles: as *Fig.* 1. *Plate* C.

Rule. Take the Dimenſion of one Side, as *a b*, and multiply it by itſelf, gives the Content.

Example. The Side of any Square being 11.28, the Content is found 127.238, as *per* Marg.

	11.28
	11.28
	31584
	12408
Anſw. =	127.23384

Prop. 2. *To find the Area of a Parallelogram.*

Defin. This is a Figure conſiſting of or bounded by 2 longer equal parallel Sides, and 2 ſhorter equal and parallel Sides, having 4 right Angles: as *Fig.* 2. *Plate* C.

Rule. Multiply the Length by the Breadth, and the Product is the Area.

Example. The Length *a b* = 23.53, Breadth *b c* = 2.15. See the Work.

	23.53
	2.15
Anſw. =	50.5895

Prop. 3. *To find the Area of a Rombus.*

Defin. This Figure is bounded by 4 equal Sides, and hath 4 Angles, the 2 oppoſite ones being equal, and the Sides parallel: as *Fig* 3. *Plate* C.

Rule. The Diſtance between 2 oppoſite Angles, as *b d*, multiplied by the Line *e c*, gives the Anſwer.

Example. The Diagonal (*b d*) = 1.34; *e c* = .85; the Anſwer is 1.139.

	1.34
	.85
	1.1390

Prop. 4. *To find the Area of the Romboides.*

Defin. This Figure is bounded with 4 right Lines, the oppoſite Sides equal and parallel, and the oppoſite Angles equal: as *Fig.* 4. *Plate* C.

Rule. Multiply the Diagonal *a b* by the Perpendicular *c n* or *m r*, and the Rectangle anſwers.

Example. The Diagonal *a b* = 12.47, the Perpendicular *c n* = 4.127, and the Anſwer is therefore, as *per* Margin, = 51.46369.

	12.47
	4.127
	33669
	51127
Anſw. =	51.46369

SECT.

Sect. II. *To meaſure the Conic Sections.*

PROP. 1. *To meaſure a Triangle.*

Defin. Right-lined Triangles are of 3 kinds (beſides the equilateral, whoſe Sides are equal) *viz.* Right-angled, as (*a r n*) or (*x r n*) right-angled at *r* in *Fig.* 5. 2*dly*, Scalenous Triangles, as (*o r o*) the Sides of which are all unequal. And, 3*dly*, Iſoceles Triangles, as (*a n x*) appears; 2 of whoſe Sides are equal. This laſt is one of the Sections of a Cone, made by cutting the Cone (*e w a g x n a*) down the Axis, as (*o p*) *Fig.* 5. *Plate* C.

Rule. Multiply the Baſe (as *x a*) by half the Perpendicular (as *n r*) or half the Baſe by the Perpendicular, or the Perpendicular by the Baſe; and taking half the Product, will give the Area of any Triangle as above mentioned: for all Triangles are half a Square of the ſame Baſe and Perpendicular.

Example. The Baſe (*a x*) being = 9.35, the Perpendicular *n r* = 12.42, the Area is 58.0635, as *per* Margin.

9.35
6.21
19635
5610
Anſw. = 58.0635

Prop. 2. *To meaſure the Circle.*

Defin. This is formed by cutting the Cone (*x w a g x n a*) parallel to the Baſe (*x w a g x*) as (*m c*) *Fig.* 5. *Plate* C.

Rule. There are ſeveral ways of finding the Area of a Circle. As,

1*ſt*, As 7 to 22, ſo the Diameter (as *r s*, *Fig.* 6.) to the Circumference.

2*dly*, As 1 to 3.1416, ſo the Diameter to the Circumference.

Then multiply the Circumference by a 4th of the Diameter, or a 4th of the Circumference by the Diameter gives the Anſwer.

Or the Area of a Circle is found without the Circumference thus:

As 1 is to .7854 :: ſo is the Square of any Diameter to the Area.

Example. Admit the Diameter (*r s*, *Fig.* 6.) be = 14.32, what is the Area?

By the laſt Method

1. .7854 :: 205.0624. 161.056 = Anſw.

14.32
14.32
45824
20048
□ = 205.0624

Prop. 3. *To meaſure the Ellipſis, or Oval.*

Defin. This is a Section of a Cone, made by cutting it through both its Sides, but not parallel to the Baſe; as the Section (*l e*) *Fig.* 5. *Plate* C. which produceth *Fig.* 7. *Plate* C.

Rule. Multiply the *Latus tranſverſum* (or tranſverſe Ax) *l t*, by the Diameter Conjugate (*c d*) and the Product is = the Square of the Diameter of a Circle equal to the Oval: then work as in the laſt.

Example. Admit *l t* = 15.34, and *c d* = 12.28, the Anſwer is found 147.95 *fere*: for

1. .7854 :: 188.3752. 147.9499

l t = 15.34
c d = 12.28

42952
18408

∠ = 188.3752

Prop. 4. *To find the Area of the Parabola.*

Defin. This Section is formed by cutting the Cone parallel to the oppoſite Side, as in the Line *t s*, *Fig.* 5. which is parallel to the Side (*a n*) and produceth a Parabola; as *Fig.* 8. *Plate* C.

Rule. This Figure being 2 Thirds of a Parallelogram made of the whole Ordinat (*p a*) and the Abſciſſa (*s s*) therefore multiplying thoſe 2 Lines together, 2 Thirds of the Rectangle is the Area.

3.84
2.54

20736
768

Example. Admit *a p* = 3.84, and *s s* = 2.54; 2 Thirds of the Rectangle is = the Anſwer = 6.5024

Rectangle = 9.7536
$\frac{1}{3}$ = 3.2512
$\frac{2}{3}$ *or Anſwer* = 6.5024

Prop. 5. *To find the Area of the Hyperbola (or Hyperbolic Space.)*

Defin. This Section of a Cone is formed by cutting it (not parallel to the Side, as in the Parabola, but) ſo that if the Axis of the Section were continued upward, it would interſect the oppoſite Side of the Cone produced: the Section's Axis is repreſented by the Line *y z*, *Fig.* 5. and produceth an Hyperbola, as *Fig.* 9. *Plate* C.

To meaſure this Figure, there is no general Rule that can be given for Practice: But the Inveſtigation is performed by the help of the Aſymptotes, an Infinite Series, and partly by the Method of Fluxions; which 'tis not proper here to inſert.

SECT. III. *To meaſure any of the Regular Polygons.*

Defin. Theſe are Figures conſiſting of above 4 equal Sides, and as many equal Angles. As,

The Pentagon having 5 equal Sides, and as many Angles.
Hexagon 6
Heptagon 7
Octogon 8

The

The Nonogon having 9 equal Sides, and as many Angles.
Decagon 10
Undecagon 11
Dodecagon 12

Rule. To meaſure any of theſe, draw 2 Lines from any 2 Angles that include a Side to the Center: then find the Content of that Triangle, as taught *Prop.* 1. § 2. above. Laſtly, Multiply the Area of that Triangle by the Number of Sides, and the Product is the Content of the whole Polygon. Or multiply half the Sum of the Sides by the neareſt Diſtance from the Center to one of the Sides, gives the Anſwer.

Example. The Side of the Pentagon (*s d, &c.*) *Fig.* 10. is = 15.34, the Perpendicular (or Radius of a Circle inſcribed) is *rc* = 10.5: So that 15.34 multiplied by 5, the Number of Sides, gives = 76.7 = Sum of the Sides; half of that is = 38.35, which multiplied by *r c* = 10.5, gives the Area = 402.675.

38.35
10.5
Anſwer = 402.675

Sect. IV. *To meaſure the Trapezium, the Parallelogramic, the Polygram, (or Multangle, or irregular Polygon) the Area or Superficies of the Cylinder, Cone, and Sphere.*

PROP. 1. *To meaſure the Trapezium.*

Defin. This is a Figure of 4 Sides and 4 Angles, uncertain whether parallel or equal, but is commonly neither; as *Fig.* 11. *Plate* C.

Rule. Multiply the Diagonal (*a n*) by half the Sum of the Perpendiculars *m p* and *c p*; or the Sum of the Perpendiculars by half the Diagonal, and either of the Products is the Anſwer.

Example. The Diagonal *a n* = 17.42; the Perpendicular *m p* = 4, and *c p* = 2.6 *r*, half the Diagonal is = 8.71, and *m p* more *c p* = 6.6 *r*, the Product of which 2 laſt is 58 06666, *&c.*

8.71
6.6*r*
5226
Anſw. = 58.061

Or half the Product of the Diagonal in the Perpendiculars gives the Anſwer.

Prop. 2. *To find the Area of the Parallelogramic, or Parallelopleuron.*

Defin. This Figure hath 4 Sides, 2 of which oppoſite are parallel, and of 4 Angles the 2 at each end being equal: as *Fig.* 12. *Plate* C.

Rule.

Rule. Multiply half the Sum of the 2 Sides ($v\,r$) and ($s\,t$) by the Length ($t\,w$) gives the Answer. Or a Diagonal ($r\,t$) multiplied by half the Sum of 2 Perpendiculars let fall on that Diagonal from s and v.

Example. The Ends $r\,u = 0.79$; $s\,t = 0.7$; Length $w\,t = .8888$, *&c.* half the Sum of .79 and .7 multiplied in $.8\,r = .662\,r\,1$.

Prop. 3. *To find the Area of the Polygram, (sometimes called an irregular Polygon, or Multiangle.)*

Defin. This is any Geometrical Figure having above 4 Sides, which are uncertain, and generally unequal; as are also its Angles: as *Fig.* 13. *Plate* C.

Rule. First divide your Figure into Trapeziums and Triangles, and then measure the same as under *Prop.* 1. §. 2. and §. 4.

Example. Figure 13 is divided into,

1. The Trapeziums ($u\,t\,m\,q\,v$)
2. . . . . and $q\,m\,p\,r\,q$
3. And the Triangle $q\,r\,a\,q$.

In the 1st, $qt = 18.26$, the $\frac{1}{2} = 9.13$
$ux = 5.17$, more $hm = 3.5 = 8.67$

Sum ru more $st = 1.49$
$\frac{1}{2} = .745$
$.8\,r$
5960
$.662\,r\,1$
61171
7304
Prod. = 79.1571

In 2d Trapez. $q\,p = 15.73$, the half $= 7.865$
$m\,n = 4.5$, more $r\,s = 2.31 = 6.81$
Prod. = 53.56065

In 3d or the $\triangle$, $q\,a = 10.72$; the half $= 5.36$
$r\,o$ = the Perpend. $= 3.27$
Prod. = 17.5272

The Sum = the Area of the whole *Fig.* 13. = 150.24495

Prop. 4. *To find the Area of a Cylinder.*

Defin. This Figure is defined under *Prop.* 1. *Chap.* 2.

Rule. Multiply the Ambit or Circumference of the Cylinder by its Length, and to that Rectangle add the double Area of the Base or End, and the Sum is the whole Area.

Area of the curved part of the Cylinder is therefore = 3.1416
Area of the Base = .7854
Ditto = .7854
Whole Area (or Sum) = 4.7124

Example. Ambit of the Cylinder = 3.1416; Altitude as also the Diameter of the Base = 1.

But there are 2 other ways of doing this, where the Diameter and Altitude are equal. As

As, 2*dly*, Multiply the Area of the Sphere having the ſame Diameter by 3, and divide by 2.

Or, 3*dly*, 6 times the Area of the Baſe or End is the Anſwer. Thus 6 times .7854 is = 4.7124, as above.

Prop. 5. *To find the Convex Superficial Content of a Sphere.*

Defin. This Body is defined where its Solidity is meaſured, *Prop.* 4. *Chap.* 2. and 'tis repreſented *Fig.* 6. *Plate* C.

Rule. The Area is 4 times that of a great Circle; therefore having by the Diameter found the Area of a great Circle of any Sphere, multiply it by 4, and you have the Anſwer.

1. .7854 :: 144.
144

Prod. = 113.0976 *Area Circle.*
4

452.3904 = *Area of the Sphere.*

Example. The Diameter of a Sphere being 12, the Area of the ſame is 452.39, as *per* Margin.

Prop. 6. *To find the Superficies (or Area) of a Pyramid.*

Defin. This is a ſolid Body, having (generally) a Square or ſome regular Polygon for its Baſe, being tapering like the Spire of a Steeple, *&c.*

Rule. Multiply the Circumference of the Baſe (or Sum of the Sides) by half the Altitude, and it gives the Superficial Content.

Example. The Side of the Baſe of a Pyramid (being an Octogon) is = 5.65, that multiplied by 8 = the Number of Sides, and that Product by half the Altitude = 30.7, the laſt Product is the Area ſought.

5.65
8

The Total Sides = 45.20
Half the Altitude = 30.7

Area = 1387.64

Prop. 7. *To find the Area of a Cone.*

Defin. The Cone is a Solid which hath a Circle for its Baſe, from whence it is gradually tapering upward till it terminate in a Point directly over the Center of its Baſe; as *Fig.* 5. *Plate* C.

Rule. As the Radius of the Baſe: Is to the Side of the Cone :: So is the Area of the Baſe to the Area of the Cone.

1. .7854 :: 36
36

28.2744 = *Area of the Baſe.*

Example. Diameter of the Baſe = 6; Side of the Cone = 15: So the Area of the Baſe of the Cone is found = 28.2744; and the Area of the Cone, as by the 2d Analogy = 141.37.

Rad.Baſ. *Side.* *Ar.Baſe* *Ar.Cone*
3. 15 :: 28.2744. 141.37

Or

Or *Note* farther, That the Area of a Cone is = that of a Triangle, whose Perpendicular is the Altitude of the Cone, and the Base = the Circumference of the Cone's Base: For in Example, 1. 3.1416 :: 6. 18.8496 = the Base or Circumference; half of which multiplied by the 15, gives the Answer as before = 141.37.

SECT. V. *To find the Area of the Parts of a Circle; as the Semicircle, Quadrant, Sector, Segment, Lune or Crescent, and the Trochoid or Cycloidal Space.*

PROP. 1. *To measure the Semicircle, Quadrant, and Sectors of a Circle, also the Segment.*

Defin. The Semi or Half-Circle is the Space between (*d m b d*) *Fig.* 14. *Plate* C; the Quadrant or Quarter of the Circle is (*dmcd*) the Sector is *cbrec* or the like, less or more than a Quadrant.

Rule. Multiply 1 fourth of the Ark, or Curve Line, by the Diameter of the Circle, and the Product is the Area required.

Example. The Circumference is = 3.1416, the Diameter of that Circle = 1.

The Ark of the Half-Circle *dcbmd* as *bmd* = 1.5708
of the Quadrant *cmdc* as *mmd* = .7854
of the Sector *cbrec* as *erb* = 1.05

So that 1 fourth of 3.1416 = .7854 × by 1, gives .7854 the Area of the Circle.
1 fourth of 1.5708 = .3927 × by 1, gives .3927 the Area of the Half-Circle.
1 fourth of .7854 = .1963 × by 1, gives .19635 the Area of the Quadrant.
1 fourth of 1.05 = .262r1 × by 1, gives .262r1 the Area of the Sector.

And the Area of the Sector (*c b r e c*) less the Area of the Triangle (*c b n e c*) gives the Area of the Segment (*e n b r e*) *Fig.* 14. *Plate* C.

Prop. 2. *To find the Area of the Lune.*

Defin. This is a Figure like a Crescent or increasing Moon; as (*a r n o a*) *Fig.* 15.

Rule. Find first the Area of the Semicircle (*n c a r n*) and from that deduct that of the Segment (*n c a o n*) and the Remainer is the Area of the Lune.

Example. The Diameter *n a* = 1, and consequently the Ark *arn* = 1.5708, and Area = .3927.

The Area of the Sector *e a o n e* is = .256, from which deducting the Area of the Triangle *n e a n* = .135,

.3927
.121
.2717

the

the reſt is the Area of the Segment = *n o a c n* = .121; which taken from the Area of the Semicircle, there reſteth the Anſwer = .2717.

Prop. 3. *To find the Area of the Trochoid, or Cycloidal Space.*

Defin. This Figure (as 16, *Plate* C.) is generated by the Circle (*l b o r*) which turneth round on the Line *a r c*, the Point at *b* beginning at *a*, and when it touches the Line again at *c*, the Cycloidal Curve *a b c* is finiſhed, and the Space from that to the Line *a r c* contains 3 times the ſaid generating Circle.

Rule. Therefore having found the Area of a Circle, whoſe Diameter is (*r b*) multiply that by 3, and it produceth the Area of the Cycloidal Space.

Example. The Line *r b* = 1; Area of the Circle = .7854, and of the Cycloidal Space = 2.356.

CHAP. II.

The Menſuration of Solids.

SECT. I. *To find the ſolid Content of a Cylinder, Parallelopiped and Priſm.*

PROP. 1. *To meaſure the Cylinder.*

Defin. A Cylinder is a ſolid Body having a Circle for its Baſe, and is of equal Circumference from one end to the other, like a Rolling-Stone for Walks, &c. as the *Figure* 17. *Plate* C.

Rule. Multiply the Area of the Baſe (found as under *Prop.* 2. of *Sect.* 2. *Chap.* 1.) by the Length, and the Product is the Anſwer.

Example. The Diameter of the Baſe, as (*a b*) or (*c d*) = 14.32, the Area of that Circle will be found 161.056; which multiplied by the Length (*b c*) = 14.32, or (*a d*) the ſolid Content, is = 2306.32192, as *per* Margin.

161.056
14.32
5153792
2254784
2306.32192

Prop. 2. *To find the ſolid Content of the Parallelopipedon, and of the Priſm.*

Defin. This Solid is bounded by 4 Parallelograms, equal and parallel, for the Sides; and 2 Squares or Parallelograms, equal and parallel, for the Ends: as *Fig.* 18. *Plate* C.

Rule. Multiply the Area of the Base or End by the Length, and the Rectangle is the Answer.

Example. The End is a Square whose Side is 3.72, and the Length = 31.44. See the Operation, where (according to the Method of working *Case* 5. of *Sect.* 5. of *Chap.* 3.) the Solidity is found 435.0793.

Area at the end 138384
Length inverted 44.13

415152.
138384
55354
5535

3.72
3.72

26784
1116

13.8384 = *Area*

435.0793 = *Solid Content.*

Note, That a *Prism* is measured also by multiplying the Area of the Base (which is either a Triangle or a Polygon) by the Length of the Prism. This or the last Figure are three times a Pyramid of the same Base and Altitude.

Prop. 3. *To find the solid Content of the Cone: And, 2dly, of a Pyramid.*

Defin. This Figure hath a Circle for its Base, and is defined *Prop.* 7. *Sect.* 4. of *Chap.* 1. of this Appendix.

Rule. Multiply the Area of the Base by one third of the perpendicular Altitude of the Cone, and that Rectangle is the Answer.

But Note, That to find the Height (it being a solid Body) must be done by this Rule, as *Euclid* 47. 1.

From the Square of the slant Height (*m l*, *Fig.* 19.) deduct the Square of the Semidiameter of the Base, as (*m a*) and the Square Root of the Remainer is the perpendicular Altitude (*x l*).

Example. The Diameter *m d* is = 6, the slant Height *m l* = 5; and consequently the true Altitude (*a l*) = 4: for 5 squared less $\frac{6}{2}$ = *m a* squared, = 16 = the Square of *a l*; that is, 5 squared less 3 squared, is = 16; whose Root is 4 = *a l*. So that as 1. .7854 :: 36. 28.274 = the Area of the Base: Which multiplied by 1 third of 4 = 1.3r, the Product is = 37.6986r1.

28.274
1.3r1

84822

94246
28274

37.6986r1 = *Answ.*

To measure the Pyramid, you must also multiply the Area of the Base by a Third of the perpendicular Altitude: for as a Cone is 1 Third of a Cylinder of the same Base and Altitude, so is a Pyramid 1 Third of a Prism or Parallelopiped. 'Tis defined *Prop.* 6. *Sect.* 4. *Chap.* 1. of this Appendix.

Prop.* 4. *To find the solid Content of a Sphere.

Defin.

Defin. This is a solid Body, every way perfectly round, so that all Lines drawn from the Convex Area to the Center are equal. It is 2 Thirds of a Cylinder of the same Diameter of the Base and Altitude with the Diameter of the Sphere, or of a Cylinder circumscribed.

Rule. Find the Area of a great Circle, as *Prop.* 5. of *Sect.* 4. *Chap.* 1. which multiply by the Diameter; and 2 Thirds of the Rectangle is the solid Content required.

Example. The Diameter of a Sphere being 12, the Area of a great Circle (as *Fig.*6. *Plate* C.) is = 113.0976; which multiplied by the Diameter (or Length of a Cylinder) 12, the Rectangle is = 1357.171, the Solidity of the Cylinder; 2 Thirds of which is the solid Content of the Sphere = 904.7808.

Sect. II. *To find the Solidity of the five Platonic, or Regular Bodys,* viz. *the Tetrahedron, Hexahedron, Octahedron, Dodecahedron, and Icosihedron.*

PROP. 1. *To measure the Tetrahedron.*

Defin. This is a solid Body bounded with 4 equal equilateral Triangles; and being measured as a Pyramid, it needs no other Rule than as under *Prop.* 3. of *Sect.* 1. *Chap.* 2. See *Fig.* 20. *Plate* C.

Prop. 2. *To find the Solidity of the Hexahedron or Cube.*

Defin. This Solid is bounded with six equal Geometrical Squares.

Rule. Multiply a Side in itself, and that Rectangle by the Side gives the Answer. See *Fig.* 21. *Plate* C.

Prop. 3. *To find the Solid Content of the Octahedron.*

Defin. This Figure is bounded with 8 equal equilateral Triangles, which are the Bases of as many Pyramids, which meet in the Center of a Sphere, (out of which this Body is cut) so that it being 8 Pyramids. (See *Fig.* 22. *Plate* C.)

Rule. First find the Area of one of the Triangles, then take with Callipers, *&c.* the Distance between the Center of that Triangle and that of the opposite; by one sixth part of which multiply the Area of any one Triangle; and that Product multiply'd by 8, this last Rectangle gives the Solidity required.

Prop. 4. *To find the solid Content of the Dodecahedron.*

Defin. This is a Solid bounded by 12 equal Pentagons, which are the Bases of so many Pyramids that meet in the Center of this Solid, or of a Sphere out of which this Body is cut. See *Fig.*23. *Plate* C.

Rule. Therefore take with a Pair of Callipers the Distance from the Center of one Pentagon to that of its opposite, and by 1 sixth of

that multiplying the Area of any of the Pentagons; 12 times that Rectangle gives the Solidity. *Or twice the Distance of the opposite Pentagons multiplied by the Area of* 1 *Pentagon*, gives the Answer.

Prop. 5. *To find the Solidity of the Icosihedron.*

Defin. This is the last of the 5 Platonic or Regular Bodies: it is bounded with 20 equal equilateral Triangles, which are Bases to so many Pyramids, whose small Ends terminate in a Point in the Center of a Sphere which circumscribeth this Solid, and out of which it is cut. (See *Fig.* 24. *Plate* C.)

Rule. Multiply the Area of one Triangle by 1 sixth of the Distance of the Centers of 2 opposite Triangles (taken with Callipers) and that Rectangle multiplied by 20 (the Number of Pyramids in the Body) gives the Solidity sought.

I have not thought it necessary to give Examples to these 5 Rules, because there is not much more therein besides the repeated Mensuration of a Pyramid, which is taught above. But I have chiefly inserted them for the sake of the Definitions, which I do not remember to be in any Tract of Measuring, no more than many other things contained in other Propositions of this Appendix.

Sect. III. *To find the solid Content of the Parabolic Conoid, the Parabolic Spindle, the Cylindroid, the Sphereoid, and the Hyperbolic Conoid.*

Prop. 1. *To find the solid Content of the Parabolic Conoid.*

Defin. This Solid is generated by the Rotation of a Semi-Parabola, as (*p s s*, *Fig.* 8. *Plate* C.) round its Axis *s s*; and is = half of the circumscribing Cylinder *p q s x a s p*, (or 1 and half of the Cone inscribed.) Therefore

Rule. Multiply the Area of the Base by the Axis or greatest Abscis, and half the Rectangle is the Answer.

Example. The Diameters of the Figure (as *a p*) = 14.32, the Area of the Base will be found 161.056; which multiplied by the Abscis (*s s*) = 14.32, the Rectangle is = 2306.32192; the half of which is = 1153.16096 the Answer.

Prop. 2. *To find the Solidity of the Parabolic Spindle.*

Defin. This Figure is generated by the Rotation of the Semi-parabola round its Ordinat, and is $\frac{8}{15}$ of the circumscribing Cylinder. Therefore

Rule. Multiply the Content found as a Cylinder by 8, and divide by 15, the Quote is the Answer, which needs no Example.

Prop. 3. *To find the solid Content of the Cylindroid.*

Defin.

Defin. This Figure only differs from that under *Prop.* 1. *Sect.* 1. of *Chap.* 2. inaſmuch as that hath a Circle, this hath an Ellipſis for its Baſe: ſo that to meaſure it,

Rule. Multiply the Baſe found as *per Prop.* 3. *Sect.* 2. *Chap.* 1. by the Length, and that Product is the Anſwer.

Example. Admit the *Latus tranſverſum* (as *l t*, *Fig.* 7. *Plate* C.) by *c d* = the Diameter Conjugate, (*i. e.* ſuppoſe *l t* = 15.34, and *c d* = 12.28) the Area is = 147.95, which multiply'd by the Length = 1.881, the Anſwer is = 279.45.

Prop. 4. *To find the ſolid Content of the Sphereoid.*

Defin. This Figure is generated by the Rotation of the Semi-Ellipſis round its Axis; or *Latus tranſverſum*; and 'tis by ſome called the Prolate Sphereoid, to diſtinguiſh it from that which is generated by the Rotation of a Semi-Ellipſis, which turns round its Diameter Conjugate, and is called an Oblate Sphereoid, (of which Figure, 'tis aſſerted by the moſt Learned, our Earth is, the Diameter at the Equator being greater than between the Poles.) This Prolate Sphereoid is 2 Thirds of a Cylinder, whoſe Baſe's Diameter is = the greateſt Diameter of the Sphereoid, and its Altitude = the *Latus tranſverſum* of the Ellipſis.

Rule. Multiply the Area of a Circle, whoſe Baſe is the Conjugate Diameter, (or here the greateſt Diameter of the Sphereoid) by the Length, and 2 Thirds of the Product is the Content of the Sphereoid.

Example. The Diameter in the middle = 14.32, as *c d*, *Fig.* 7. the Area of that Circle 161.056, which multiply'd in the Length *l t* = 14, gives 200.48; 2 Thirds of which = 133.65, the Anſwer.

Prop. 5. *To find the Solidity of the Hyperbolic Conoid.*

Defin. This Figure is generated by the Rotation of the Semi-Hyperbola (*z h r i*) *Fig.* 9. round the Abſciſſa *z i*; which formeth the Hyperbolic Conoid *h r i n z h*.

Rule. The *Latus tranſverſum* = *i o*, multiply'd by 6; more the Axis, or the Abſciſſa *i z* multiply'd by 6 is = the Diviſor.

2*dly*, Multiply (*h a i s n z h*) = the Content of the Cylinder (whoſe Diameter is equal to the whole Ordinat *h n*, and its Length = the Abſciſſa *z i*) in 3 times the *Latus tranſverſum*, more 2 times the Abſciſſa, and that Rectangle is = a Dividend; which divided, there ariſeth the Quotient, which is the Solidity required.

Example. The whole Ordinat (or Diameter) *h n* = 3.51, the Abſciſs or Axis *i z* = *h a* = 2.43, and the *Latus tranſverſum i o* = 2.16.

The

The Content of the Cylinder *basnz h*, is therefore = 23.513
The *Latus tranſverſum* mult. in 3 = 6.48
The Axis or Abſciſſa mult. in 2 = 4.82
Sum = 11.3 } Mult.
The *Latus tranſverſum* mult. in 6 = 12.96
The Axis multiplied in 6 is ... = 14.46
= Sum or Diviſor = 27.42
265.6969 = *Product*
18916
24649
9.69 = *the Anſw.*

SECT. IV. *To find the Solidity of the Fruſtums of a Cone, Sphere, and Sphereoid.*

PROP. 1. *To find the ſolid Content of the Fruſtum of a Cone or Pyramid.*

Defin. This may be done ſeveral ways: as to find the Content of that part of the Cone (*drrmad*) *Fig.* 19. *Plate* C.

Rule. 1. Find the Content of the whole Cone (*dlmd*) then deducting the Content of the upper part (*rlr*) the reſt is the Content of the Fruſtum (*drrmd.*)

All the difficulty (more than as *Prop.* 3. *Sect.* 1. of this *Chap.*) is to find the Height (*al*) which is done by this Rule, That the homogeneal Sides of ſimilar Triangles are in proportion: ſo that as *mn. mr :: ma. ml*; whence a Perpendicular let fall to *a*, is the true Altitude of the Cone.

Rule 2. *But more general for a Cone, Pyramid,* &c. without the upper part: Multiply the Areas of the greater and leſſer Ends together, extract the Square Root of the Product; then multiply the Sum of the ſaid Root, and 2 Areas of the Ends by one third of the Altitude of the Fruſtum, and the Product is the Content.

Example. In *Fig.* 19. *Plate* C. the Area of the greater Baſe (*modxm*) is = 36.72; of the leſſer End (as *rsrsr*) = 15.5; the Product of which is = 569.16, whoſe Square Root is 23.857; which added to the 2 Areas, gives = 76.077; which multiply'd by 1 third of the Altitude *nr* = 6.6*r*, the Product is the ſolid Content of the Fruſtum = 507.17971.

Prop. 2. *To find the ſolid Content of the Fruſtum of a Sphere or Globe.*

Defin. By a Segment is meant a part of the Sphere cut off parallel to the Diameter by a right Line: as *bnerb*, *Fig.* 14. *Plate* C.

Rule. Find the Area of the Segment's Baſe, which multiply by the Altitude, reſerving the Product. Then

To half the Altitude of the greater Fruſtum add 1 ſixth of the Altitude of the leſſer, and multiply the Sum by the reſerv'd Rectangle, and divide this laſt by the greater Fruſtum's Altitude, and the Quote is the leſſer Fruſtum's Solidity required.

Example.

Example. Diameter of the leſſer Fruſtum = 6 = *b e*, Area = 28.2744
Its Altitude = *n s* . . . . . = 5

Altit. of the greater Fruſtum = *n o* = 7 | Reſerv'd Rectang. = 14.3720
Half that Altitude is . . . . . = 3.5
1 Sixth of the Altit. of the leſſer = .83 r 1 } Sum multipl. = 4.3r1
Product = 612.6119r1

1 Seventh of which laſt Product is = 87.516 = the Solidity of the Fruſtum required.

Note, That if you divide the Square of the Diameter of the Fruſtum's Baſe by the Altitude of either Fruſtum, the Quote is the other Fruſtum's Altitude.

Prop. 3. *To find the ſolid Content of the middle Fruſtum of a Sphereoid.*

Defin. This is repreſented by *r a i i c r*, *Fig.*7. like a Cask.

Rule. The eaſieſt Rule, and what is near the Truth, is this: Multiply the Difference of the Bung and Head Diameters by .7, and add the Product to the Head Diameter; to that Sum find the Area of a Circle, and multiply it by the Length of the Cask, and that Product is the Anſwer.

Example. The Bung Diameter 20 = *c d*, Head 15 = *r a* or *i i*, Length *o o* = 12. The Difference between the Bung and Head = 5, which multiply'd by .7 is 3.5; which added to the Head 15, gives 18.5 for a mean Diameter, whoſe Area is = 268.803; which multiply'd by the Length = 12, the Solidity is = 3225.638 = the Content of the ſaid Fruſtum (*r a d i o i r.*)

Note, That if the Sphereoid be cut in the middle of (*t m*) by a Plain (*r a*) that part (*r s t a o r*) is 5 Ninths of a Cylinder, whoſe Baſe is (*r o a*) and the Altitude (*o t.*)

I believe I have, in the Pages preceding of this Appendix, ſhewn the beſt Ways of Meaſuring a greater Variety of Superficies and Solids than any Book wrote purely on that Subject contains: and to cloſe this Chapter of Solid Meaſure, I may farther add, There is no Body, tho never ſo irregular, but what may have its Content diſcover'd by the help of Water. For example, I would know the ſolid Inches in a Faggot.

Firſt, I put Water into a Veſſel (ſuppoſe 36 Inch by 20, and is a Parallelopipedon; and taking the Depth, I find it 30 Inches deep) into which I immerſe the Faggot, ſo as to cover it with Water; and taking the Depth again, I find it 33 Inches deep, or 3 more than before the Faggot was in: ſo the ſaid 36 by 20 = 720 multiply'd in this 3, produceth 2160, the Inches ſolid contained in the Faggot.

Brief-

Brief and Useful Observations on the most Proper INSTRUMENTS *to be used in the Mensuration of several things, &c.*

Bricklayers Work is best measured by the Foot and 100 Parts: for the Divisor being 272 and a quarter, (which is the Feet in one Rod, or the Square of 16 Foot and a half) to reduce Feet into Rods, by which Denomination they value their Work; it is necessary that the Superficies of the Walls being the Dividends, should be Feet and Decimal Parts, as the Divisor is ——— = 272.25 Feet.

Or without the Rod, the shortest Way is to multiply any Number of Feet by the Value of 1 Foot at the Rate agreed on *per* Rod, as at *l.*5 : 10 : —, the Foot is in Value *l.* .020202, &c.

	.02
	5.4450
	5.49991

Here the Multiplier being .020202, &c. or .02r2, I only multiply the Feet on the flat Wall by 2, and add every other Figure of the Product till there are but 2 : which are the Integers; and the Fractions to the right hand.

Example 2. What is the Value of 1361.25 Foot, Brick and half thick, at *l.* .0202020202, &c. or .02r2 *l. per* Foot, or *l.*5 : 10 *per* Rod. See the Operation.

Foot 1361.25
Multiply .... = .02r

As *per* Rule, add every other of = 27.2250
Sum or Answer = 27.4999, &c. or *l.* 27.5

Carpenters Work. The common two Foot serves as well as any other Instrument to measure Flooring, Partitioning, and Roofing; they being each valued by the Square of 100 Foot : so that Feet multiply'd by Feet, and 2 Figures cut off towards the right hand, gives the Squares towards the left hand.

Painters and Plasterers giving in and valuing their Work by the Yard, I think the best Instrument for them to take their Dimensions with, to be a Yard divided into 100 equal Parts.

Glaziers Work and *Glass* being valued by the Foot, and Crown-Glass being to be done accurately, ought to be measured by the Foot, and Decimal, or 100 Parts of a Foot.

Joiners Work (as Wainscot, especially Oak) being valued by the Yard, ought to be measured by the Foot divided into 100 Parts, (as being of considerable Value) and the Sum of the Feet divided by 9, (the Square Feet in a Yard.)

Land. The best Instrument to measure it with, is a 4 Pole Chain, divided into 100 Parts or Links : for 40 single Pole long and 4 broad making 160 square Perches or 1 Acre; if the Chain be 4 Pole long, then 10 long and 1 broad make an Acre. So that whatever the Dimensions be in square Chains and Links (or hundred Parts) instead of dividing them by 160, if they were measured by a 1 Pole Chain, here you have nothing to do but to cut off 5 places from any Product to give the Acres. As suppose a Field be 40 Chains and 30 Links long, and the Breadth 30.40 Chains; the Content, by dividing the Product by 10, (or cutting off 1 place besides the 4 which are Decimals) is 122.512 Acres

Chains.
40.30
30.40
122.51200 *Acres.*

FINIS.

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